

Optics and Modern Physics

CHAPTER 7

Nuclear Physics

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Nuclear Physics

NUCLEAR STRUCTURE

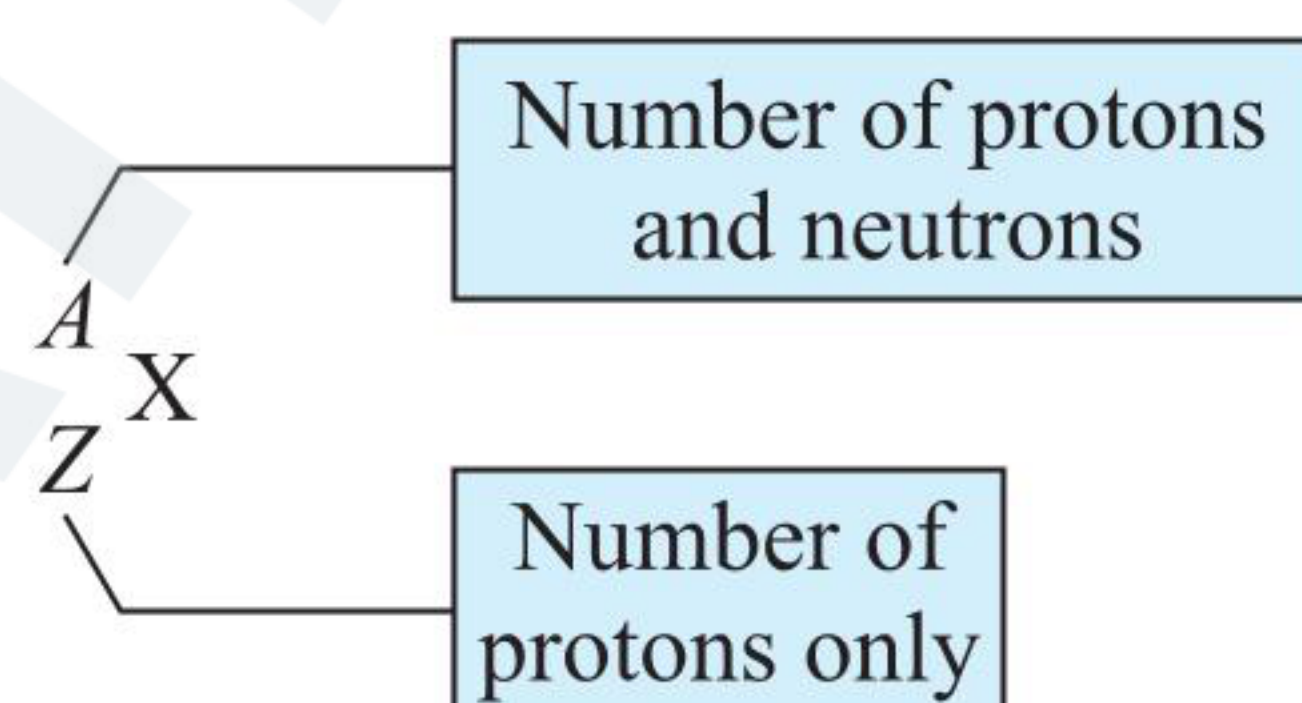
Nuclear physics is the branch of physics which deals with the study of nucleus. Atoms consist of electrons in orbit about a central nucleus. The electron orbits are quantum mechanical in nature and have interesting characteristics. Little has been said about the nucleus, however. Since the nucleus is interesting in its own right, we now consider it in greater detail.

The nucleus of an atom consists of neutrons and protons, collectively referred to as nucleons. The neutron, discovered in 1932 by the English physicist James Chadwick (1891–1974), carries no electrical charge and has a mass slightly larger than that of a proton.

The number of protons in the nucleus is different in different elements and is given by the atomic number Z . In an electrically neutral atom, the number of nuclear protons equals the number of electrons in orbit around the nucleus. The number of neutrons in the nucleus is N . The total number of protons and neutrons is referred to as the atomic mass number A because the total nuclear mass is approximately equal to A times the mass of a single nucleon:

$$\begin{array}{l} A \\ \text{[Number of proton} \\ \text{and neutrons} \\ \text{(atomic mass} \\ \text{number or nucleon} \\ \text{number)]} \end{array} = \begin{array}{l} Z \\ \text{[Number of} \\ \text{protons (atomic} \\ \text{number)]} \end{array} + \begin{array}{l} N \\ \text{[Number of} \\ \text{neutrons]} \end{array} \quad \dots(i)$$

Sometimes A is also called the nucleon number. A shorthand notation is often used to specify Z and A along with the chemical symbol for the element. For instance, the nuclei of all naturally occurring aluminum atoms have $A = 27$, and the atomic number for aluminum is $Z = 13$. In shorthand notation, then, the aluminium nucleus is specified as ${}^{27}_{13}\text{Al}$. The number of neutrons in an aluminium nucleus is $N = A - Z = 14$. In general, for an element whose chemical symbol is X , the symbol for the nucleus is shown in figure.



For a proton the symbol is ${}^1_1\text{H}$, since the proton is the nucleus of a hydrogen atom. A neutron is denoted by ${}^1_0\text{n}$. In the case of an electron we use ${}^0_{-1}\text{e}$, where $A = 0$ because an electron is not composed of protons or neutrons and $Z = -1$ because the electron has a negative charge.

The number of protons in the nucleus characterizes the species of the atom—the element to which the atom belongs. Electrons can be added to or removed from an atom to form an

ion, but this does not change its species. For example, removing two electrons from calcium results in calcium ion Ca^{2+} , but its species is still calcium. The number of electrons can vary by ionization, but the proton number cannot vary without changing an atom of a different element, the chemical symbol implies the proton number.

Each nuclear species with a given Z and A is called a nuclide. Each Z characterizes a chemical element. Nucleus mass is roughly the sum of its constituent proton and neutron masses. The nuclear charge is Z (atomic number) times the charge of a proton ($e = 1.6 \times 10^{-19} \text{ C}$).

The chemical properties of an atom are determined by its electron configuration. The number of electrons and protons are equal in number in a neutral atom; the chemical properties are essentially determined by Z . The dependence of the chemical properties on N is negligible.

ILLUSTRATION 7.1

How many electrons, protons, and neutrons are there in 12 g of ${}^6_6\text{C}^{12}$ and in 14 g of ${}^6_6\text{C}^{14}$?

Sol. Mass number (or atomic weight) of ${}^6_6\text{C}^{12}$ is 12.

Therefore, the number of atoms in 12 g of ${}^6_6\text{C}^{12}$ is

$$\text{Avogadro number} = 6 \times 10^{23}$$

The number of electrons in 12 g of ${}^6_6\text{C}^{12}$ is

$$6 \times 6 \times 10^{23} = 36 \times 10^{23}$$

The number of protons in 12 g of ${}^6_6\text{C}^{12}$ is 36×10^{23} .

The number of neutrons in 12 g of ${}^6_6\text{C}^{12}$ is

$$(A - Z) \times 6 \times 10^{23} = (12 - 6) \times 6 \times 10^{23} = 36 \times 10^{23}$$

Similarly, number of electrons in 14 g of ${}^6_6\text{C}^{14}$ is 36×10^{23} .

Number of protons in 14 g of ${}^6_6\text{C}^{14}$ is 36×10^{23} .

$$\begin{aligned} \text{Number of neutrons in 14 g of } {}^6_6\text{C}^{14} &= (A - Z) \times 6 \times 10^{23} \\ &= (14 - 6) \times 6 \times 10^{23} \\ &= 48 \times 10^{23} \end{aligned}$$

ATOMIC MASS NUMBER

It is the nearest integer value of mass represented in amu (atomic mass unit). Atomic masses refer to the masses of neutral atoms. Thus, an atomic mass always includes the mass of its Z electrons. The atomic mass of an atom is measured relative to the mass of an atom of the neutral carbon-12 isotope (the nucleus plus six electrons). Atomic masses are measured in atomic mass units, which are denoted by the symbol ‘u’. Atomic mass units are defined in terms of the mass of the isotope ${}^{12}\text{C}$, whose atomic mass is defined to be exactly 12 u.

$$1 \text{ amu} = \frac{1}{12} \times [\text{mass of one atom of } {}_6\text{C}^{12} \text{ atom at rest and in ground state}]$$

$$= 1.6603 \times 10^{-27} \text{ kg}$$

Properties of particles in the atom

Particle	Electric charge (C)	Mass	
		Kilogram (kg)	Atomic mass units (u)
Electron	-1.60×10^{-19}	9.109390×10^{-31}	5.485799×10^{-4}
Proton	$+1.60 \times 10^{-19}$	1.672623×10^{-27}	1.007276
Neutron	0	1.674929×10^{-27}	1.008665

SOME DEFINITIONS

Isotopes: The nuclei having the same number of protons but different number of neutrons are called isotopes.

Isotopes of carbon: ${}^{11}_6\text{C}$, ${}^{12}_6\text{C}$, ${}^{13}_6\text{C}$, ${}^{14}_6\text{C}$

Isotopes of hydrogen: ${}^1_1\text{H}$, ${}^2_1\text{H}$, ${}^3_1\text{H}$

The isotopes ${}^1_1\text{H}$ is called ordinary hydrogen or simply hydrogen; ${}^2_1\text{H}$ is called deuterium. Deuterium, sometimes known as heavy hydrogen, can combine with oxygen to form heavy water (D_2O). The third isotope of hydrogen, ${}^3_1\text{H}$, is called tritium, which is unstable.

Isotones: Nuclei with the same neutron number N but different atomic number Z are called isotones.



Isobars: The nuclei with the same mass number but different atomic number are called isobars.



SIZE OF NUCLEI

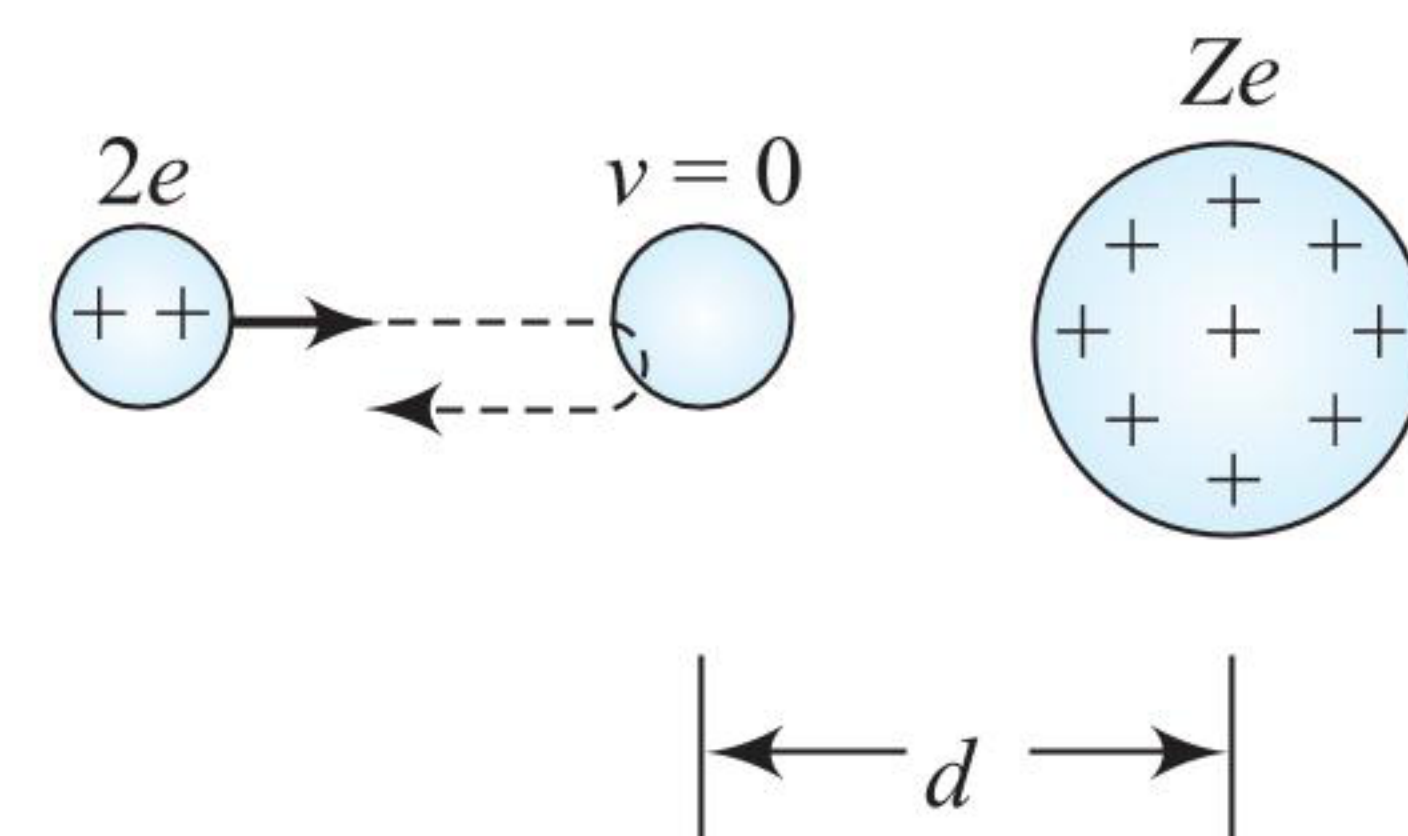
The size and structure of nuclei were first investigated in the scattering experiments of Rutherford. Using the principle of conservation of energy, Rutherford found an expression for how close an alpha particle moving directly toward the nucleus can come to the nucleus before being turned around by Coulomb repulsion.

In such a head-on collision, the kinetic energy of the incoming alpha particle must be converted completely to electrical potential energy when the particle stops at the point of closest approach and turns around (refer figure). If we equate the initial kinetic energy of the alpha particle to the maximum electrical potential energy of the system (alpha particle + target nucleus), we have

$$\frac{1}{2}mv^2 = k_e \frac{q_1 q_2}{r} = k_e \frac{(2e)(Ze)}{d}$$

where d is the distance of closest approach. Solving for d , we get

$$d = \frac{4k_e Ze^2}{mv^2}$$



An alpha particle on a head-on collision with a nucleus of charge Ze . Because of the Coulomb repulsion between charges of the same sign, the alpha particle approaches to a distance d from the target nucleus called the distance of closest approach

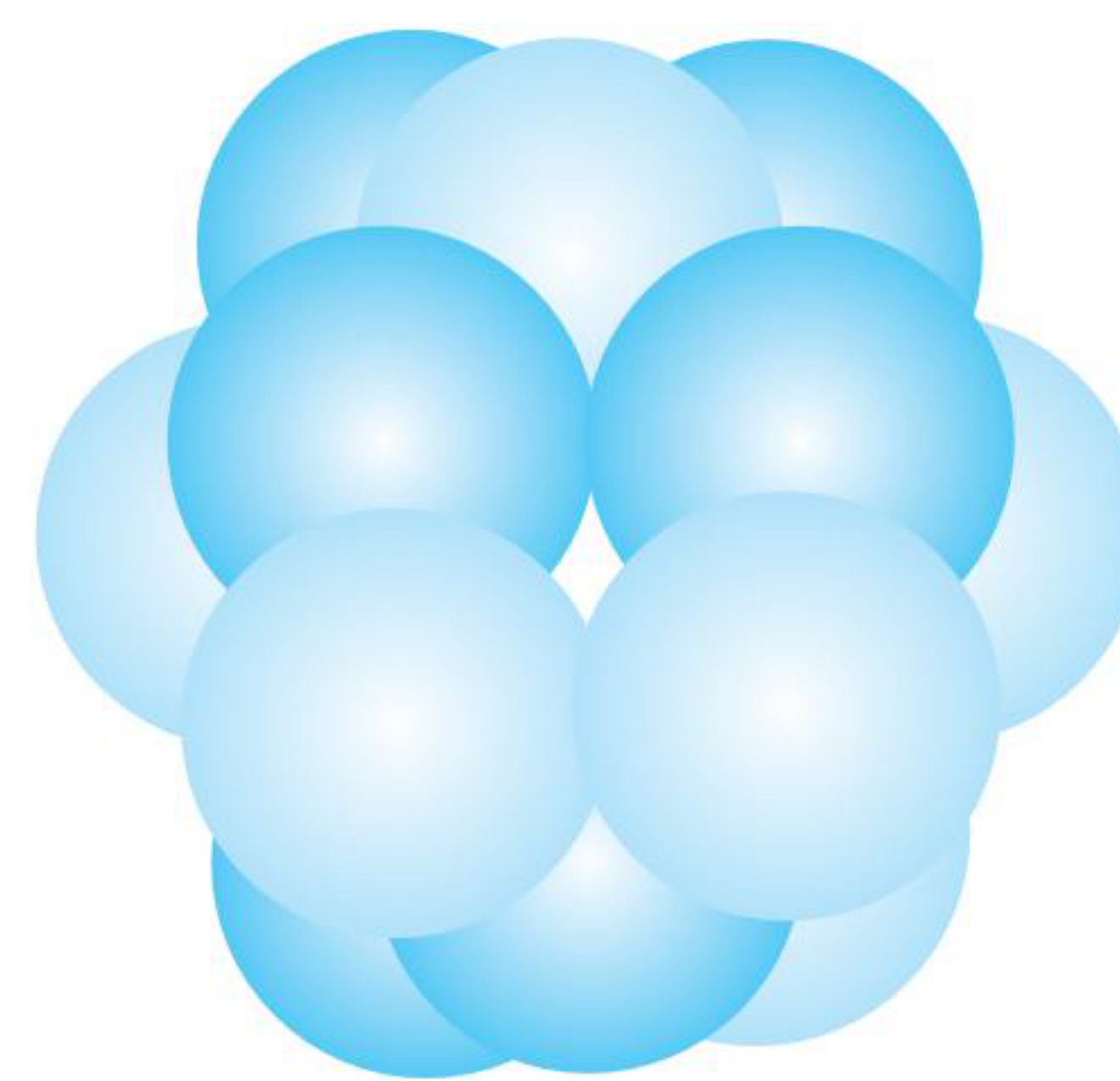
From this expression, Rutherford found that alpha particles approached to within $3.2 \times 10^{-14} \text{ m}$ of a nucleus when the foil was made of gold. Thus, the radius of the gold nucleus must be less than this value. For silver atoms, the distance of closest approach was $2 \times 10^{-14} \text{ m}$. From these results, Rutherford concluded that the positive charge in an atom is concentrated in a small sphere, which he called the nucleus, whose radius is no greater than about 10^{-14} m . Because such small lengths are common in nuclear physics, a convenient unit of length is the femtometer (fm), sometimes called the Fermi, defined as

$$1 \text{ fm} = 10^{-15} \text{ m}$$

Since the time of Rutherford's scattering experiments, a couple of other experiments have shown that most nuclei are approximately spherical and have an average radius given by

$$r = r_0 A^{1/3} \quad \dots(i)$$

where A is the total number of nucleons and r_0 is a constant equal to $1.2 \times 10^{-15} \text{ m}$. Because the volume of a sphere is proportional to the cube of its radius, it follows from Eq. (i) that the volume of a nucleus (assumed to be spherical) is directly proportional to A , the total number of nucleons. This suggests that all nuclei have nearly the same density. Nucleons combine to form a nucleus as though they were tightly packed spheres (see figure).



A nucleus can be modelled as a cluster of tightly packed spheres, each of which is a nucleon

ILLUSTRATION 7.2

- Find an approximate expression for the mass of a nucleus of mass number A .
- Find an expression for the volume of this nucleus in terms of the mass number.
- Find a numerical value for its density.

Sol.

- The mass of the proton is approximately equal to that of neutron. Thus, if the mass of one of these particles is m , the mass of the nucleus is approximately Am .

- (b) Assuming the nucleus is spherical and using Eq. (i), we find that the volume is

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi r_0^3 A$$

- (c) The nuclear density can be found as follows:

$$\rho_n = \frac{\text{mass}}{\text{volume}} = \frac{Am}{\frac{4}{3}\pi r_0^3 A} = \frac{3m}{4\pi r_0^3}$$

The fact that A cancels out of our expression for nuclear density proves our statement above that all nuclei have roughly the same density. Taking $r_0 = 1.2 \times 10^{-15}$ m and $m = 1.67 \times 10^{-27}$ kg, we find that

$$\rho_n = \frac{3(1.67 \times 10^{-27} \text{ kg})}{4\pi(1.2 \times 10^{-15} \text{ m})^3} = 2.3 \times 10^{17} \text{ kg m}^{-3}$$

Note that the nuclear density is about 2.3×10^{14} times greater than the density of water ($1 \times 10^3 \text{ kg m}^{-3}$).

ILLUSTRATION 7.3

Calculate the radius of ^{70}Ge .

Sol. We have,

$$R = R_0 A^{1/3} = (1.2 \text{ fm}) (70)^{1/3} = (1.2 \text{ fm}) (4.12) = 4.94 \text{ fm}$$

ILLUSTRATION 7.4

The most common kind of iron nucleus has a mass number of 56. Find the radius, approximate mass, and approximate density of the nucleus.

Sol. We use two key ideas. The radius and mass of a nucleus depend on the mass number A and density is mass divided by volume.

We use Eq. (i) to determine the radius of the nucleus. The mass of the nucleus in atomic mass units is approximately equal to the mass number. The radius is

$$R = R_0 A^{1/3} = (1.2 \times 10^{-15} \text{ m})(56)^{1/3} = 4.6 \times 10^{-15} \text{ m} = 4.6 \text{ fm}$$

Since $A = 56$, the mass of the nucleus is approximately 56 u, or $m \approx (56)(1.66 \times 10^{-27} \text{ kg}) = 9.3 \times 10^{-26} \text{ kg}$. The volume is

$$V = \frac{4}{3}\pi R^3 = \frac{4}{3} \times 3.14 \times (4.6 \times 10^{-15})^3 \approx 4.1 \times 10^{-43} \text{ m}^3$$

And the density ρ is approximately

$$\rho = \frac{m}{V} \approx \frac{9.3 \times 10^{-26} \text{ kg}}{4.1 \times 10^{-43} \text{ m}^3} = 2.3 \times 10^{17} \text{ kg m}^{-3}$$

The density of solid iron is about 700 kg m^{-3} , so we see that the iron nucleus is more than 10^{13} times as dense as the bulk material. Densities of this magnitude are also found in neutron stars, which are similar to gigantic nuclei made almost entirely of neutrons. A 1 cm cube of material with this density would have a mass of $2.3 \times 10^{11} \text{ kg}$ or 230 million metric tons.

ILLUSTRATION 7.5

Calculate the electric potential energy of interaction due to the electric repulsion between two nuclei of ^{12}C when they “touch” each other at the surface.

Sol. The radius of a ^{12}C nucleus is

$$R = R_0 A^{1/3} = (1.2 \text{ fm}) (12)^{1/3} = 2.75 \text{ fm}$$

The separation between the centers of the nuclei is $2R = 5.50 \text{ fm}$.

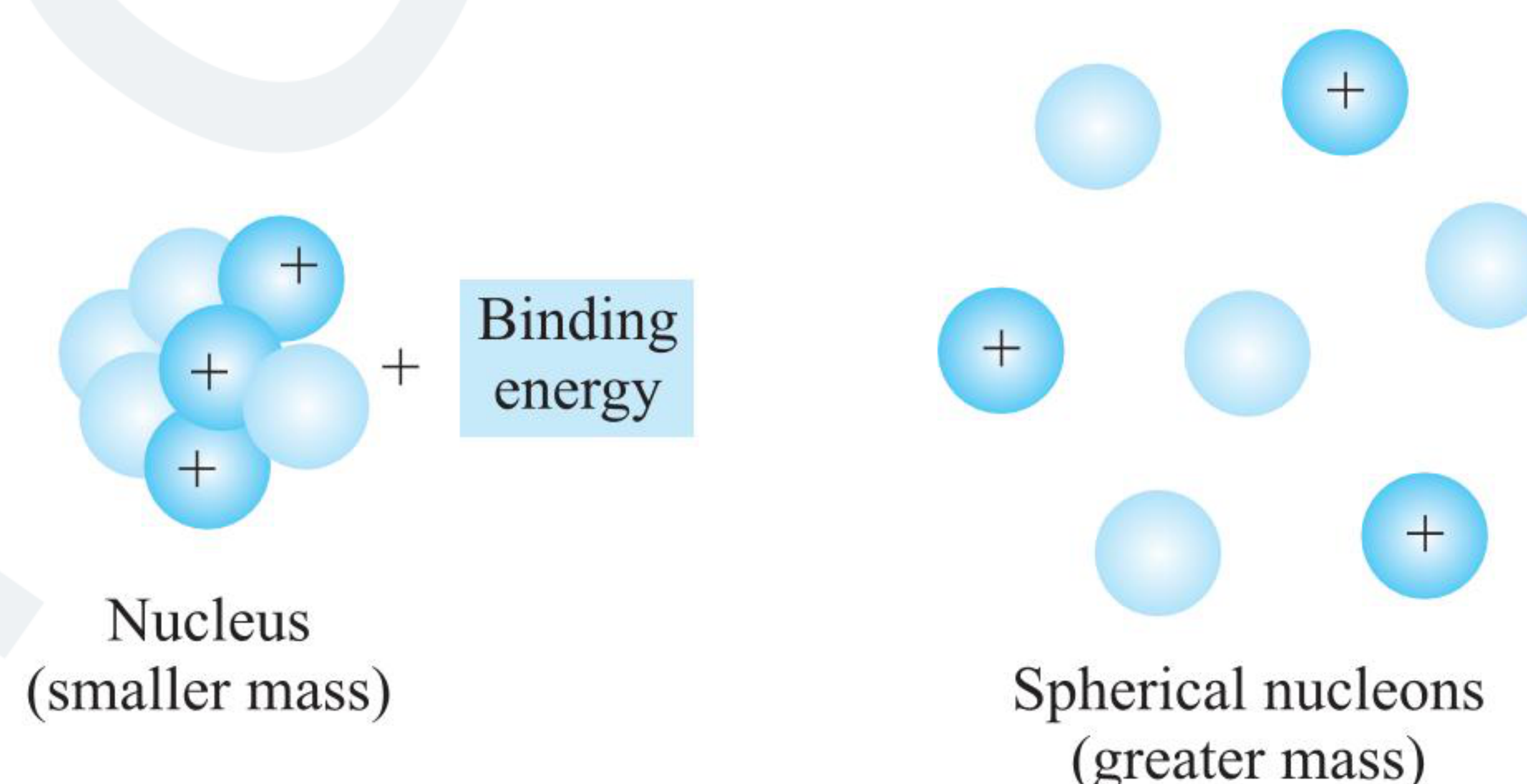
The potential energy of the pair is

$$U = \frac{q_1 q_2}{4\pi\epsilon_0 r} = (9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}) \frac{(6 \times 1.6 \times 10^{-19} \text{ C})^2}{5.50 \times 10^{-15} \text{ m}} \\ = 1.50 \times 10^{-12} \text{ J} = 9.39 \text{ MeV}$$

NUCLEAR BINDING ENERGY

We know that in a stable nucleus, because of strong nuclear force of attraction, the nucleons are held tightly together in a small volume. As the system is stable, we can relatively say that the total potential energy of the system is negative and to separate all the nucleons from each other, some energy must be supplied to break the nucleus. The more stable the nucleus is, the greater is the energy needed to break it apart. This required energy is called binding energy of the nucleus.

The mass of each element atom is less than the sum of masses of its constituent particles.



For example, let us discuss the ^4_2He atom. We compare the mass of atom to that of its constituents. To calculate the mass of the components, we can either add the masses of two protons, two neutrons, and two electrons or we can add the masses of two H atoms and two neutrons. Using the values from table, we have

$$2m_{\text{H}} + 2m_{\text{N}} = 2(1.007825) + 2(1.008665) = 4.0329804 \dots \text{(i)}$$

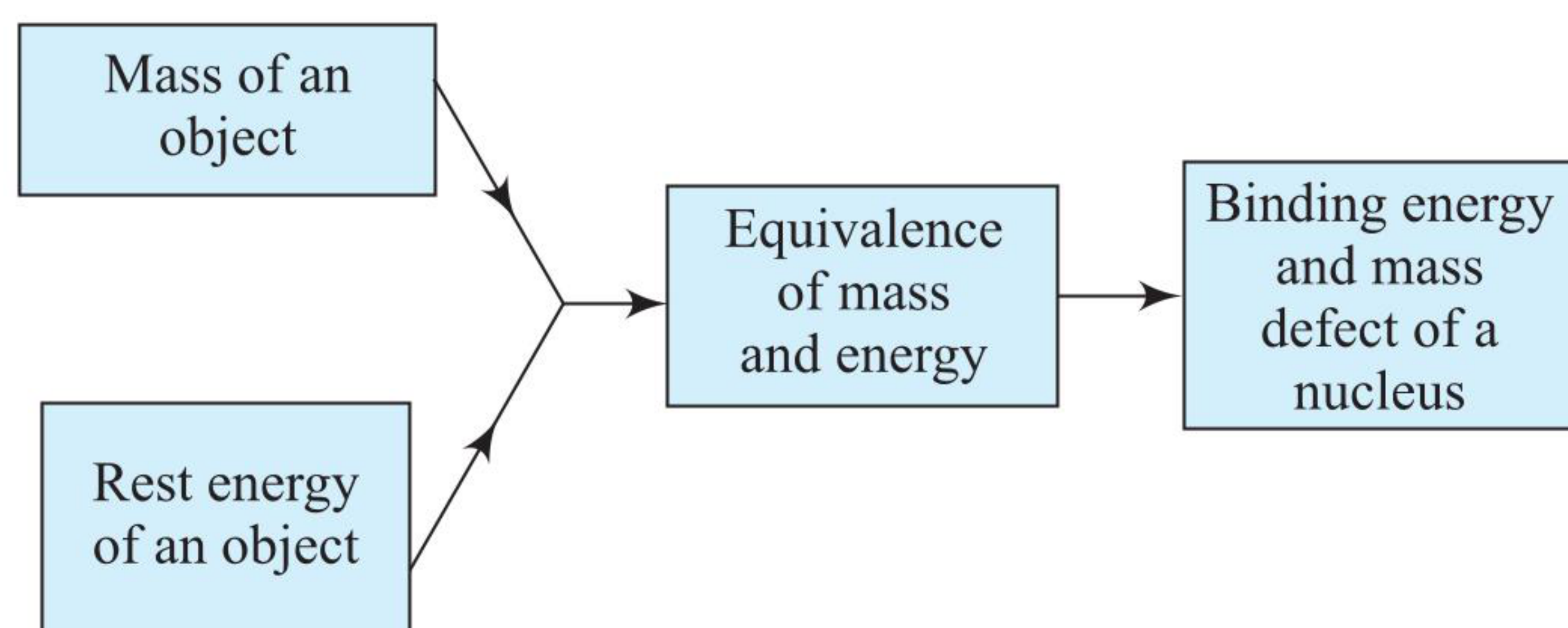
Now, we can see that in the experimental mass of ^4_2He atom is 4.0026034 which is less than the value given in Eq. (i), the sum of masses of constituent particles. Similarly, this can be verified for all the other elements. Also, the reason for this can be explained by Eq. (ii) which is a basic nuclear reaction for formation of a nuclei ^A_ZX . In this reaction, we can see that when Z protons and $A - Z$ neutrons fuse together to form a nucleus ^A_ZX , some amount of energy must be released.

$$Zm_p + (A - Z)m_n \rightarrow M_{\text{X}} + \text{Energy} \dots \text{(ii)}$$

If we think from where this energy comes, we can simply say by Einstein's mass-energy relationship, some amount of mass from independent nucleons is converted into energy and released when nucleons bind with each other to form a stable nucleus. This is the energy that hold the nucleons together in a nucleus, we call this binding energy of nucleus. So, when this energy is supplied to a nucleus, it splits into its constituent particles.

The binding energy used to disassemble the nucleus appears as extra mass of the separated nucleons. In other words, the sum of the individual masses of the separated protons and

neutrons is greater by an amount Δm than the mass of the stable nucleus. The difference in mass Δm is known as the mass defect of the nucleus.



For the reaction given in Eq. (ii), if we calculate the difference in masses of nucleons and that of nucleus X, then it is given as

$$\Delta m = Zm_p + (A - Z)m_n - M_X \quad \dots(iii)$$

This difference in masses of independent nucleons and mass of nucleus is called mass defect of the nuclear reaction. Using mass defect Δm , we can find the energy released in a nuclear reaction, e.g., the binding energy of the above nucleus X can be given as

$$\Delta E_{BE} = \Delta mc^2 \quad \dots(iv)$$

In a similar way, we can find the binding energy for any nucleus in nature for a known composition.

MASS DEFECT

It has been observed that there is a difference between expected mass and actual mass of a nucleus.

$$M_{\text{expected}} = Zm_p + (A - Z)m_n$$

$$M_{\text{observed}} = M_{\text{atom}} - Zm_e$$

It is found that $M_{\text{observed}} < M_{\text{expected}}$

Hence, mass defect is defined as

$$\text{Mass defect, } \Delta m = M_{\text{expected}} - M_{\text{observed}}$$

$$\Rightarrow \Delta m = [Zm_p + (A - Z)m_n] - [M_{\text{atom}} - Zm_e]$$

MASS-ENERGY EQUIVALENCE

We have discussed in previous section that using mass defect of a nuclear reaction, we can find the energy released in the nuclear process. We know generally nuclear masses are given in atomic mass unit where

$$1 \text{ amu} = 1.656 \times 10^{-27} \text{ kg}$$

If in a reaction 1 amu mass is converted into energy, then using Einstein's mass-energy relationship the amount of energy released is

$$\Delta E' = \Delta mc^2 = (1.656 \times 10^{-27}) (3 \times 10^8)^2 \text{ J}$$

$$= \frac{(1.656 \times 10^{-27})(3 \times 10^8)^2}{1.6 \times 10^{-19}} \text{ eV}$$

$$= 931.5 \times 10^6 \text{ eV} = 931.5 \text{ MeV}$$

Thus, we can say that 1 amu mass is equivalent to 931.5 MeV energy.

ILLUSTRATION 7.6

The most abundant isotope of helium has a ${}^4_2\text{He}$ nucleus whose mass is $6.6447 \times 10^{-27} \text{ kg}$. For this nucleus, find (a) the mass defect and (b) the binding energy.

Given: Mass of the electron: $m_e = 5.485799 \times 10^{-4} \text{ u}$, mass of the proton: $m_p = 1.007276 \text{ u}$ and mass of the neutron: $m_n = 1.008665 \text{ u}$.

Sol. The symbol ${}^4_2\text{He}$ indicates that the helium nucleus contains $Z = 2$ protons and $N = 4 - 2 = 2$ neutrons. To obtain the mass defect Δm , we first determine the sum of the individual masses of the separated protons and neutrons. Then, we subtract from this sum the mass of the nucleus.

(a) We find that the sum of the individual masses of the nucleons is

$$\underbrace{2(1.6726 \times 10^{-27} \text{ kg})}_{\text{Two protons}} + \underbrace{2(1.6749 \times 10^{-27} \text{ kg})}_{\text{Two neutrons}}$$

$$= 6.6950 \times 10^{-27} \text{ kg}$$

This value is greater than the mass of the intact He nucleus, and the mass defect is

$$\Delta m = 6.6950 \times 10^{-27} \text{ kg} - 6.6447 \times 10^{-27} \text{ kg}$$

$$= 0.0503 \times 10^{-27} \text{ kg}$$

(b) According to Eq. (iv), the binding energy is given by

$$\Delta E_{BE} = (\Delta m)c^2 = (0.0503 \times 10^{-27} \text{ kg})(3.00 \times 10^8 \text{ m s}^{-1})^2$$

$$= 4.53 \times 10^{-12} \text{ J}$$

Usually, binding energies are expressed in energy units of electron volt instead of joule ($1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$).

$$\therefore \text{Binding energy} = (4.53 \times 10^{-12} \text{ J}) \left(\frac{1 \text{ eV}}{1.60 \times 10^{-19} \text{ J}} \right)$$

$$= 2.83 \times 10^7 \text{ eV} = 28.3 \text{ MeV}$$

In this result, one million electron volt is denoted by the unit MeV. The value of 28.3 MeV is more than two million times greater than the energy required to remove an orbital electron from an atom.

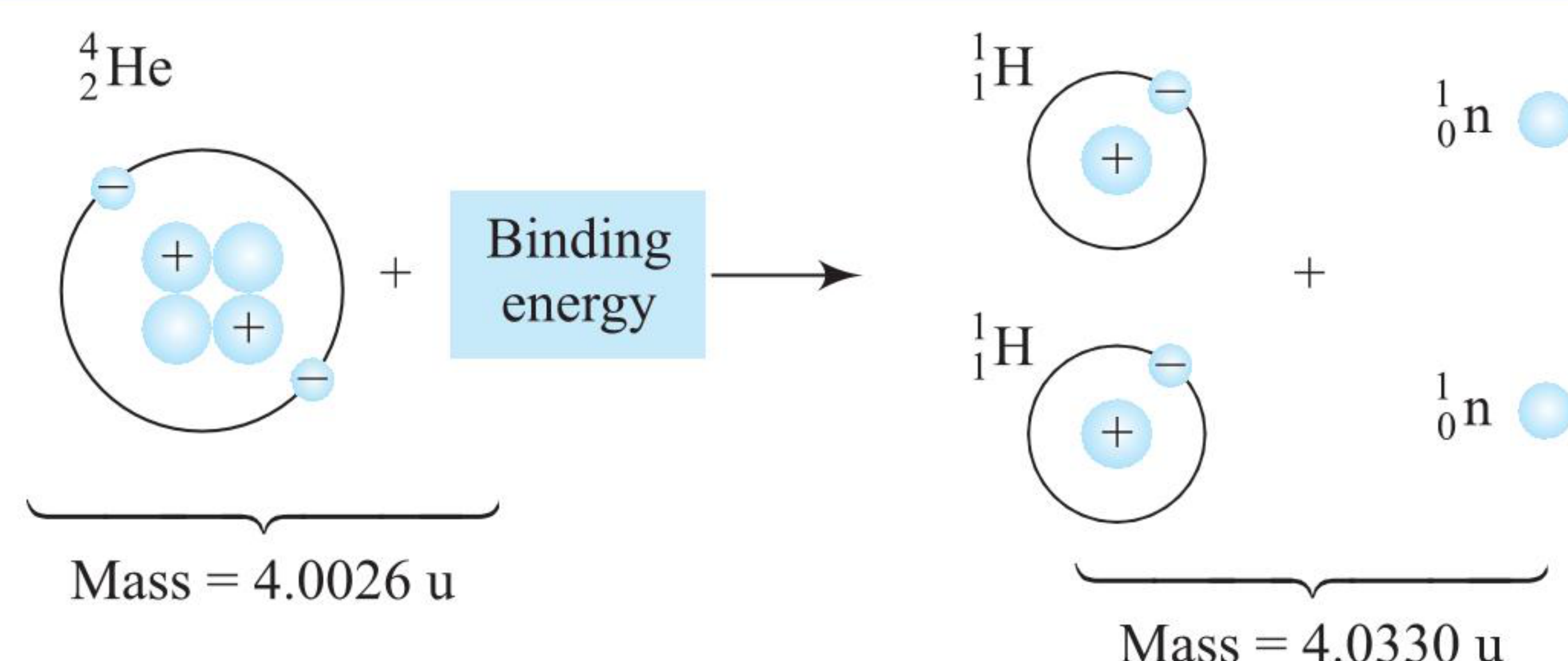
ILLUSTRATION 7.7

The atomic mass of ${}^4_2\text{He}$ is 4.0026 u and the atomic mass of ${}^1_1\text{H}$ is 1.0078 u. Using atomic mass units instead of kilograms, obtain the binding energy of the ${}^4_2\text{He}$ nucleus.

Sol. To determine the binding energy, we calculate the mass defect in atomic mass units and then use the fact that one atomic mass unit is equivalent to 931.5 MeV of energy. The mass of 4.0026 u for ${}^4_2\text{He}$ includes the mass of the two electrons in the neutral helium atom. To calculate the mass defect, we must subtract 4.0026 u from the sum of the individual masses of the nucleons, including the mass of the electrons. As the figure illustrates, the electron mass will be included if the masses of two hydrogen atoms are used in the calculation instead of the masses of two protons. The mass of a hydrogen atom is given as 1.0078 u, and the mass of a neutron is given in table as 1.0087 u.

The sum of the individual masses is

$$\underbrace{2(1.0078 \text{ u})}_{\text{Two hydrogen atoms}} + \underbrace{2(1.0087 \text{ u})}_{\text{Two neutrons}} = 4.0330 \text{ u}$$



The mass defect is $\Delta m = 4.0330 \text{ u} - 4.0026 \text{ u} = 0.0304 \text{ u}$. Since 1 u is equivalent to 931.5 MeV, the binding energy is 28.3 MeV.

ILLUSTRATION 7.8

The nucleus of the deuterium atom, called the deuteron, consists of a proton and a neutron. Calculate the deuteron's binding energy, given atomic mass, i.e., the mass of a deuterium nucleus plus an electron is measured to be 2.014102 u.

Sol. We know that the proton and neutron masses are

$$m_p = 1.007825 \text{ u}, \quad m_n = 1.008665 \text{ u}$$

Note that the masses used for the proton and neutron in this example are actually those of the neutral atoms. We are able to use atomic masses for these calculations because the electron masses cancel out. That is, when m_d containing one electron mass is subtracted from $(m_p + m_n)$ containing one electron mass, the electron masses cancel out. We have,

$$m_p + m_n = 2.016490 \text{ u}$$

To calculate the mass difference, we subtract the deuteron mass from this value.

$$\begin{aligned} \therefore \Delta m &= (m_p + m_n) - m_d \\ &= 2.016490 \text{ u} - 2.014102 \text{ u} = 0.002388 \text{ u} \end{aligned}$$

Because 1 u corresponds to an equivalent energy of 931.494 MeV (i.e., $1 \text{ u} \cdot c^2 = 931.494 \text{ MeV}$), the mass difference corresponds to the binding energy

$$E_b = (0.002388 \text{ u}) (931.494 \text{ MeV/u}) = 2.224 \text{ MeV}$$

This result tells us that to separate a deuteron into a proton and a neutron, it is necessary to add 2.224 MeV of energy to the deuteron to overcome the attractive nuclear force between the proton and neutron. One way of supplying the deuteron with this energy is by bombarding it with energetic particles.

If the binding energy of a nucleus were zero, the nucleus would separate into its constituent protons and neutrons without the addition of any energy, i.e., it would spontaneously break apart.

BINDING ENERGY PER NUCLEON

If we wish to see how nuclear binding energy varies from nucleus to nucleus, it is necessary to compare the binding energy per nucleon basis. The binding energy per nucleon for a given element can be given as binding energy divided by the nucleon number A as

$$(\text{BE})_N = \frac{\Delta mc^2}{A} \quad \dots(i)$$

This value $(\text{BE})_N$ in Eq. (i) gives the criterion of stability among different elements. We can define binding energy per

nucleon theoretically as the amount of energy needed to remove a nucleon from the nucleus of an element. For example, let us consider two elements X and Y with mass numbers A_X and A_Y ($A_X > A_Y$) and binding energies ΔE_X and ΔE_Y such that $\Delta E_X > \Delta E_Y$. Here, one can say that as for element X binding energy ΔE_X is more as compared to that for element Y, nucleus X is more stable than nucleus Y. But if we find the $(\text{BE})_N$ values for both elements, it is given as

$$(\text{BE})_N)_X = \frac{\Delta E_X}{A_X}; (\text{BE})_N)_Y = \frac{\Delta E_Y}{A_Y}$$

These values are such that $(\text{BE})_N)_X < (\text{BE})_N)_Y$ which implies that to remove a nucleon from element X requires less energy than from element Y. This implies that for the nucleus of Y it is difficult to remove one nucleon from its nucleus, thus structure of Y is more stable than X.

To understand this in a better way, let us consider an example of ^{56}Fe and ^{209}Bi . Their binding energies are given as

$$\Delta E_{\text{Fe}} = 492.8 \text{ MeV}; \Delta E_{\text{Bi}} = 1640 \text{ MeV}$$

From the above values, it seems that to break the nucleus of Bi more energy is required, hence it is more stable than that of Fe. But we should not ignore the bigger size of bismuth nucleus. If we find binding energy per nucleon for both of these nuclei, we get

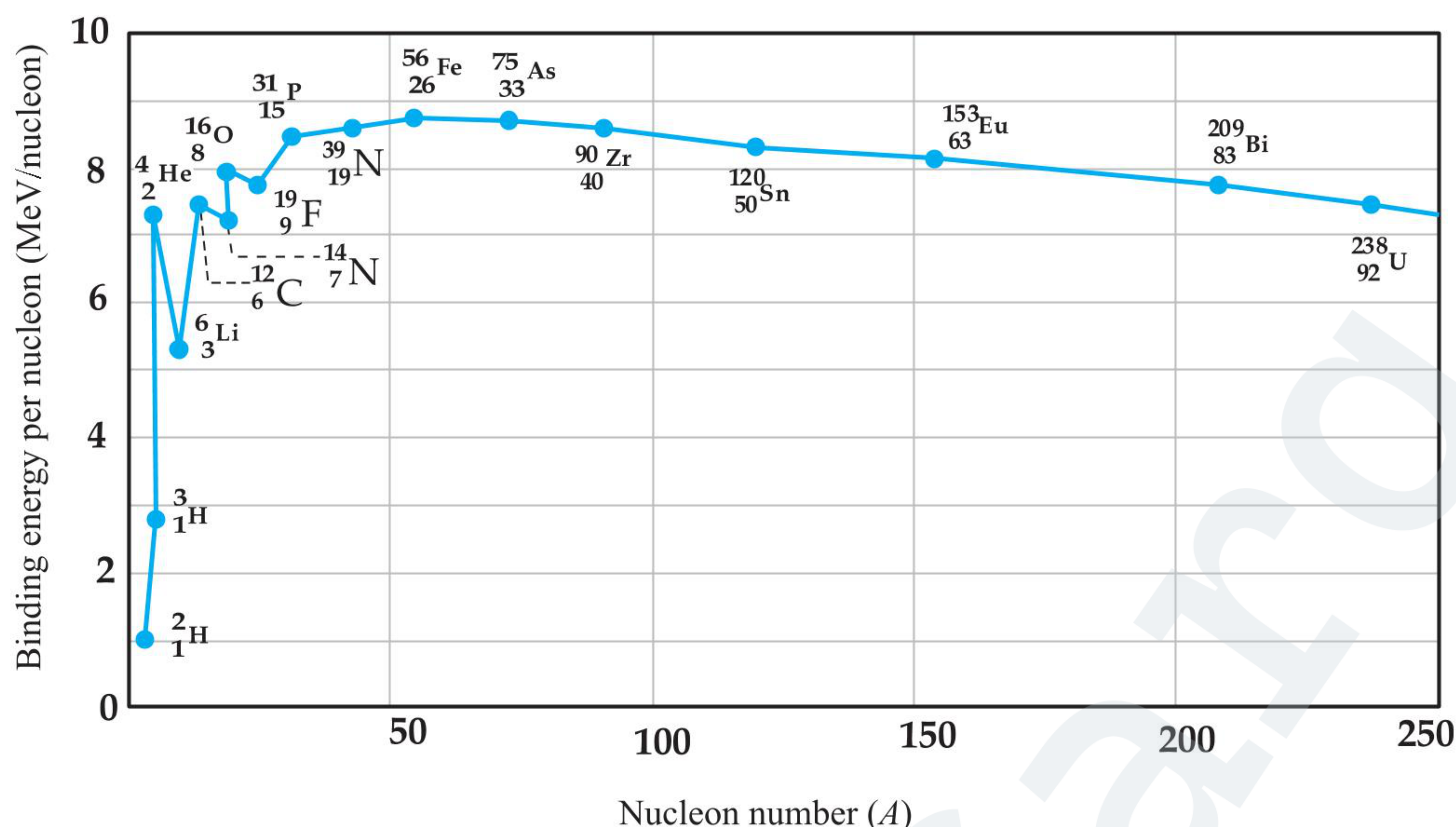
$$(\text{BE})_N)_{\text{Fe}} = \frac{492.8}{56} = 8.8 \text{ MeV}; (\text{BE})_N)_{\text{Bi}} = \frac{1640}{209} = 7.84 \text{ MeV}$$

Now, we can see that removal of one electron from Fe nucleus is more difficult as compared to that from Bi nucleus. So, Fe nuclei are more stable than Bi. Thus, to judge or compare the stability of different nuclei, we compare the binding energy per nucleon and not the nuclear binding energy.

To see how the nuclear binding energy varies from nucleus to nucleus, it is necessary to compare the binding energy for each nucleus on a per-nucleon basis. Figure shows a graph in which the binding energy divided by the nucleon number A is plotted against the nucleon number itself. In the graph, the peak for the ^4_2He isotope of helium indicates that the ^4_2He nucleus is particularly stable. The binding energy per nucleon increases rapidly for nuclei with small masses and reaches a maximum of approximately 8.7 MeV/nucleon for a nucleon number of about $A = 60$. For greater nucleon numbers, the binding energy per nucleon decreases gradually. Eventually, the binding energy per nucleon decreases enough so there is insufficient binding energy to hold the nucleus together. Nuclei more massive than the $^{209}_{83}\text{Bi}$ nucleus of bismuth are unstable and hence radioactive.

VARIATION OF BINDING ENERGY PER NUCLEON WITH MASS NUMBER

The binding energy per nucleon is a characteristic property of elements. The graph in figure shows the variation of binding energy per nucleon with mass number for all the elements of periodic table. In the graph, we can see that the binding energy per nucleon increases rapidly for nuclei with small masses and reaches a maximum of approximately 8.8 MeV/nucleon for iron ($^{56}_{26}\text{Fe}$). That is, nuclei with mass



A plot of binding energy per nucleon versus the nucleon number A

numbers greater or less than 60 are not as strongly bound as those near the middle of the periodic table. As we shall see later, this fact allows energy to be released in fission and fusion reactions. The curve is slowly varying for $A > 40$, which suggests that the nuclear force saturates. In other words, a particular nucleon can interact with only a limited number of other nucleons.

For greater nucleon numbers, the binding energy per nucleon decreases gradually. Later, the binding energy per nucleon decreases enough so there is insufficient binding energy to hold the nucleon together in the nucleus. It is observed that nuclei with $A > 209$ are unstable and hence radioactive.

In the figure, in the beginning there are some fluctuations in the graph. We can see that binding energies per nucleon for ${}^4\text{He}$, ${}^{12}\text{C}$, and ${}^{16}\text{O}$ are relatively higher as compared to their neighboring elements or these elements have nuclei which are relatively more stable than their neighbors. This is because of the existence of nuclear energy levels in the nucleus. Each nuclear energy level can contain two neutrons of opposite spins or two protons of opposite spins. Energy levels in nuclei are filled in sequence, just as energy levels in atoms, to achieve configurations of minimum energy and therefore maximum stability. Similar to the case of atomic orbitals, here also the configuration of opposite spins in nucleons with pairs in nuclear energy levels are more stable. This concept can be used to explain the reason of more stability of ${}^4\text{He}$, ${}^{12}\text{C}$, and ${}^{16}\text{O}$ compared to their neighboring elements.

Note: If binding energy per nucleon is more for a nucleus, then it is more stable. For example, if

$$\left(\frac{\text{BE}_1}{A_1}\right) > \left(\frac{\text{BE}_2}{A_2}\right)$$

then nucleus 1 would be more stable.

Important observations from the graph:

1. The maximum binding energy per nucleon is 8.8 MeV and it is for Fe^{56} . Thus, iron nucleus is the most stable and the most tightly bound.

2. The fact that (BE/A) varies by less than 10 percent above $A = 10$ suggests that each nucleon in the nucleus interacts only with the neighbors, independently of the total number of nucleons present in the nucleus.
3. A small decrease after $A = 56$ is due to the destabilizing effect of the long-range repulsive Coulomb force.
4. Not only the Coulomb force reduces the BE, it also shifts the neutron–proton ratio in heavy nuclei toward the neutron excess, by an amount that will increase with A .

ILLUSTRATION 7.9

Calculate the binding energy for nucleon of ${}^{12}_6\text{C}$ nucleus, if mass of proton $m_p = 1.0078$ u, mass of neutron $m_n = 1.0087$ u, mass of C_{12} , $m_C = 12.0000$ u; and $1 \text{ u} = 931.4 \text{ MeV}$.

Sol. In the nucleus of carbon, there are 6 protons and 6 neutrons.

$$\begin{aligned} \text{Now, } 6m_p &= 6(1.0078) = 6.0468 \text{ u} \\ 6m_n &= 6(1.0087) = 6.05224 \text{ u} \\ \text{Total mass} &= 6m_p + 6m_n = 12.0990 \text{ u} \\ m_C &= 12.0000 \text{ u} \\ \text{Mass defect, } \Delta m &= 0.0990 \text{ u} \\ \text{BE} &= \Delta m \times 931.4 = (0.0990) 931.4 = 92.2 \text{ MeV} \end{aligned}$$

ILLUSTRATION 7.10

Obtain the binding energy of the nuclei ${}^{56}_{26}\text{Fe}$ and ${}^{209}_{83}\text{Bi}$ in units of MeV from the following data:

$$\begin{aligned} m_p &= 1.007825 \text{ u}, m_n = 1.008665 \text{ u} \\ m({}^{56}_{26}\text{Fe}) &= 55.934939 \text{ u}, m({}^{209}_{83}\text{Bi}) = 208.980388 \text{ u} \end{aligned}$$

Which nucleus has greater binding energy per nucleon?

Sol. In ${}^{56}_{26}\text{Fe}$ nucleus, there are 26 protons and $(56 - 26) = 30$ neutrons.

$$\begin{aligned} \text{Mass defect} &= 26 \times m_p + 30 m_n - m({}^{56}_{26}\text{Fe}) \\ &= 26 (1.007825 \text{ u}) + 30 (1.008665 \text{ u}) - 55.934939 \text{ u} \\ &= 56.463400 \text{ u} - 55.934939 \text{ u} = 0.528461 \text{ u} \end{aligned}$$

$$\text{Binding energy} = 0.528461 \times 931.5 \text{ MeV} = 492.26 \text{ MeV} \\ (\text{as } 1 \text{ u} = 931.5 \text{ MeV})$$

$$\text{Binding energy per nucleon} = \frac{492.26 \text{ MeV}}{56} = 8.79 \text{ MeV}$$

In $^{209}_{83}\text{Bi}$ nucleus, there are 83 protons and $(209-83) = 126$ neutrons.

$$\begin{aligned} \text{Mass defect} &= 83 \times m_p + 126 \times m_n - m(^{209}_{83}\text{Bi}) \\ &= 83(1.007825 \text{ u}) + 126(1.008665 \text{ u}) - 208.980388 \text{ u} \\ &= 1.760872 \text{ u} \end{aligned}$$

$$\text{Binding energy} = 1.760872 \times 931.5 \text{ MeV} = 164.3 \text{ MeV}$$

$$\text{Binding energy per nucleon} = \frac{1640.3}{209} = 7.85 \text{ MeV}$$

Thus, $^{56}_{26}\text{Fe}$ has greater binding energy per nucleon than $^{209}_{83}\text{Bi}$.

ILLUSTRATION 7.11

Find the binding energy of an α -particle from the following data.

Mass of the helium nucleus = 4.001265 amu

Mass of proton = 1.007277 amu

Mass of neutron = 1.00866 amu (take 1 amu = 931.4813 MeV)

Sol. Mass of two protons is $2 \times 1.007277 = 2.014554$ amu

Mass of two neutrons is $2 \times 1.008666 \text{ amu} = 2.017332$ amu

Total initial mass of two protons and two neutrons is

$$2.014554 + 2.017332 = 4.031886 \text{ amu}$$

Mass defect, $\Delta m = 4.031886 - 4.001265 \text{ amu} = 0.030621 \text{ amu}$

Now, binding energy of α -particle in MeV is given as

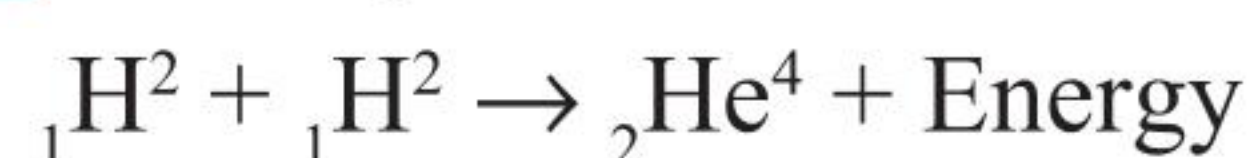
$$\Delta E_n = \Delta m \times 931.2 \text{ MeV} = 0.030621 \times 931.2 = 28.5142 \text{ MeV}$$

Binding energy per nucleon = $28.5142/4 = 7.12855 \text{ MeV}$

ILLUSTRATION 7.12

The binding energies per nucleon for deuteron (${}_1\text{H}^2$) and helium (${}_2\text{He}^4$) are 1.1 MeV and 7.0 MeV, respectively. Calculate the energy released when two deuterons fuse to form a helium nucleus (${}_2\text{He}^4$).

Sol. The equation of fusion is represented as



The binding energy per nucleon of deuteron is 1.1 MeV

Therefore, the net initial binding energy = $4 \times 1.1 = 4.4 \text{ MeV}$

The binding energy per nucleon of ${}_2\text{He}^4$ is 7.0 MeV.

Therefore, the net final binding energy is $4 \times 7.0 = 28.0 \text{ MeV}$

Hence, energy released = $28.0 \text{ MeV} - 4.4 \text{ MeV} = 23.6 \text{ MeV}$

Q-VALUES

We have just examined some nuclear reactions for which mass numbers and atomic numbers must be balanced in the equations. We shall now consider the energy involved in these reactions, because energy is another important quantity that must be conserved.

Let us illustrate this procedure by analyzing the following nuclear reaction:



The total mass on the left side of the equation is the sum of the mass of ${}_1^2\text{H}$ (2.014102 u) and the mass of ${}_7^{14}\text{N}$ (14.003074 u), which equals 16.017176 u. Similarly, the mass on the right side of the equation is the sum of the mass of ${}_6^{12}\text{C}$ (12.000000 u) plus the mass of ${}_2^4\text{He}$ (4.002602 u), for a total of 16.002602 u. Thus, the total mass before the reaction is greater than the total mass after the reaction. The mass difference in this reaction is greater than the total mass after the reaction. The mass difference in this reaction is equal to $16.017176 \text{ u} - 16.002602 \text{ u} = 0.014574 \text{ u}$. This “lost” mass is converted to the kinetic energy of the nuclei present after the reaction. In energy units, 0.014574 u is equivalent to 13.576 MeV of kinetic energy carried away by the carbon and helium nuclei.

The energy required to balance the equation is called the Q value of the reaction. In Eq. (i), the Q value is 13.576 MeV. Nuclear reactions in which there is a release of energy—that is, positive Q values—are said to be exothermic reactions.

The energy balance sheet is not complete; however, we must also consider the kinetic energy of the incident particle before the collision. As an example, let us assume that the deuteron in Eq. (i) has a kinetic energy of 5 MeV. Adding this to our Q value, we find that the carbon and helium nuclei have a total kinetic energy of 18.576 MeV following the reaction. Now, consider the reaction



Before the reaction, the total mass is the sum of the masses of the alpha particle and the nitrogen nucleus: $4.002602 \text{ u} + 14.003074 \text{ u} = 18.005676 \text{ u}$. After the reaction, the total mass is the sum of the masses of the oxygen nucleus and the proton: $16.999133 \text{ u} + 1.07825 \text{ u} = 18.006958 \text{ u}$. In this case, the total mass deficit is 0.001282 u, equivalent to an energy deficit of 1.194 MeV. This deficit is expressed by the negative Q value of the reaction, -1.194 MeV . Reactions with negative Q values are called endothermic reactions. Such reactions will not take place unless the incoming particle has at least enough kinetic energy to overcome the energy deficit.

At first it might appear that the reaction in Eq. (ii) could take place if the incoming alpha particle had a kinetic energy of 1.194 MeV. In practice, however, the alpha particle must have more energy than this. If it had an energy of only 1.194 MeV, energy would be conserved but careful analysis would show that momentum was not. This can easily be understood by recognizing that the incoming alpha particle has some momentum before the reaction. However, if its kinetic energy were only 1.194 MeV, the products (oxygen and a proton) would be created with zero kinetic energy and thus zero momentum. It can be shown that, in order to conserve both energy and momentum, the incoming particle must have a minimum kinetic energy given by

$$\text{KE}_{\min} = \left(1 + \frac{m}{M}\right) |Q| \quad \dots(\text{iii})$$

where m is the mass of the incident particle, M is the mass of the target, and the absolute value of the Q value is used. For the reaction given by Eq. (ii), we find

$$\begin{aligned} \text{KE}_{\min} &= \left(1 + \frac{4.00262}{14.003074}\right) |-1.194 \text{ MeV}| \\ &= 1.535 \text{ MeV} \end{aligned}$$

This minimum value of the kinetic energy of the incoming particle is called the threshold energy. The nuclear reaction shown in Eq. (ii) will not occur if the incoming alpha particle has a kinetic energy of less than 1.535 MeV, but can occur if its kinetic energy is equal to or greater than 1.535 MeV.

ILLUSTRATION 7.13

Find the minimum kinetic energy of an α -particle to cause the reaction $^{14}\text{N}(\alpha, p)^{17}\text{O}$. The masses of ^{14}N , ^4He , ^1H and ^{17}O are respectively 14.00307 u, 4.00260 u, 1.00783 u and 16.99913 u.

Sol. Since, the masses are given in atomic mass units, it is easiest to proceed by finding the mass difference between reactants and products in the same units and then multiplying by 931.5 MeV/u. Thus, we have

$$Q = (14.00307 \text{ u} + 4.00260 \text{ u} - 1.00783 \text{ u} - 16.99913 \text{ u}) \left(931.5 \frac{\text{MeV}}{\text{u}} \right) = -1.20 \text{ MeV}$$

Q value is negative. It means reaction is endothermic. So, the minimum kinetic energy of α -particle to initiate this reaction would be

$$K_{\min} = |Q| \left(\frac{m_{\alpha}}{m_{\text{N}}} + 1 \right) = (1.20) \left(\frac{4.00260}{14.00307} + 1 \right) = 1.54 \text{ MeV}$$

ILLUSTRATION 7.14

Show that $^{230}_{92}\text{U}$ does not decay by emitting a neutron or proton. Given:

$$\begin{aligned} M(^{229}_{92}\text{U}) &= 230.033927 \text{ amu}, M(^{230}_{92}\text{U}) = 229.033496 \text{ amu}, \\ M(^{229}_{91}\text{Pa}) &= 229.032089 \text{ amu}, m(n) = 1.008665 \text{ amu}, \\ m(p) &= 1.007825 \text{ amu}. \end{aligned}$$

Sol. The corresponding decay equations would be



In the first equation, the energy released can be given by

$$Q = [230.033927 - 229.033496 - 1.008665] \times 931.5 = -7.7 \text{ MeV}$$

Because $Q < 0$, neutron decay is not possible spontaneously.

Similarly, in the second equation, the energy released can be given as

$$Q = [230.033927 - 229.032089 - 1.007825] \times 931.5 = -5.6 \text{ MeV}$$

As $Q < 0$, proton decay is also not possible spontaneously.

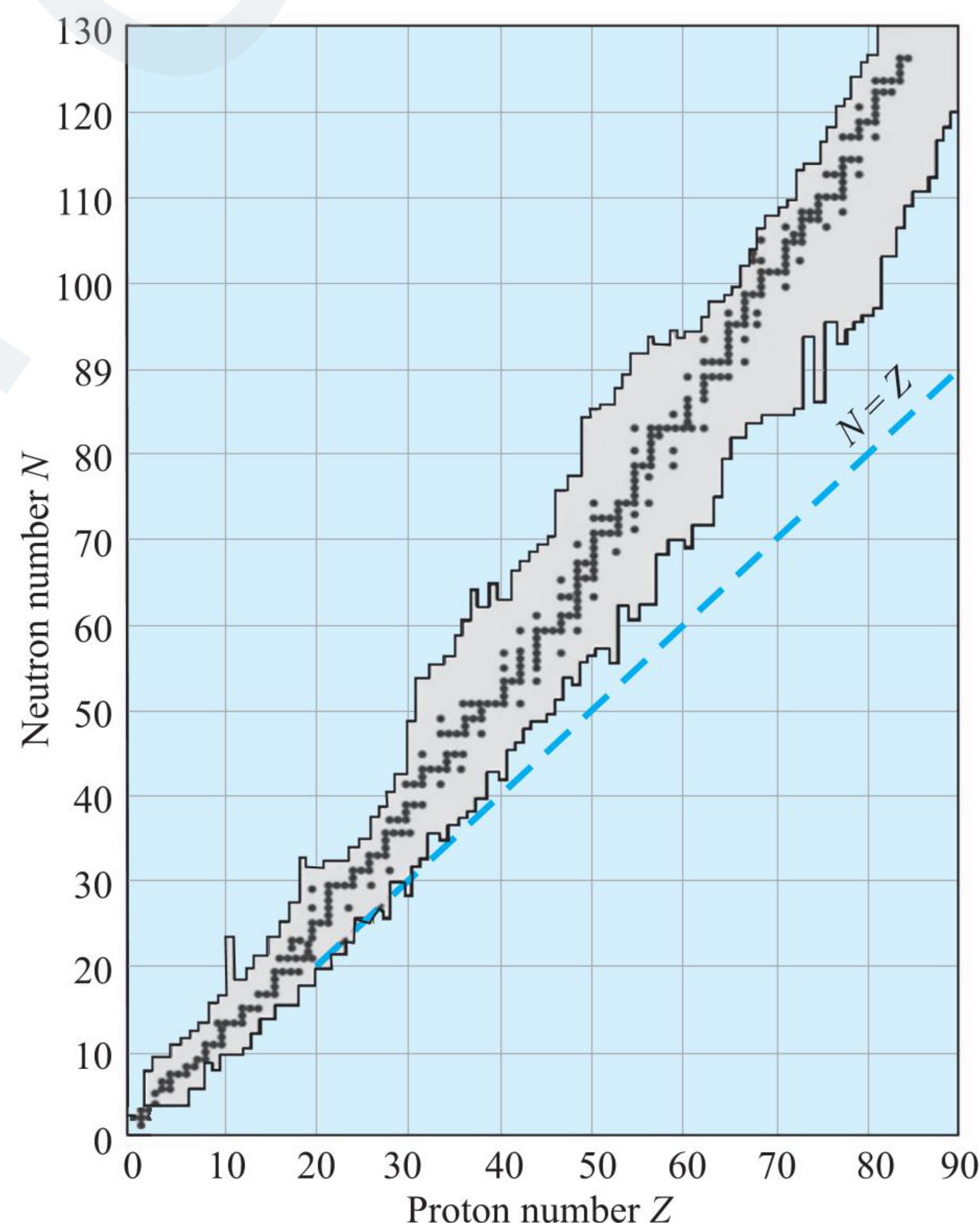
NUCLEAR STABILITY

Given that the nucleus consists of a closely packed collection of protons and neutrons, you might be surprised that it can exist. The very large repulsive electrostatic forces between protons should cause the nucleus to fly apart. However, nuclei are stable because of the presence of another, short-range (about 2 fm) force, the **nuclear force**. This is an attractive force that acts between all nuclear particles. The protons attract each other via the nuclear force, and at the same time they repel each other through the Coulomb force. The attractive nuclear force also acts between pairs of neutrons and between neutrons and protons.

The nuclear attractive force is stronger than the Coulomb repulsive force within the nucleus (at short ranges). If this were

not the case, stable nuclei would not exist. Moreover, the strong nuclear force is nearly independent of charge. In other words, the nuclear forces associated with the proton–proton, proton–neutron, and neutron–neutron interactions are approximately the same.

There are about 260 stable nuclei; hundreds of others have been observed but are unstable. A plot of N versus Z for a number of stable nuclei is given in figure. Note that light nuclei are most stable if they contain equal number of protons and neutrons—that is, if $N = Z$ —but heavy nuclei are more stable if $N > Z$. This can be partially understood by recognizing that, as the number of protons increases, the strength of the Coulomb force increases, which tends to break the nucleus apart. As a result, more neutrons are needed to keep the nucleus stable, because neutrons experience only the attractive nuclear forces. In effect, the additional neutrons “dilute” the nuclear charge density. Eventually, when $Z = 83$, the repulsive forces between protons cannot be compensated by the addition of more neutrons. Elements that contain more than 83 protons do not have stable nuclei, but decay or disintegrate into other particles in various amounts of time. The masses of several stable particles are given in table.



CONCEPT APPLICATION EXERCISE 7.1

1. How many electrons, protons, and neutrons are there in a nucleus of atomic number 11 and mass number 24?
2. Calculate the average binding energy per nucleon of $^{93}_{41}\text{Nb}$ having mass 92.906 u.
3. Protons and neutrons exist together in an extremely small space within the nucleus. How is this possible when protons repel each other?
4. Following data are available about three nuclei P , Q , and R . Arrange them in decreasing order of stability.

	<i>P</i>	<i>Q</i>	<i>R</i>
Atomic mass number (<i>A</i>)	10	5	6
Binding energy (MeV)	100	60	66

5. A nucleus has binding energy of 100 MeV. It further releases 10 MeV energy. Find the new binding energy of the nucleus.
6. A nuclear reaction is given as $A + B \rightarrow C + D$. Binding energies of *A*, *B*, *C*, and *D* are given as B_1 , B_2 , B_3 and B_4 . Find the energy released in the reaction.
7. Calculate the binding energy of an alpha particle from the following data:
 Mass of ${}^1_1\text{H}$ atom = 1.007826 u
 Mass of neutron = 1.008665 u
 Mass of ${}^4_2\text{He}$ atom = 4.00260 u
 (take 1 u = 931 MeV/ c^2)
8. Find the binding energy of ${}^{56}_{26}\text{Fe}$. Atomic mass of ${}^{56}_{26}\text{Fe}$ is 55.9349 u and that of ${}^1_1\text{H}$ is 1.00783 u. Mass of neutron is 1.00867 u.
9. Use Avogadro's number to show that the atomic mass unit is 1 u = 1.66×10^{-27} kg.
10. Take a sample of lead and oxygen. They contain different atoms and the density of solid lead is much greater than that of gaseous oxygen. Decide whether the density of the nucleus in a lead atom is greater than, approximately equal to, or less than that in an oxygen atom.
11. Show that the nuclide ${}^8\text{Be}$ has a positive binding energy but is unstable with respect to decay into two alpha particles, where masses of neutron, ${}^1_1\text{H}$, and ${}^8\text{Be}$ are 1.008665 u, 1.007825 u, and 8.005305 u, respectively.

ANSWERS

1. 11, 11, 13 2. 8.66 MeV/nucleon 5. 110 MeV
 6. $(B_3 + B_4) - (B_1 + B_2)$ 7. 28.3 MeV
 8. 492 MeV 10. approximate equal to

RADIOACTIVITY

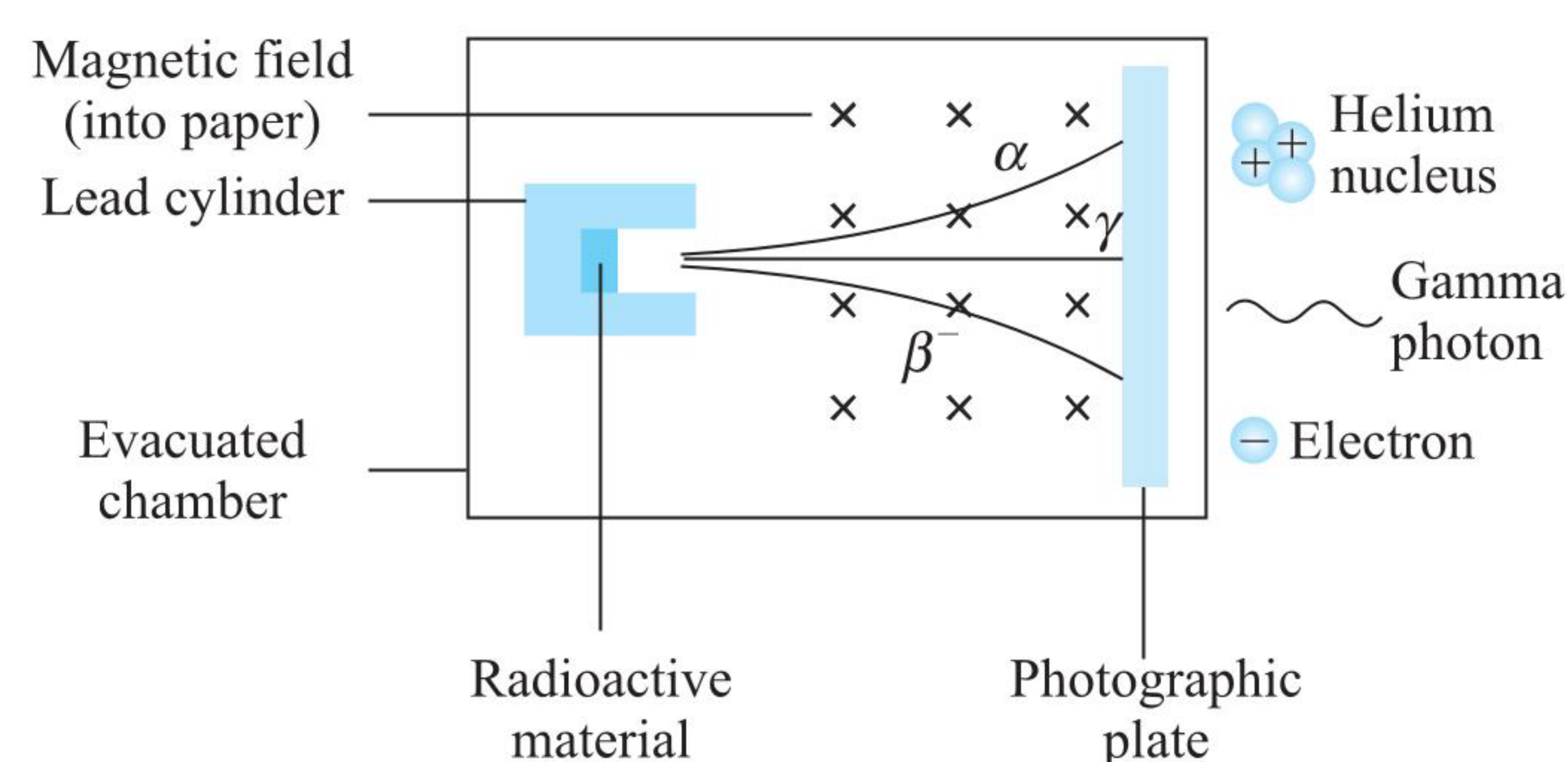
We have discussed that inside a nucleus electrostatic attraction is counterbalanced by short-range strong nuclear forces and the nucleus becomes stable. Despite the forces being balanced, many nuclides are unstable because of nuclear size or the limited range of nuclear forces. Due to slight imbalance in small sized nuclides, these nuclides spontaneously disintegrate into other nuclides. This phenomenon of spontaneous disintegration is called radioactivity. Further, in this chapter, we will discuss the aspects of radioactivity by which unstable nuclides disintegrate to achieve stability.

When an unstable or radioactive nucleus disintegrates spontaneously, certain kinds of particles and/or high-energy photons are released. These particles and photons are collectively called “rays”. Three kinds of rays are produced by naturally occurring radioactivity: α -rays, β -rays, and γ -rays. They are named according to the first three letters of the Greek alphabet, alpha (α), beta (β), and gamma (γ), to indicate the extent of their ability to penetrate matter. α -rays are the least penetrating,

being blocked by a thin (≈ 0.01 mm) sheet of lead, while β -rays penetrate lead to a much greater distance (≈ 0.1 mm). γ -rays are the most penetrating and can pass through an appreciable thickness (≈ 100 mm) of lead.

The nuclear disintegration process that produces α -, β -, and γ -rays must obey the laws of physics; these laws are called conservation laws because each of them deals with a property (such as mass/energy, electric charge, linear momentum, and angular momentum) that is conserved or does not change during a process. To these conservation laws, we now add a fifth, the conservation of nucleon number. In all radioactive decay processes, it has been observed that the number of nucleons present before the decay is equal to the number of nucleons after the decay. Therefore, the number of nucleons is conserved during a nuclear disintegration. As applied to the disintegration of a nucleus, the conservation laws require that the energy, electric charge, linear momentum, angular momentum, and nucleon number that a nucleus possesses must remain unchanged when it disintegrates into nuclear fragments and accompanying α -, β -, or γ -rays.

The three types of radioactivity that occur naturally can be observed in a relatively simple experiment. A piece of radioactive material is placed at the bottom of a narrow hole in a lead cylinder. The cylinder is located within an evacuated chamber, as figure illustrates. A magnetic field is directed perpendicular to the plane of the paper, and a photographic plate is positioned to the right of the hole. Three kinds of spots appear on the developed plate, which are associated with the radioactivity of the nuclei in the material. Since moving particles are deflected by a magnetic field only when they are electrically charged, this experiment reveals that two types of radioactivity (α - and β -rays, as it turns out) consist of charged particles, while the third type (γ -rays) does not.

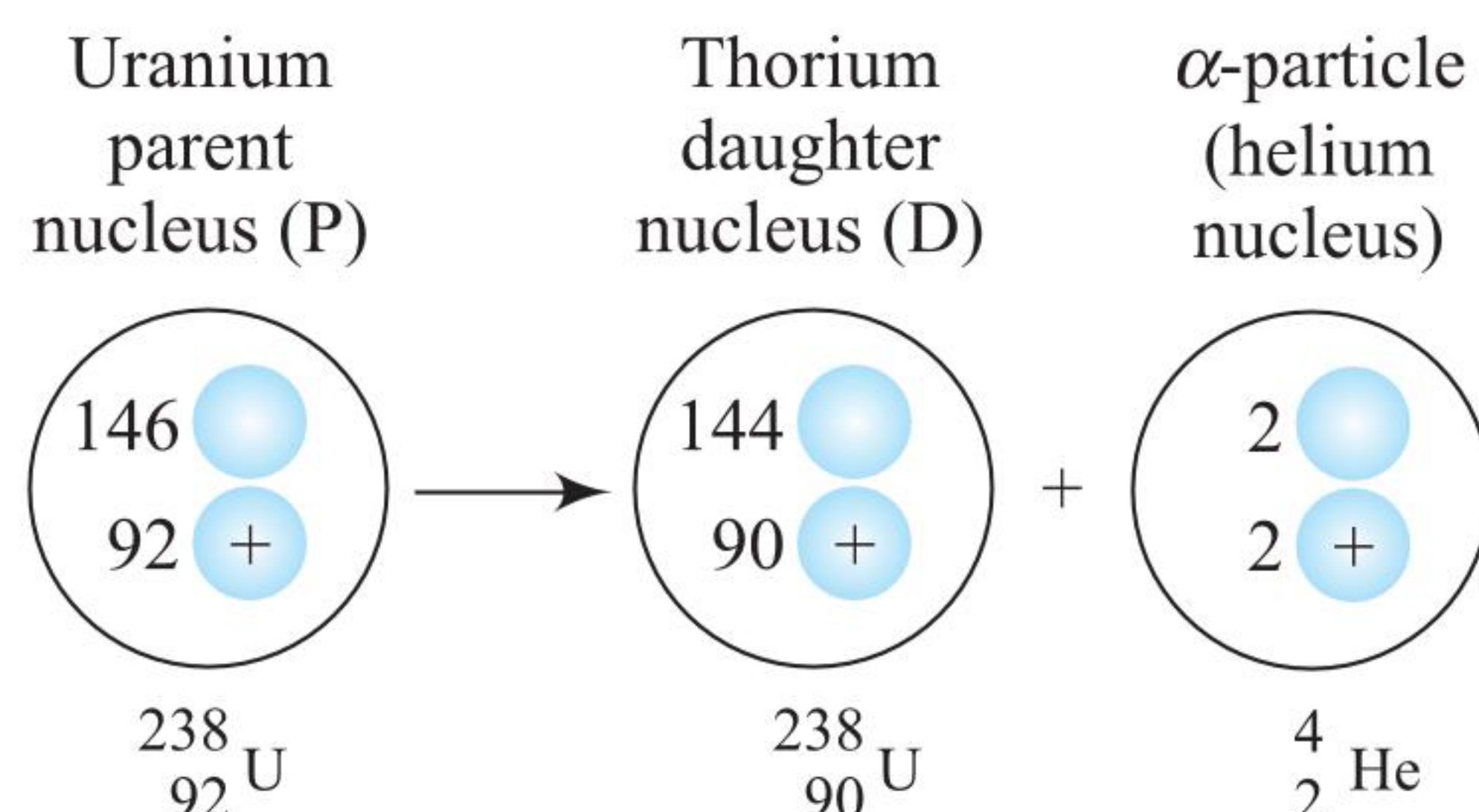
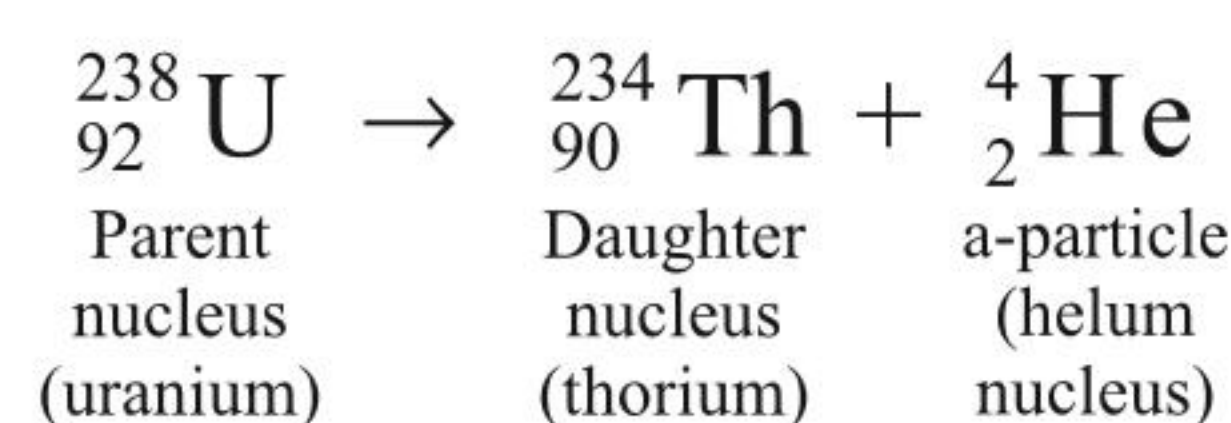


In figure, α - and β -ray are deflected by a magnetic field and, therefore, consist of moving charged particles. γ -rays are not deflected by a magnetic field and, consequently, must be uncharged.

α -DECAY

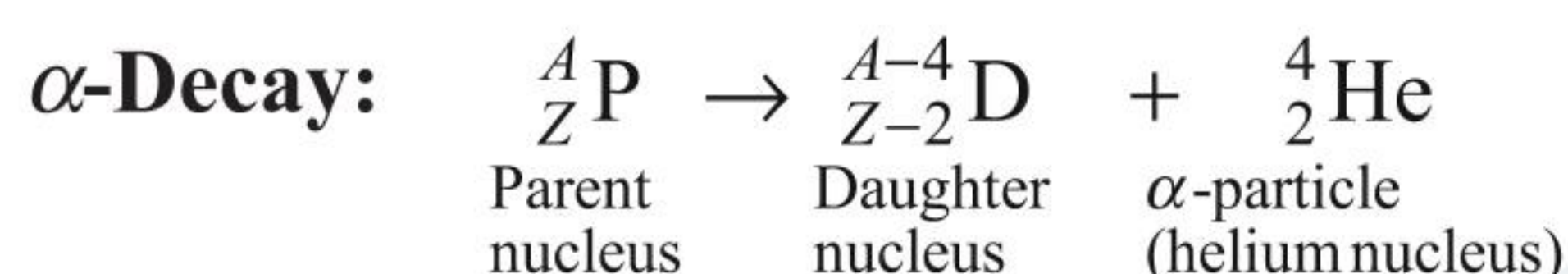
When a nucleus disintegrates and produces α -rays, it is said to undergo α -decay. Experimental evidence shows that α -rays consist of positively charged particles, each one being the ${}^4_2\text{He}$ nucleus of helium. Thus, an α -particle has a charge of $+2e$ and a nucleon number of $A = 4$. Since the grouping of 2 protons and 2 neutrons in a ${}^4_2\text{He}$ nucleus is particularly stable, it is not surprising that an α -particle can be ejected as a unit from a more massive unstable nucleus.

Figure shows the disintegration process as an example of α -decay.



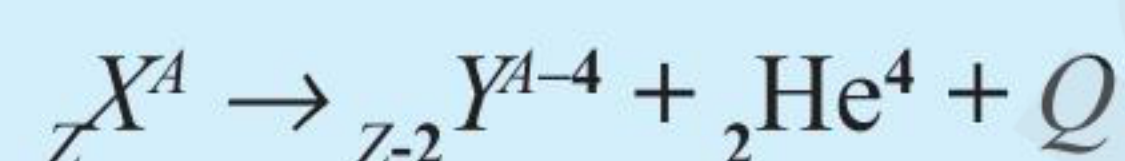
The original nucleus is referred to as the parent nucleus (P), and the nucleus remaining after disintegration is called the daughter nucleus (D). Upon emission of an α -particle, the uranium ${}_{92}^{238}\text{U}$ parent is converted into the ${}_{90}^{234}\text{Th}$ daughter, which is an isotope of thorium. The parent and daughter nuclei are different, so α -decay converts one element into another, a process known as transmutation.

Electric charge is conserved during α -decay. In figure, for instance, 90 of the 92 protons in the uranium nucleus end up in the thorium nucleus, and the remaining two protons are carried off by the α -particle. The total number of 92, however, is the same before and after disintegration. α -decay also conserves the number of nucleons, for the number is the same before (238) and after ($234 + 4$) disintegration. Consistent with the conservation of electric charge and nucleon number, the general form for α -decay is



When a nucleus releases an α -particle, the nucleus also releases energy. In fact, the energy released by radioactive decay is responsible, in part, for keeping the interior of the earth hot and, in some places, even molten. The following example shows how the conservation of mass/energy can be used to determine the amount of energy released in α -decay.

α -Decay:



Q value: It is defined as the energy released during the decay process.

Q value = rest mass energy of reactants – rest mass energy of products.

This energy is available in the form of increase in KE of the products. Let us consider the following:

$$M_x = \text{mass of atom } {}_Z^AX^A$$

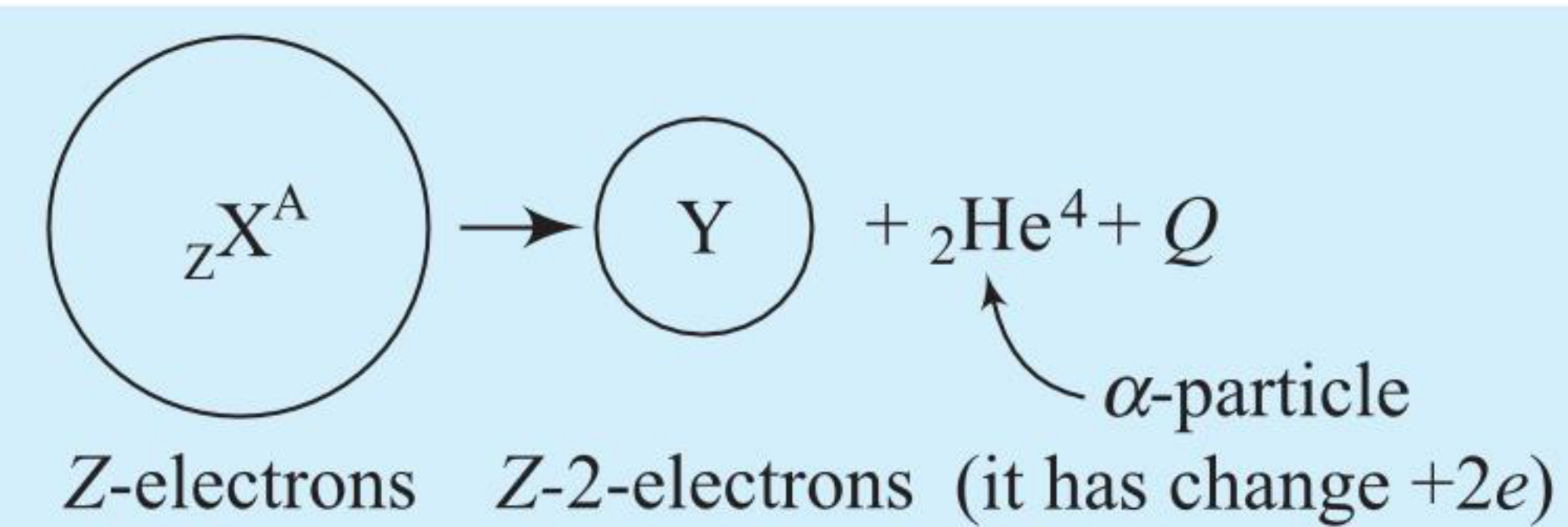
$$M_Y = \text{mass of atom } {}_{Z-2}Y^{A-4}$$

$$M_{\text{He}} = \text{mass of atom } {}_2\text{He}^4$$

$$Q \text{ value} = [(M_X - Zm_e) - \{(M_Y - (Z-2)m_e) - (M_{\text{He}} - 2m_e)\}]c^2$$

$$= [M_X - M_Y - M_{\text{He}}]c^2$$

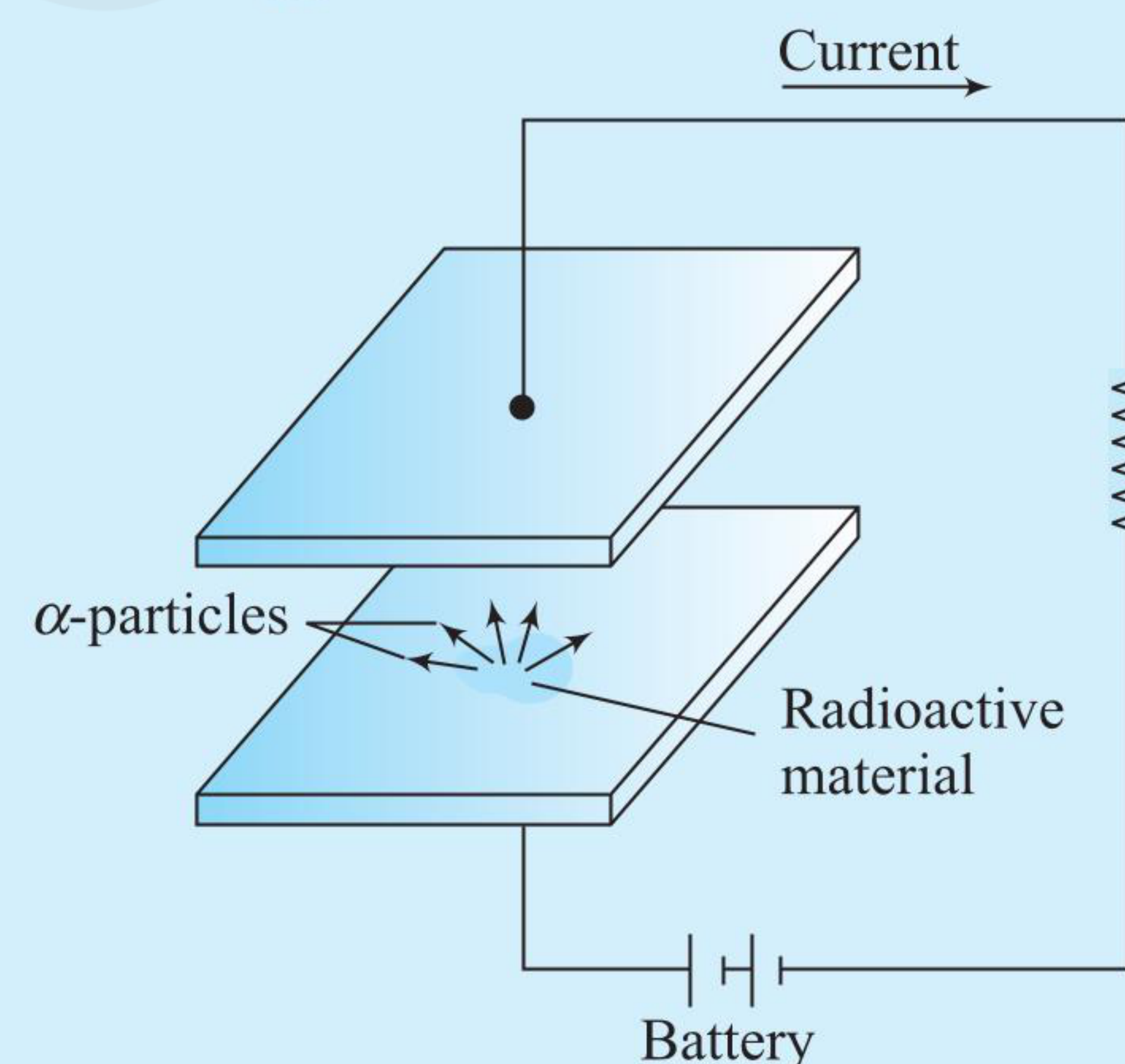
Considering actual number of electrons in α -decay



$$Q \text{ value} = [M_X - (M_Y + 2m_e) - (M_{\text{He}} - 2m_e)]c^2$$

$$= [M_X - M_Y - M_{\text{He}}]c^2$$

One widely used application of α -decay is in smoke detectors. Figure illustrates how a smoke detector operates. Two small and parallel metal plates are separated by a distance of about one centimetre. A tiny amount of radioactive material at the center of one of the plates emits α -particles, which collide with air molecules. During the collisions, the air molecules are ionized to form positive and negative ions. The voltage from a battery causes one plate to be positive and the other negative, so that each plate attracts ions of opposite charge. As a result there is a current in the circuit attached to the plates. The presence of smoke particles between the plates reduces the current, since the ions that collide with a smoke particle are usually neutralized. The drop in current that smoke particles cause is used to trigger an alarm.



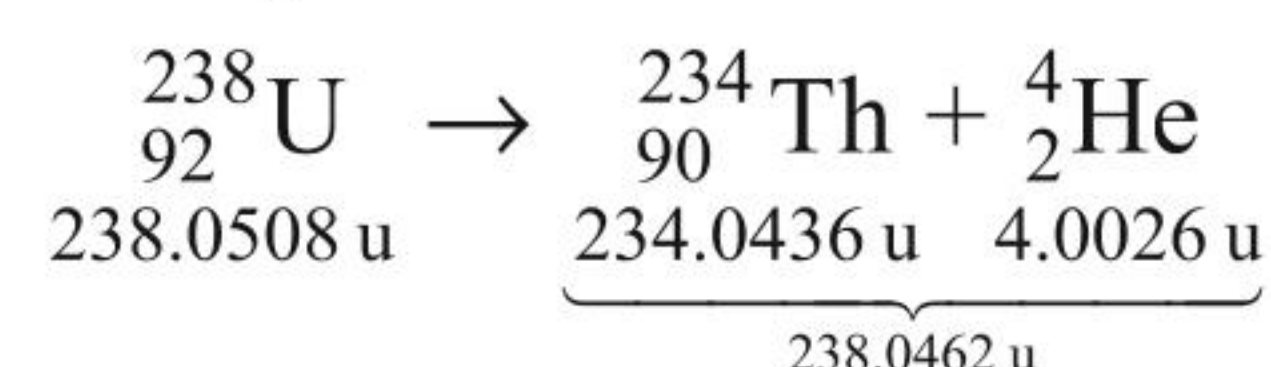
A smoke detector

ILLUSTRATION 7.15

The atomic mass of uranium ${}_{92}^{238}\text{U}$ is 238.0508 u, that of thorium ${}_{90}^{234}\text{Th}$ is 234.0436 u and that of an alpha particle ${}_{2}^{4}\text{He}$ is 4.0026 u. Determine the energy released when α -decay converts ${}_{92}^{238}\text{U}$ into ${}_{90}^{234}\text{Th}$.

Sol. Since energy is released during the decay, the combined mass of the ${}_{90}^{234}\text{Th}$ daughter nucleus and the α -particle is less than the mass of the ${}_{92}^{238}\text{U}$ parent nucleus. The difference in mass is equivalent to the energy released. We will determine the difference in mass in atomic mass units and then use the fact that 1 u is equivalent to 931.5 MeV.

The decay and the masses are shown below:



The decrease in mass is $238.0508 \text{ u} - 238.0462 \text{ u} = 0.0046 \text{ u}$. As usual, the masses are atomic masses and include the mass

of the orbital electrons. But this causes no error here because the same total number of electrons is included for ${}_{92}^{238}\text{U}$, on the one hand, and for ${}_{90}^{234}\text{Th}$ plus ${}^4_2\text{He}$, on the other. Since 1 u is equivalent to 931.5 MeV, the released energy is 4.3 MeV.

When α -decay occurs as discussed in Illustration 7.15, the energy released appears as kinetic energy of the recoiling ${}_{90}^{234}\text{Th}$ nucleus and the α -particle, except for a small portion carried away as a γ -ray. Illustration 7.16 discusses how the ${}_{90}^{234}\text{Th}$ nucleus and the α -particle share the released energy.

ILLUSTRATION 7.16

Refer to Illustration 7.15, the energy released by the α -decay of ${}_{92}^{238}\text{U}$ is found to be 4.3 MeV. Since this energy is carried away as kinetic energy of the recoiling ${}_{90}^{234}\text{Th}$ nucleus and the α -particle, it follows that $\text{KE}_{\text{Th}} + \text{KE}_{\alpha} = 4.3 \text{ MeV}$. However, KE_{Th} and KE_{α} are not equal. Which particle carries away more kinetic energy, the ${}_{90}^{234}\text{Th}$ nucleus or the α -particle?

Sol. Kinetic energy depends on the mass m and speed v of a particle, since $\text{KE} = \frac{1}{2}mv^2$. The ${}_{90}^{234}\text{Th}$ nucleus has a much greater mass than the α -particle, and since the kinetic energy is proportional to the mass, it is tempting to conclude that the ${}_{90}^{234}\text{Th}$ nucleus has greater kinetic energy. This conclusion is not correct, however, since it does not take into account the fact that the ${}_{90}^{234}\text{Th}$ nucleus and the α -particle have different speeds after the decay. In fact, we expect the thorium nucleus to recoil with smaller speed precisely because it has the greater mass.

The decaying of ${}_{92}^{238}\text{U}$ is like a father and his young daughter on ice skates, pushing off against one another. The more massive father recoils with much less speed than the daughter. We can use the principle of conservation of linear momentum to verify our explanation.

As we know, the conservation principle states that the total linear momentum of an isolated system remains constant. An isolated system is one for which the vector sum of the external forces acting on the system is zero, and the decaying ${}_{92}^{238}\text{U}$ nucleus fits this description. It is stationary initially, and since momentum is mass times velocity, its initial momentum is zero. In its final form, the system consists of the ${}_{90}^{234}\text{Th}$ nucleus and the α -particle and has a final total momentum of $m_{\text{Th}}v_{\text{Th}} + m_{\alpha}v_{\alpha}$. According to momentum conservation, the initial and final values of the total momentum of the system must be the same, so that $m_{\text{Th}}v_{\text{Th}} + m_{\alpha}v_{\alpha} = 0$. Solving this equation for the velocity of the thorium nucleus, we find that $v_{\text{Th}} = -m_{\alpha}v_{\alpha}/m_{\text{Th}}$. Since m_{Th} is much greater than m_{α} , we can see that the speed of the thorium nucleus is less than the speed of the α -particle. Moreover, the kinetic energy depends on the square of the speed and only the first power of the mass. As a result of its much greater speed, the α -particle has the greater kinetic energy.

ILLUSTRATION 7.17

Find the Q -value and the kinetic energy of the emitted α -particle in the α -decay of (a) ${}_{88}^{226}\text{Ra}$ and (b) ${}_{86}^{220}\text{Rn}$.

Given: $m({}_{88}^{226}\text{Ra}) = 226.02540 \text{ u}$

$$m({}_{86}^{222}\text{Rn}) = 222.01750 \text{ u}$$

$$m({}_{84}^{216}\text{Po}) = 216.00189 \text{ u}$$

Sol.

$$(a) \text{ As } {}_{88}^{226}\text{Ra} \rightarrow {}_{86}^{222}\text{Rn} + {}^4_2\text{He} + Q$$

$$\begin{aligned} Q &= [m({}_{88}^{226}\text{Ra}) - m({}_{86}^{222}\text{Rn}) - m({}^4_2\text{He})] \\ &= [226.02540 - 222.01750 - 4.00260] \text{ u} = 0.0053 \text{ u} \\ &= 0.0053 \times 931.5 \text{ MeV} = 4.94 \text{ MeV} \end{aligned}$$

KE of the α -particle

$$K_{\alpha} = \frac{(A-4)}{A} \times Q = \frac{(226-4)}{226} \times 4.94 \text{ MeV} = 4.85 \text{ MeV}$$

$$(b) \text{ Proceeding as in (a), } Q = 0.00688 \text{ u} = 0.00688 \times 931.5 \text{ MeV} = 6.41 \text{ MeV}$$

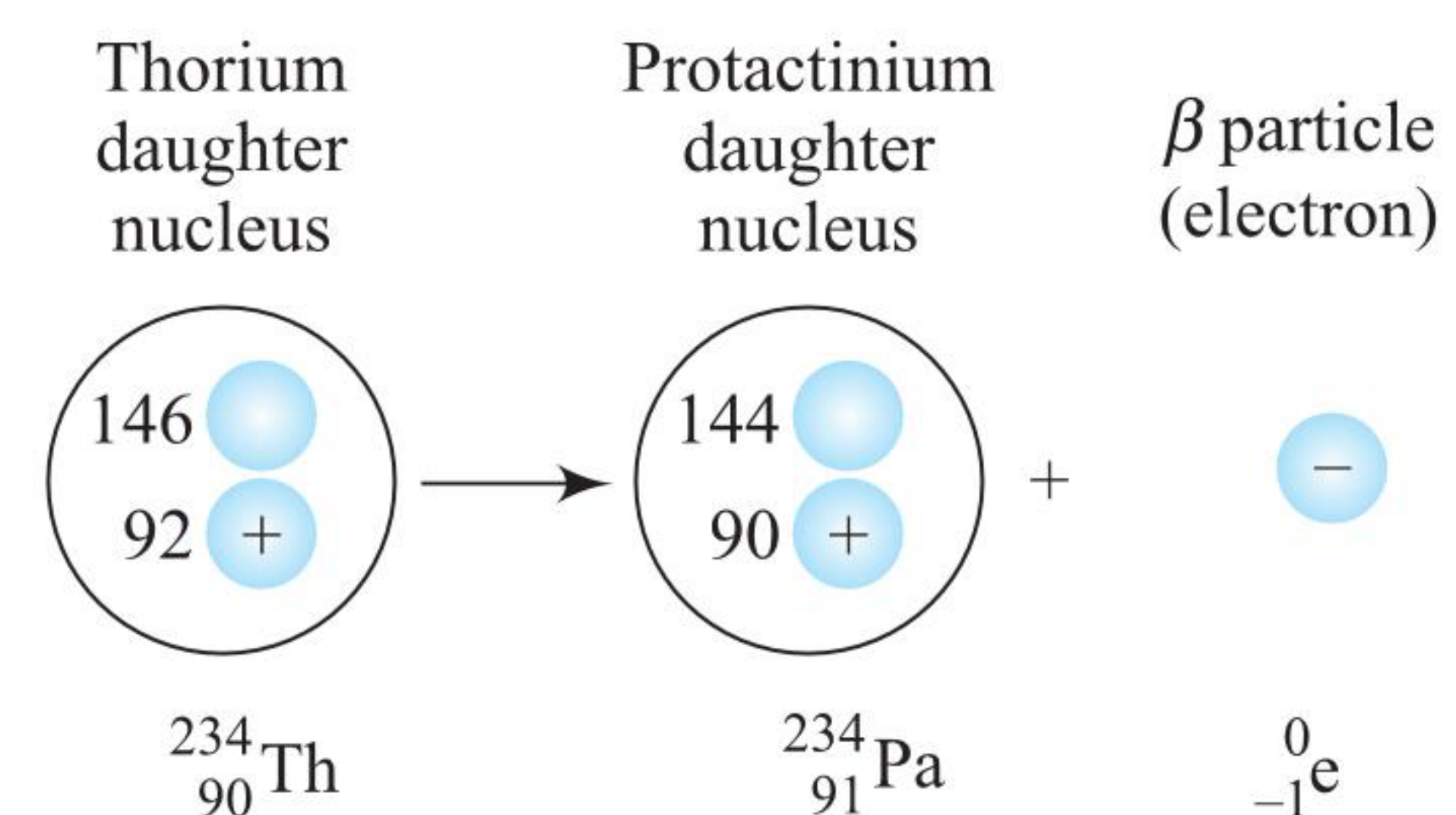
KE of the α -particle,

$$K_{\alpha} = \frac{(A-4)}{A} Q = \left(\frac{220-4}{220} \right) \times 6.41 \text{ MeV} = 6.29 \text{ MeV}$$

β -DECAY

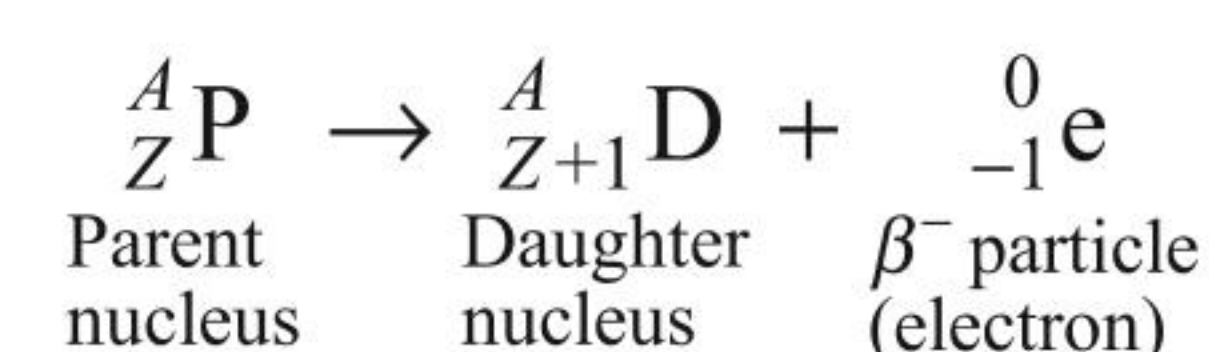
The β -rays are deflected by the magnetic field in a direction opposite to that of the positively charged α -rays. Consequently, these β -rays, which are the most common kind, consist of negatively charged particles or β -particles. Experiment shows that β -particles are electrons. When a radioactive nucleus undergoes β -decay, the daughter nucleus has the same number of nucleons as the parent nucleus, but the atomic number is changed by 1.

As an illustration of β -decay, consider the thorium ${}_{90}^{234}\text{Th}$ nucleus, which decays by emitting a β -particle, as shown in figure. β -decay occurs when a neutron in an unstable parent nucleus decays into a proton and an electron, the electron being emitted as the β -particle. In the process, the parent nucleus is transformed into the daughter nucleus.



β^- decay, like α -decay, causes a transmutation of one element into another. In this case, thorium ${}_{90}^{234}\text{Th}$ is converted into protactinium ${}_{91}^{234}\text{Pa}$. The law of conservation of charge is obeyed, since the net number of positive charges is the same before (90) and after (91 – 1) the β^- emission. The law of conservation of nucleon number is obeyed, since the nucleon number remains at $A = 234$.

β^- Decay

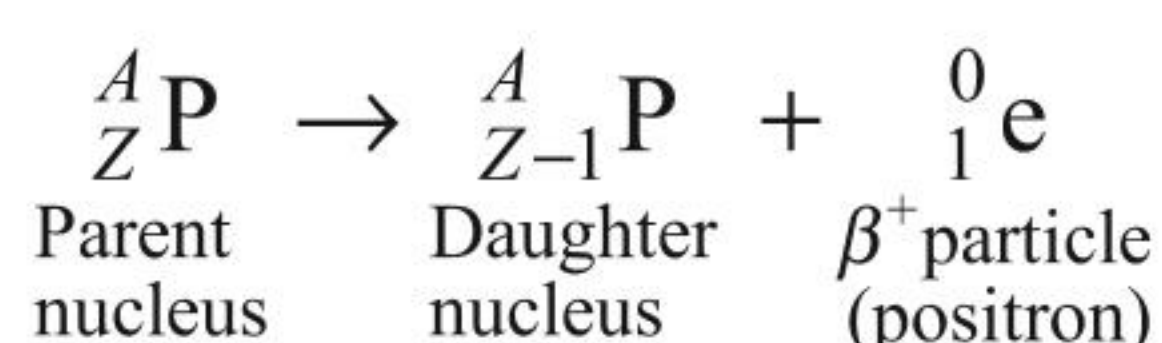


The electron emitted in β^- decay does not actually exist within the parent nucleus and is not one of the orbital electrons. Instead, the electron is created when a neutron decays into a proton and

an electron; when this occurs, the proton number of the parent nucleus increases from Z to $Z + 1$ and the nucleon number remains unchanged. The electron is usually fast-moving and escapes from the atom, leaving behind a positively charged atom.

β^+ Decay

A second kind of β -decay sometimes occurs. In this process, the particle emitted by the nucleus is a positron rather than an electron. A positron, also called a β^+ particle, has the same mass as an electron but carries a charge of $+e$ instead of $-e$. The disintegration process for β^+ decay is



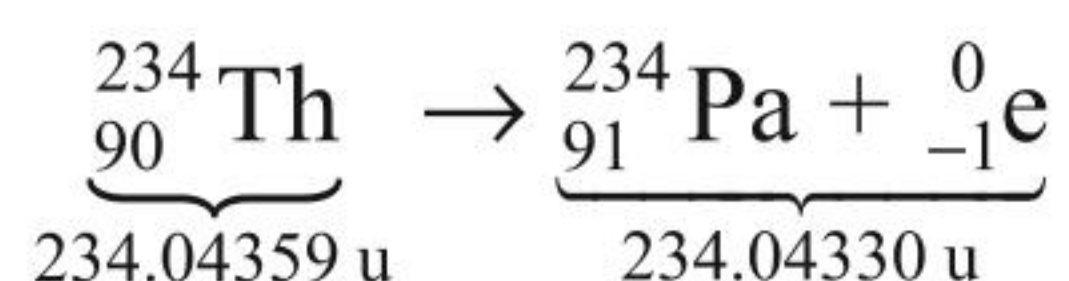
The emitted positron does not exist within the nucleus but, rather, is created when a nuclear proton is transformed into a neutron. In the process, the proton number of the parent nucleus decreases from Z to $Z - 1$, and the nucleon number remains the same. As with β^- decay, the laws of conservation of charge and nucleon number are obeyed, and there is a transmutation of one element into another.

In the following illustration, we will discuss how energy is released during β^- decay, just as it is during α -decay, and that the conservation of mass/energy applies.

ILLUSTRATION 7.18

The atomic mass of thorium ${}_{90}^{234}\text{Th}$ is 234.04359 u, while that of protactinium ${}_{91}^{234}\text{Pa}$ is 234.04330 u. Find the energy released when β^- decay changes ${}_{90}^{234}\text{Th}$ into ${}_{91}^{234}\text{Pa}$.

Sol. To find the energy released, we follow the usual procedure of determining how much the mass has decreased because of the decay and then calculating the equivalent energy. The decay and the masses are shown below:



When the $^{234}_{90}\text{Th}$ nucleus of a thorium atom is converted into a $^{234}_{91}\text{Pa}$ nucleus, the number of orbital electrons remains the same, so the resulting protactinium atom is missing one orbital electron. However, the given mass includes all 91 electrons of a neutral protactinium atom. In effect, then, the value of 234.04330 u for $^{234}_{91}\text{Pa}$ already includes the mass of the β^- particle. The mass decrease that accompanies the β^- decay is $234.04359\text{ u} - 234.04330\text{ u} = 0.00029\text{ u}$. The equivalent energy ($1\text{ u} = 931.5\text{ MeV}$) is 0.27 MeV. This is the maximum kinetic energy that the emitted electron can have.

ILLUSTRATION 7.19

Consider the beta decay



where $^{198}\text{Hg}^*$ represents a mercury nucleus in an excited state at energy 1.088 MeV above the ground state. What can be the maximum kinetic energy of the electron emitted? The atomic mass of ^{198}Au is 197.968233 u and that of ^{198}Hg is 197.966760 u.

Sol. If the product nucleus ^{198}Hg is formed in its ground state, the kinetic energy available to the electron and the antineutrino is

$$Q = [m(^{198}\text{Au}) - m(^{198}\text{Hg})]c^2$$

As $^{198}\text{Hg}^*$ has energy 1.088 MeV more than ^{198}Hg in ground state, the kinetic energy actually available is

$$\begin{aligned} Q &= [m(^{198}\text{Au}) - m(^{198}\text{Hg})]c^2 - 1.088 \text{ MeV} \\ &= (197.968233 \text{ u} - 197.966760 \text{ u}) \left(931 \frac{\text{MeV}}{\text{u}} \right) - 1.088 \text{ MeV} \\ &= 1.3686 \text{ MeV} - 1.088 \text{ MeV} = 0.2806 \text{ MeV} \end{aligned}$$

This is also the maximum possible kinetic energy of the electron emitted.

ILLUSTRATION 7.20

Calculate the Q value in the following decays:



The atomic masses needed are as follows:

^{19}O	^{19}F	^{25}Al	^{25}Mg
19.003576 u	18.998403 u	24.990432 u	24.985839 u

Sol.

(a) The Q value of β^- -decay is

$$\begin{aligned} Q &= [m(^{19}\text{O}) - m(^{19}\text{F})]c^2 \\ &= [19.003576 \text{ u} - 18.998403 \text{ u}] (931 \text{ MeV/u}) \\ &= 4.816 \text{ MeV} \end{aligned}$$

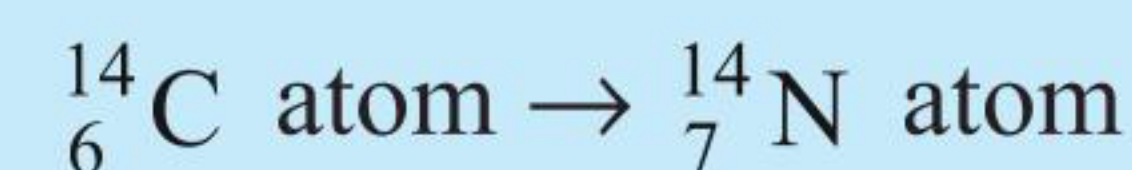
(b) The Q value of β^+ -decay is

$$\begin{aligned} Q &= [m(^{25}\text{Al}) - m(^{25}\text{Mg}) - 2m_e]c^2 \\ &= \left[24.99032 \text{ u} - 24.985839 \text{ u} - 2 \times 0.511 \frac{\text{MeV}}{c^2} \right] c^2 \\ &= (0.004593 \text{ u}) (931 \text{ MeV/u}) - 1.022 \text{ MeV} \\ &= 4.276 \text{ MeV} - 1.022 \text{ MeV} = 3.254 \text{ MeV} \end{aligned}$$

ILLUSTRATION 7.21

Find the energy liberated in the beta decay of ${}^{14}_6\text{C}$ to ${}^{14}_7\text{N}$ as represented by equation: ${}^{14}_6\text{C} \rightarrow {}^{14}_7\text{N} + e^- + \bar{\nu}$

Equation refers to nuclei. Adding six electrons to both sides gives



Sol. We know that ${}^{14}_6\text{C}$ has a mass of 14.003242 u and ${}^{14}_7\text{N}$ has a mass of 14.003074 u. Here, the mass difference between the initial and final states is

$$\Delta m = 14.003242 \text{ u} - 14.003074 \text{ u} = 0.000168 \text{ u}$$

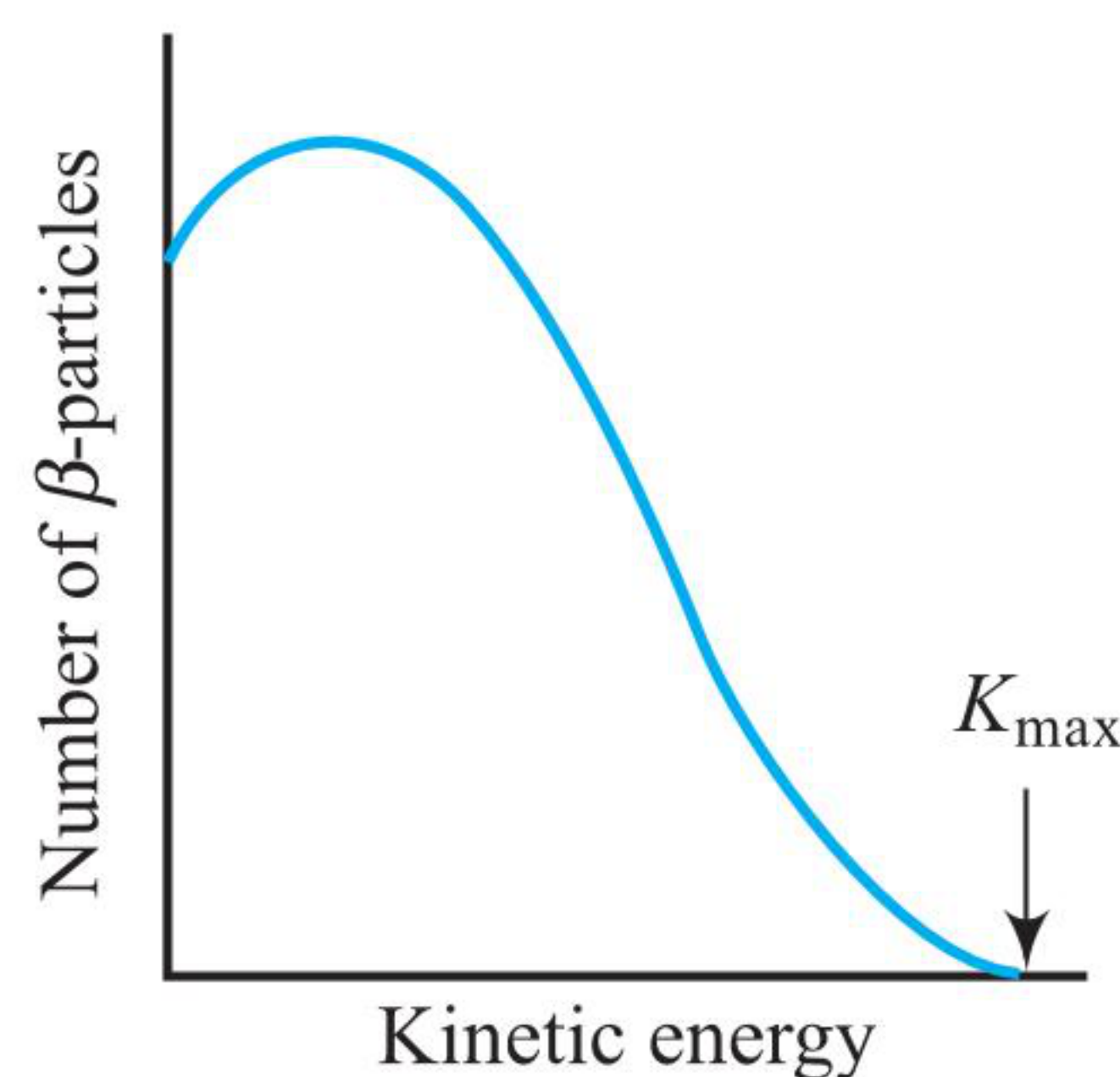
This corresponds to an energy release of

$$E = (0.000168 \text{ u})(931.494 \text{ MeV/u}) = 0.156 \text{ MeV}$$

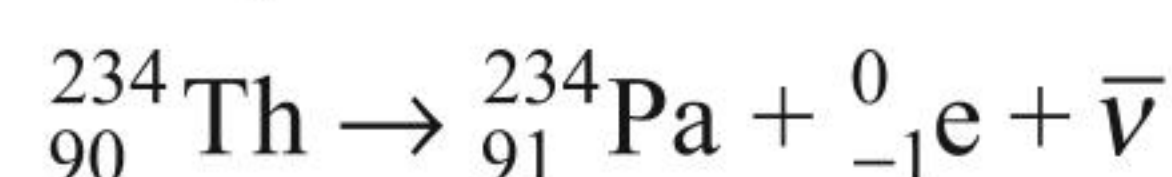
Neutrino

From illustrations discussed in previous section, we see that the energy released in the beta decay of ^{14}C is approximately 0.16 MeV. As with alpha decay, we expect the electron to carry away virtually all of this as kinetic energy because apparently it is the lightest particle produced in the decay. However, as figure shows, only a small number of electrons have this maximum kinetic energy, represented as K_{max} on the graph; most of the

electrons emitted have kinetic energies lower than this predicted value. If the daughter nucleus and the electron are not carrying away this liberated energy, then the energy conservation requirement leads to the question, what accounts for the missing energy? As an additional complication, further analysis of beta decay shows that the principles of conservation of both angular momentum and linear momentum appear to be violated.



When a β -particle is emitted by a radioactive nucleus, energy is simultaneously released. Experimentally, however, it is found that most β -particles do not have enough kinetic energy to account for all the energy released. If a β -particle carries away only part of the energy, where does the remainder go? In 1930, Pauli proposed that a third particle must be present to carry away the “missing” energy and to conserve momentum. Enrico Fermi later developed a complete theory of beta decay and named this particle the neutrino (“little neutral one”) because it had to be electrically neutral and have little or no mass. Although it eluded detection for many years, the neutrino (ν) was finally detected experimentally in 1956. For instance, the β -decay of thorium $^{234}_{90}\text{Th}$ is more correctly written as



The bar above the ν is included because the neutrino emitted in this particular decay process is an antimatter neutrino or antineutrino. A normal neutrino (ν without the bar) is emitted when β^+ decay occurs.

The neutrino has zero electrical charge and is extremely difficult to detect because it interacts very weakly with matter.

The neutrino has the following properties:

1. Zero electric charge.
2. A mass much smaller than that of the electron, but probably not zero. (Recent experiments suggest that the neutrino definitely has mass, but the value is uncertain—perhaps less than $1 \text{ eV}/c^2$.)
3. A spin of $\frac{1}{2}$.
4. Very weak interaction with matter, making it quite difficult to detect.

Thus, with the introduction of the neutrino, we are now able to represent the beta decay process of equation in its correct form:



where the bar in the symbol $\bar{\nu}$ indicates an antineutrino. To explain what an antineutrino is, let us first consider the following decay:



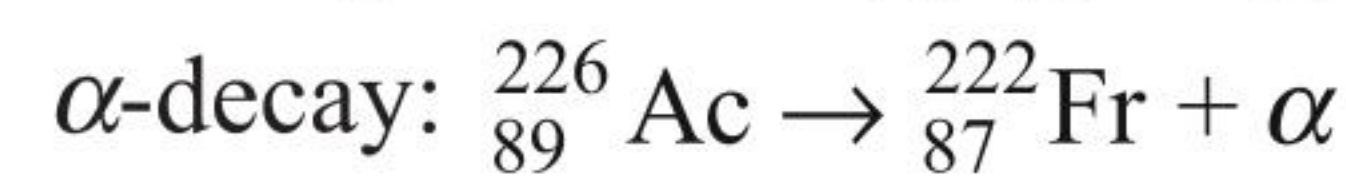
Here, we see that when ^{12}N decays into ^{12}C , a particle is produced that is identical to the electron except that it has a positive charge of $+e$. This particle is called a positron. Because it is like the electron in all respects except charge, the positron is said to be the antiparticle to the electron. We shall discuss antiparticles further in the chapter. For now, suffice it to say that

in beta decay, an electron and an antineutrino are emitted or a positron and a neutrino are emitted.

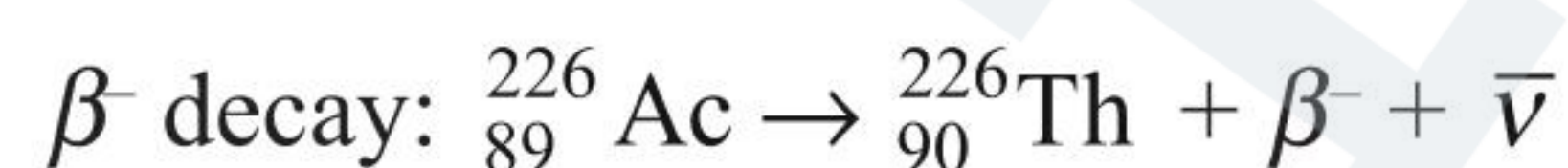
ILLUSTRATION 7.22

Find whether α -decay or any of the β -decay are allowed for $^{226}_{89}\text{Ac}$.

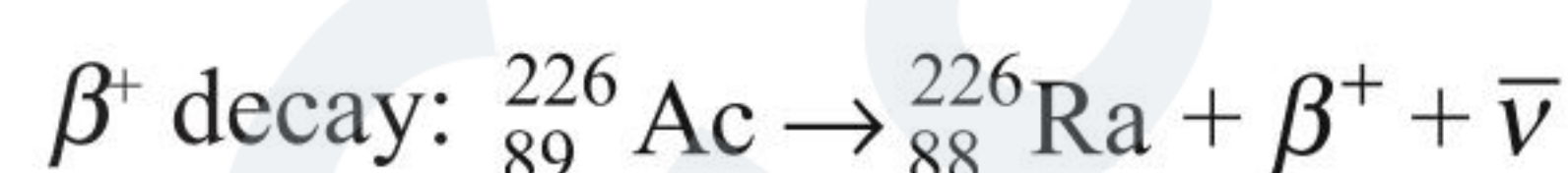
Sol. Our first step will be to write the reaction, then find the disintegration energy Q . If $Q > 0$, the decay is allowed.



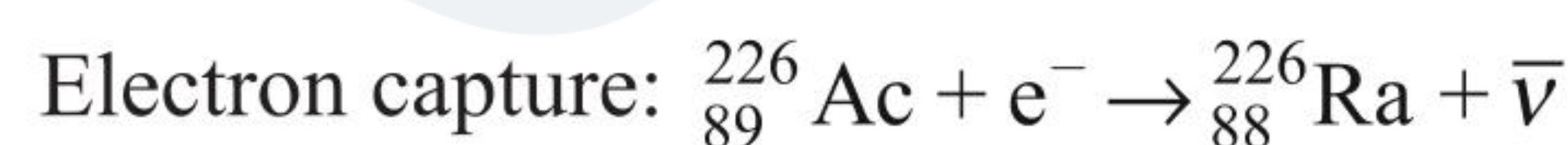
$$Q = [M(^{226}_{89}\text{Ac}) - M(^{222}_{87}\text{Fr}) - M(^4_2\text{He})]c^2 = 5.50 \text{ MeV} \quad (\alpha \text{ decay is allowed})$$



$$Q = [M(^{226}_{89}\text{Ac}) - M(^{226}_{90}\text{Th})]c^2 = 1.12 \text{ MeV} \quad (\beta^- \text{ decay is allowed})$$



$$Q = [M(^{226}_{89}\text{Ac}) - M(^{226}_{88}\text{Ra}) - 2m_e]c^2 = -0.38 \text{ MeV} \quad (\beta^+ \text{ decay is not allowed})$$



$$Q = [M(^{226}_{89}\text{Ac}) - M(^{226}_{88}\text{Ra})]c^2 = 0.64 \text{ MeV} \quad (\text{Electron capture is allowed})$$

ILLUSTRATION 7.23

The nucleus $^{23}_{10}\text{Ne}$ decays by β^- -emission. Write down the β^- decay equation and determine the maximum kinetic energy of the electrons emitted from the following data:

$$m(^{23}_{10}\text{Ne}) = 22.994466 \text{ u}, \quad m(^{23}_{11}\text{Na}) = 22.989770 \text{ u}$$

Sol. The equation representing the β^- -emission of $^{23}_{10}\text{Ne}$ is as follows: $^{23}_{10}\text{Ne} \rightarrow ^{23}_{11}\text{Na} + ^0_{-1}\text{e} + \bar{\nu} + Q$

Ignoring the rest mass of antineutrino ($\bar{\nu}$),

$$Q = [m_N(^{23}_{10}\text{Ne}) - m_N(^{23}_{11}\text{Na}) - m_e]c^2$$

If the mass defect due to electronic binding energy is ignored. We can replace the nuclear masses (m_N) by the respective atomic masses (m). That is,

$$Q = [\{m(^{23}_{10}\text{Ne}) - 10m_e\} - \{m(^{23}_{11}\text{Na}) - 11m_e\}]c^2$$

$$\text{or } Q = [m(^{23}_{10}\text{Ne}) - m(^{23}_{11}\text{Na})]c^2 = [22.994466 \text{ u} - 22.989770 \text{ u}]c^2$$

$$\text{or } Q = (0.004696 \text{ u}) \times 931.5 \text{ MeV} = 4.37 \text{ MeV} \quad (\text{as } 1 \text{ u } c^2 = 931.5 \text{ MeV})$$

This energy is shared, mainly by e^- and $\bar{\nu}$ as $^{23}_{10}\text{Ne}$ is far heavier than both these particles. Further, the energy carried by the electron is maximum (i.e., 4.37 MeV) and this happens when the neutrino carries no energy.

γ -DECAY

Very often a nucleus that undergoes radioactive decay is left in an excited energy state. The nucleus can then undergo a second

decay to a lower energy state, perhaps to the ground state, by emitting one or more high-energy photons. The process is very similar to the emission of light by an atom. An atom emits radiation to release some extra energy when an electron “jumps” from a state of higher energy to a state of lower energy. Likewise, the nucleus uses essentially the same method to release any extra energy it may have following a decay or some other nuclear event. The photons emitted in such a de-excitation process are called gamma rays, which have very high energy relative to the energy of visible light.

A nucleus may reach an excited state as the result of a violent collision with another particle. However, it is more common for a nucleus to be in an excited state as a result of alpha or beta decay. The following sequence of events represents a typical situation in which gamma decay occurs:



Equation (i) represents beta decay in which ${}^{12}\text{B}$ decays to ${}^{12}\text{C}^*$ where the asterisk indicates that the carbon nucleus is left in an excited state following the decay. The excited carbon nucleus then decays to the ground state by emitting a gamma ray, as indicated by Eq. (ii). Note that gamma emission does not result in any change in either Z or A .

ILLUSTRATION 7.24

What is the wavelength of the 0.186 MeV γ -ray photon emitted by radium ${}^{226}_{88}\text{Ra}$?

Sol. The photon energy is the difference between two nuclear energy levels. Equation $E_i - E_f = hf$ gives the relation between the energy level separation ΔE and the frequency f of the photon as $\Delta E = hf$. Since $f\lambda = c$, the wavelength of the photon is $\lambda = hc/\Delta E$.

First, we must convert the photon energy into joule:

$$\Delta E = (0.186 \times 10^6 \text{ eV}) \left(\frac{1.60 \times 10^{-19}}{1 \text{ eV}} \right) = 2.98 \times 10^{-14} \text{ J}$$

The wavelength of the photon is

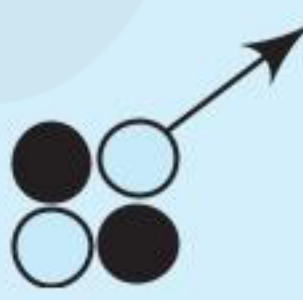
$$\lambda = \frac{hc}{\Delta E} = \frac{(6.63 \times 10^{-34} \text{ Js})(3.00 \times 10^8 \text{ m s}^{-1})}{2.98 \times 10^{-14} \text{ J}} = 6.67 \times 10^{-12} \text{ m}$$

STABILITY AND NEUTRON-PROTON RATIO (n/p)

Stable nuclei have the ratio n/p (neutron/proton) either equal to 1 or more than 1. The ratio is nearly 1 for light nuclei upto calcium (${}^{40}_{20}\text{Ca}$). It increases up to 1.6 for heavy nuclei. The following table shows n/p ratio for some stable nuclei.

Isotope	n	p	n/p
${}^{12}_6\text{C}$	6	6	1
${}^{14}_7\text{N}$	7	7	1

${}^{16}_8\text{O}$	8	8	1
${}^{20}_{10}\text{Ne}$	10	10	1
${}^{40}_{20}\text{Ca}$	20	20	1
${}^{64}_{30}\text{Zn}$	34	30	1.13
${}^{90}_{40}\text{Zr}$	50	40	1.40
${}^{202}_{80}\text{Hg}$	122	80	1.53

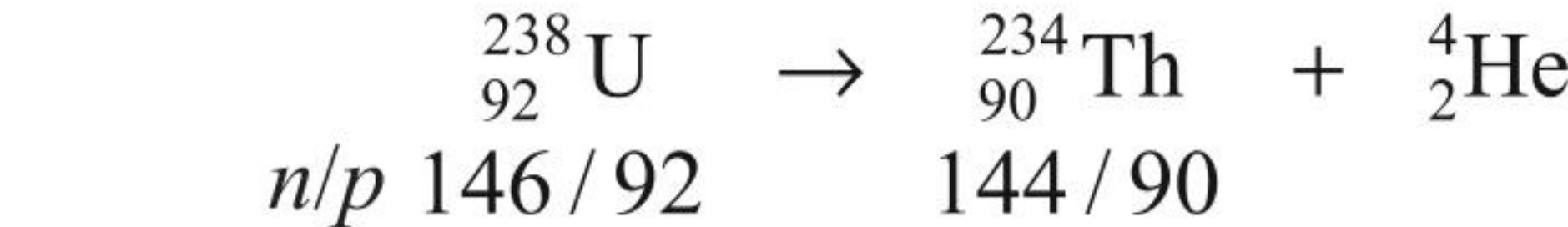
	Decay event	Reason for instability
Gamma decay	Emission of gamma ray reduces energy of nucleus	Nucleus has excess energy
Alpha decay	 Emission of alpha particle reduces size of nucleus	Nucleus too large
Beta decay	 Emission of electron by neutron in nucleus changes the neutron to a proton	Nucleus has too many neutrons relative to number of protons
Electron capture	 Capture of electron by proton in nucleus changes the proton to a neutron	Nucleus has too many protons relative to number of neutrons
Positron emission	 Emission of positron by proton in nucleus changes the proton to a neutron	Nucleus has too many protons relative to number of neutrons

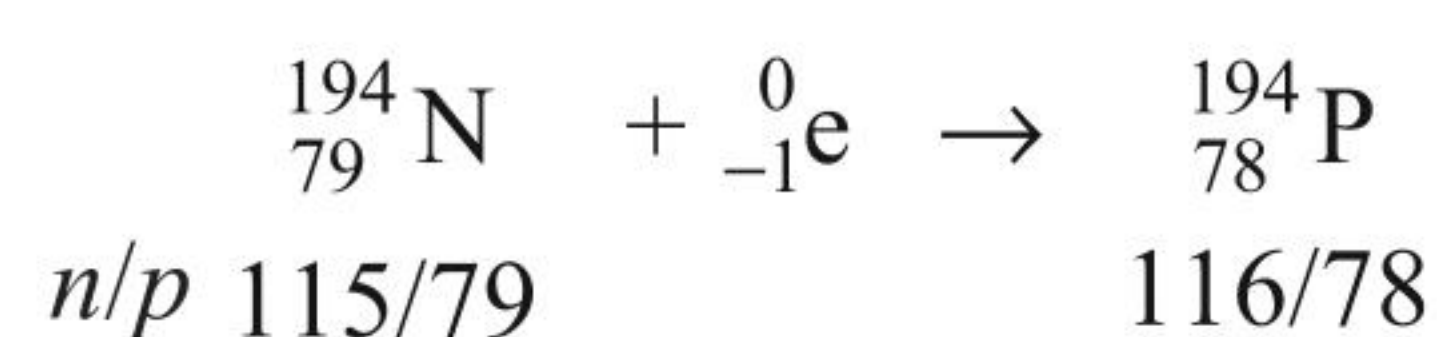
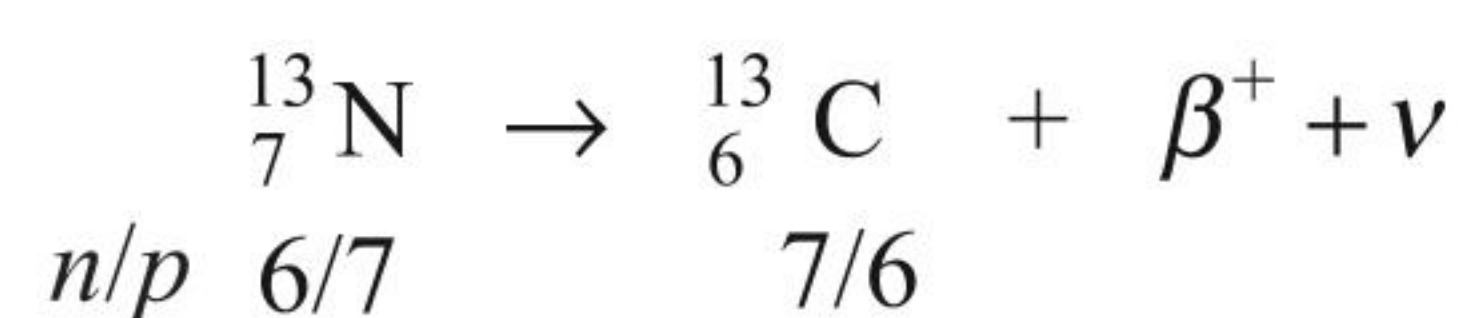
-  Proton (charge = +e)

 Electron (charge = -e)
-  Neutron (charge = 0)

 Positron (charge = +e)

1. When n/p ratio is higher than that required for stability, β -emission takes place. It decrease n/p ratio, e.g., when ${}^{14}_6\text{C}$ decays to ${}^{14}_7\text{N}$, and n/p ratio decreases from 1.33 to 1.
$${}^{14}_6\text{C} \rightarrow {}^{14}_7\text{N} + \beta^- + \bar{\nu}$$
2. When n/p ratio is less than that required for stability, the nuclei can either emit α -particle, emit β^+ or undergo K-electron capture.





CONCEPT APPLICATION EXERCISE 7.2

1. A uranium nucleus (atomic number 92, mass number 231) emits an α -particle and the resultant nucleus emits a β -particle. What are the atomic and mass numbers of the final nucleus?
2. A radioactive nucleus undergoes a series of decay according to the scheme

$$A \xrightarrow{\alpha} A_1 \xrightarrow{\beta} A_2 \xrightarrow{\gamma} A_3 \rightarrow A_4$$

If the mass number and atomic number of A are 180 and 72, respectively, what are these number for A_4 ?
3. A nucleus, absorbing a neutron, emits an electron to go over to neptunium which on further emitting an electron goes over to plutonium. How would you represent the resulting plutonium?
4. A uranium nucleus U-238 of atomic number 92 emits two α -particles and two β -particles and transforms into a thorium nucleus. What is the mass number and atomic number of the thorium nucleus so produced?
5. How many electrons, protons, and neutrons are there in 12 g of ${}^{12}_6\text{C}$ and in 14 g of ${}^{14}_6\text{C}$?
6. Determine the product of the reaction: ${}^7_3\text{Li} + {}^4_2\text{He} \rightarrow ? + n$
7. How many alpha and beta particles are emitted when uranium (${}^{238}_{92}\text{U}$) decays to lead ${}^{206}_{82}\text{Pb}$?
8. An α -particle strikes an aluminium atom ${}^{27}_{13}\text{Al}$. As a result, an unknown nucleus ${}^A_Z\text{X}$ and a neutron are formed.

$${}^4_2\text{He} + {}^{27}_{13}\text{Al} \rightarrow {}^A_Z\text{X} + {}^1_0\text{n}$$

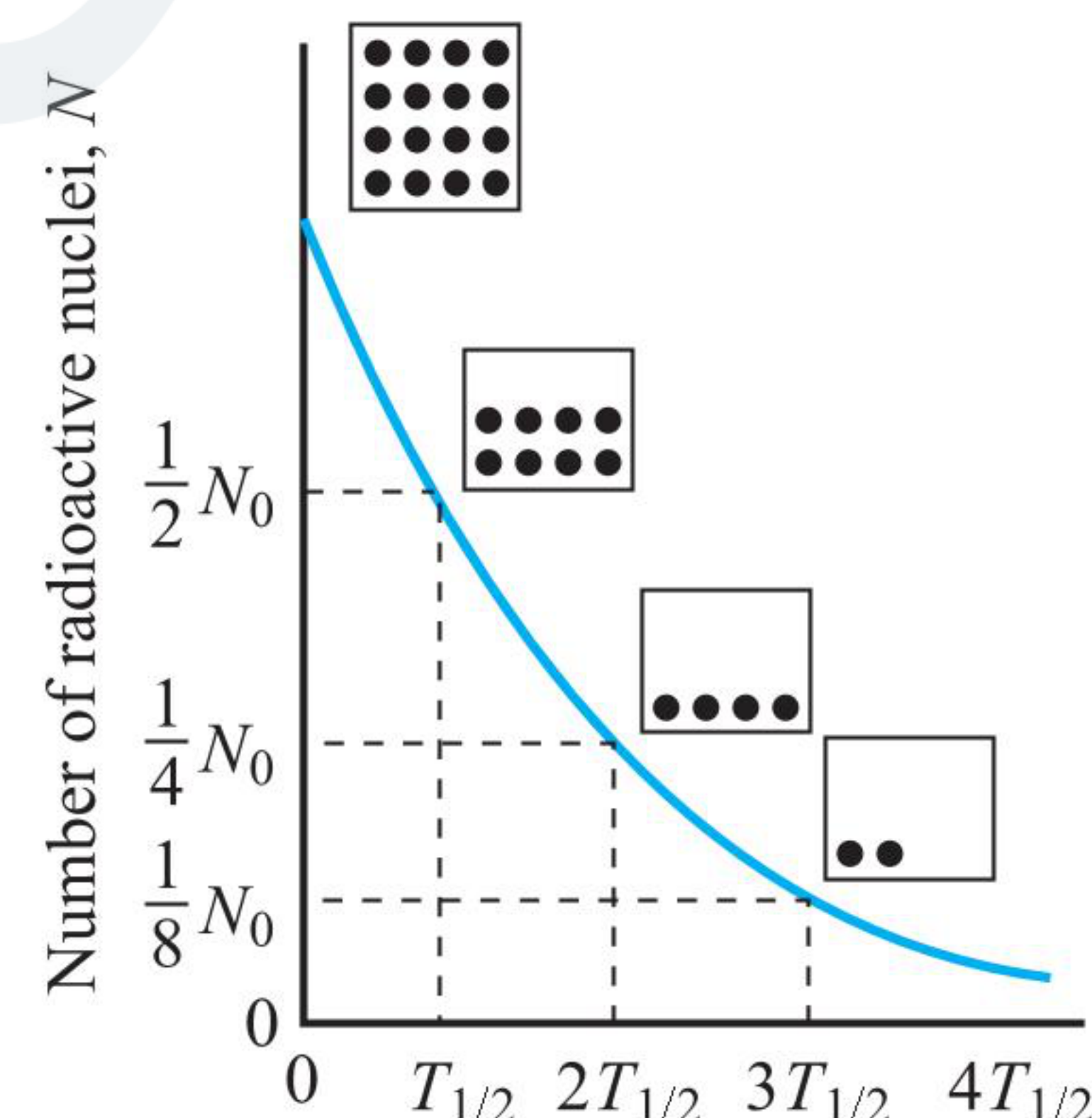
Identify the nucleus produced, indicating its atomic number Z (the number of protons) and its mass number A (the number of nucleons).
9. A neutron is observed to strike an ${}^{16}_8\text{O}$ nucleus and a deuteron is given off. What is the nucleus that result?
10. A ${}^{238}_{92}\text{U}$ undergoes alpha decay. What is the resulting daughter nucleus?
11. Thorium ${}^{238}_{90}\text{Th}$ produces a daughter nucleus that is radioactive. The daughter, in turn, produces its own radioactive daughter and so on. This process continues until bismuth ${}^{812}_{83}\text{Bi}$ is reached. What are the total number N_α of α -particles and the total number N_β of β -particles that are generated in this series of radioactive decays?

ANSWERS

- | | | | | |
|------------|---------------------------|-----------------------------|------------------------------|------------------------|
| 1. 91, 234 | 2. 69 | 3. ${}^{239}_{94}\text{Pu}$ | 4. 230, 90 | 5. ${}^{10}_5\text{B}$ |
| 7. 8, 6 | 8. ${}^{30}_{15}\text{P}$ | 9. ${}^{15}_7\text{N}$ | 10. ${}^{234}_{90}\text{Th}$ | 11. 4, 1 |

RADIOACTIVE DECAY AND ACTIVITY

The question of which radioactive nucleus in a group of nuclei disintegrates at a given instant is decided like the drawing of numbers in a state lottery; individual disintegrations occur randomly. As time passes, the number N of parent nuclei decreases, as figure shows. This graph of N versus time indicates that the decrease occurs in a smooth fashion, with N approaching zero after enough time has passed. To help describe the graph, it is useful to define the half-life $T_{1/2}$ of a radioactive isotope as the time required for one-half of the nuclei present to disintegrate. For example, radium ${}^{226}_{88}\text{Ra}$ has a half-life of 1600 years, for it takes this amount of time for one-half of a given quantity of this isotope to disintegrate. In another 1600 years, one-half of the remaining radium atoms will disintegrate, leaving only one-fourth of the original number intact. In the figure, the number of nuclei present at time $t = 0$ s is $N = N_0$, while the number present at $t = T_{1/2}$ is $N = \frac{1}{2}N_0$. The number present at $t = 2T_{1/2}$ is $N = \frac{1}{4}N_0$ and so forth. The value of the half-life depends on the nature of the radioactive nucleus.



The half-life $T_{1/2}$ of a radioactive decay is the time in which one-half of the radioactive nuclei disintegrate.

MEASUREMENT OF RADIOACTIVITY

Radioactivity of an element is measured in terms of “activity”. The activity of a sample of any radioactive nuclide is the rate at which the nuclei of its constituent atoms disintegrate. If N are the number of nuclei present in a radioactive sample at an instant, then activity of this sample is given as

$$A_c = -\frac{dN}{dt} \quad \dots(i)$$

Here, dN/dt is negative as with time the number of elements always decreases due to disintegration; due to negative sign, A_c is always taken positive. The activity of a substance is measured in terms of “dps” or disintegrations per second. The SI unit of activity is named after Becquerel, defined as

$$1 \text{ becquerel} = 1 \text{ Bq} = 1 \text{ disintegration/second}$$

Generally, activities of radioactive samples in nature are very high. That is why bequerel being a very small unit, in normal practice, more often MBq or GBq is used. For the same traditional units, Curie (Ci) and Rutherford (Rd) are also used. These are defined as

$$1 \text{ Ci} = 3.7 \times 10^{10} \text{ disintegrations s}^{-1}$$

$$1 \text{ Rd} = 10^6 \text{ disintegrations s}^{-1}$$

Curie was originally defined as roughly the activity of 1 g of $^{226}_{88}\text{Ra}$. Similarly, it was observed that 1 kg of ordinary potassium has an activity of about 1 mCi (10^{-3} Ci) because in ordinary potassium small proportion of radioisotope $^{40}_{19}\text{K}$ is also present.

FUNDAMENTAL LAWS OF RADIOACTIVITY

On the basis of experiments performed by Rutherford and Soddy, some conclusions were made for behaviour radioactive elements and the properties of radioactivity. These conclusions are summarized as fundamental laws of radioactivity. These are discussed as follows.

1. Radioactivity is purely a nuclear process. It is not concerned in any manner with the electrons orbiting the nucleus.
2. As radioactivity is a nuclear process, it is independent from any chemical property of the element. As we have discussed that radioactive property of an element is only the process concerned with nucleus of the element, it does not affect the electronic configuration of the element. If this element takes part in a chemical reaction, the product formed will also have the radioactive property in the same fraction by which the radioactive atom is present in the molecule of product.
3. As radioactivity is a random process, its study is only possible by laws of probability. In a group of several radioactive atoms, which one will disintegrate first is just a matter of chance.
4. As radioactivity is a random process, the disintegration density throughout the volume of a radioactive element remains constant. If an element X decays to a daughter nuclide Y, then in a given volume of the element, all portions of volume will have same ratio of number of atoms of Y to that of X. Thus, homogeneity is maintained.

Thus, due to randomness, the amount of disintegrations per unit volume per second (called disintegration density) remains approximately constant in the whole volume of the substance.

RADIOACTIVE DECAY LAW

This law relates the activity of a substance with the number of active or undecayed atoms present in a group of radioactive atoms at an instant of time. This law is stated as follows.

“The activity of a radioactive element at any instant is directly proportional to the number of undecayed active atoms (parent atoms) present at that instant.”

Let us consider that at $t = 0$, there are N_0 parent atoms in a substance and after a time t , N atoms are left undecayed. This implies that in the duration from $t = 0$ to $t = t$, $N_0 - N$ atoms are decayed to their daughter element. If in further time from $t = t$ to $t = t + dt$, dN more atoms will decay then at time $t = t$, we can say that the activity of the element is given as

$$A_c = \left| \frac{dN}{dt} \right| \quad \dots(i)$$

Now, according to radioactive decay law, we have

$$\left| \frac{dN}{dt} \right| \propto N \quad \text{or} \quad A_c = \left| \frac{dN}{dt} \right| = \lambda N \quad \dots(ii)$$

Here, λ is the proportionality constant, called decay constant for the decay process. The value of decay constant differs for different elements. From Eq. (ii), we can see that if λ is high the element will have high value of activity and if λ is less, the activity will be relatively less. Thus, we can say that the decay constant for a radioactive element gives a relative criteria of its stability as well as rate of reaction. If the value of λ for an element is more, it is more active or relatively less stable and if for an element λ is less, it is more stable. From Eq. (ii), we can also write

$$\lambda = \frac{\left| \frac{dN}{dt} \right|}{N} \quad \dots(iii)$$

Thus, decay constant of a process can be given as “activity per atom” (as given in Eq. (iii)). This shows that for a given radioactive element the activity per atom always remains constant whereas we have already discussed that with time the overall activity of a substance decreases with time as the number of parent elements continuously decreases with time.

Now, from Eq. (iii), we can write

$$\frac{dN}{dt} = -\lambda N \quad \dots(iv)$$

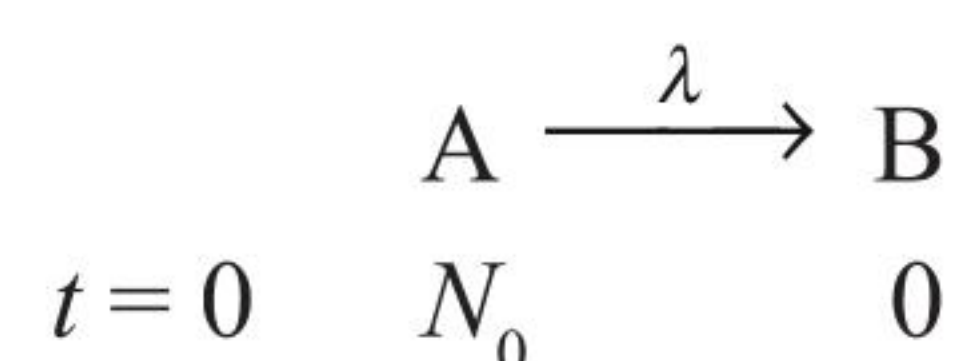
Here, negative sign shows that $\left| \frac{dN}{dt} \right|$, the rate at which the activity elements are disintegrating, is negative or number of active elements are decreasing with time. Now, we have from Eq. (iv),

$$\frac{dN}{N} = -\lambda dt$$

Decay constant is different for different radioactive substances, but it does not depend on the amount of substance and time. SI unit of λ is s^{-1} .

If $\lambda_1 > \lambda_2$, then the first substance is more radioactive (less stable) than the second one.

For the case, if A decays to B with decay constant λ



where N_0 = number of active nuclei of A at $t = 0$.

$$\begin{array}{ccc} t = t & N & N' \end{array}$$

where N = Number of active nuclei of A at $t = t$.

Integrating the above expression within time limits from $t = 0$ to $t = t$, we have

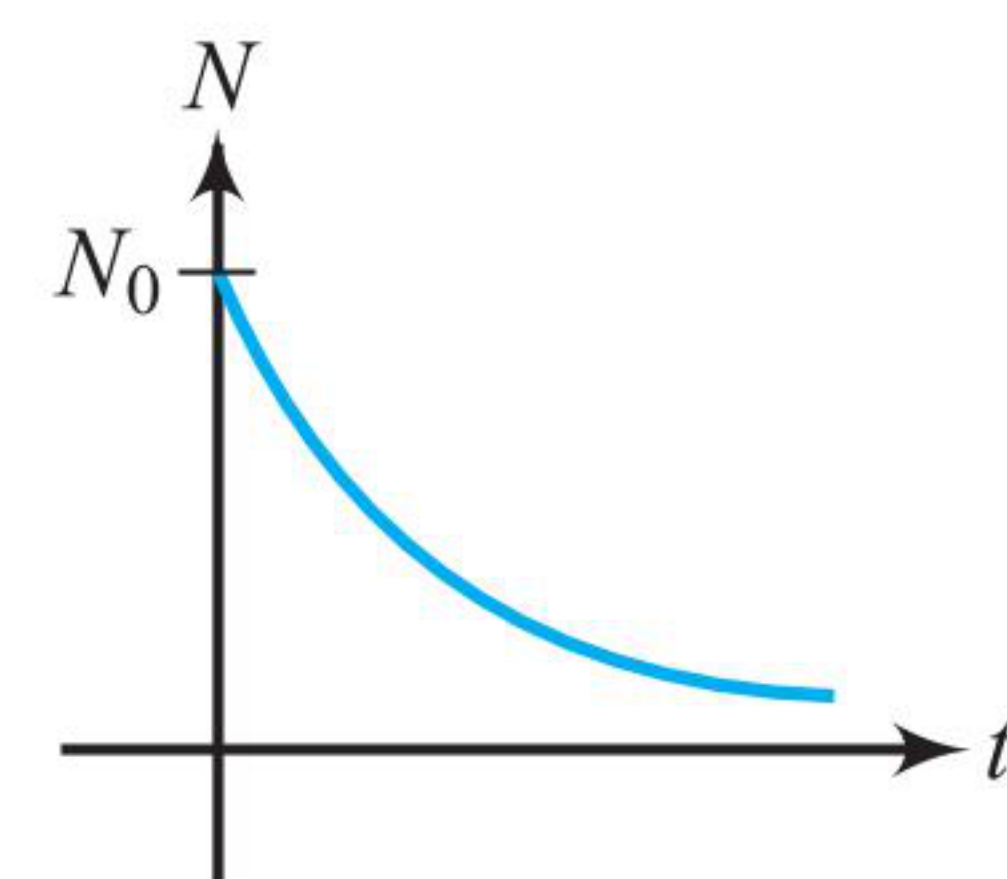
$$\int_{N_0}^N \frac{dN}{N} = -\int_0^t \lambda dt \quad \text{or} \quad [\ln N]_{N_0}^N = -\lambda t$$

$$\text{or} \quad \ln \left(\frac{N}{N_0} \right) = -\lambda t \quad \text{or} \quad N = N_0 e^{-\lambda t} \quad \dots(v)$$

Equation (v) gives the number of active parent atoms N present at time t in the mixture. This equation is called radioactive decay equation.

From Eq. (v), we can have

$$\begin{aligned} \lambda N &= \lambda N_0 e^{-\lambda t} \\ \text{or} \quad A_c &= A_{c_0} e^{-\lambda t} \quad \dots(vi) \end{aligned}$$

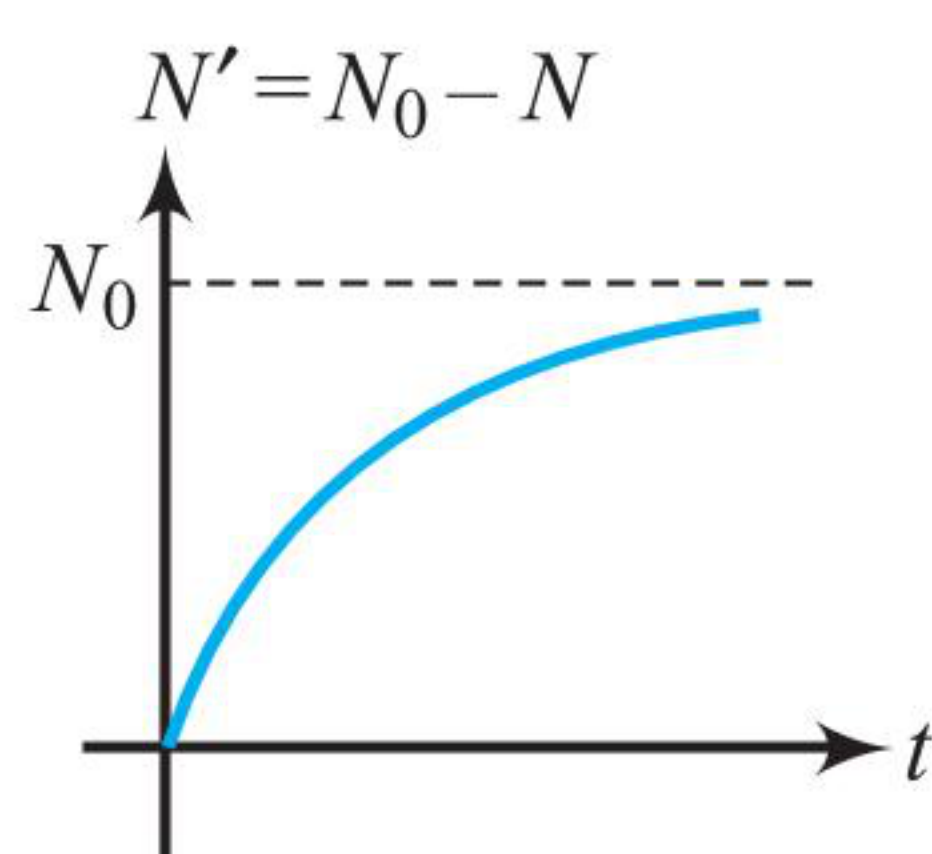


Here, $A_{c_0} = \lambda N_0$ is the initial activity of the substance at $t = 0$. Equation (vi) is another form of radioactive decay equation. This equation can be used to find the activity of a radioactive substance at any instant of time.

Number of nuclei decayed (i.e., the number of nuclei of B formed),

$$N' = N_0 - N = N_0 - N_0 e^{-\lambda t}$$

$$\text{or } N' = N_0(1 - e^{-\lambda t})$$



ACTIVITY

The activity of a radioactive sample is the number of disintegrations per second that occur. Each time a disintegration occurs, the number N of radioactive nuclei decreases. As a result, the activity can be obtained by dividing ΔN (the change in the number of nuclei) by Δt (the time interval during which the change takes place); the average activity over the time interval Δt is the magnitude of $\Delta N/\Delta t$. Since the decay of any individual nucleus is completely random, the number of disintegrations per second that occur in a sample is proportional to the number of radioactive nuclei present, so that

$$\frac{\Delta N}{\Delta t} = -\lambda N$$

where λ is a proportionality constant referred to as the decay constant. The minus sign is present in this equation because each disintegration decreases the number N of nuclei originally present.

The SI unit for activity is the becquerel (Bq); one becquerel equals one disintegration per second. Activity is also measured in terms of a unit called the curie (Ci), in honour of Marie Curie (1867–1934) and Pierre Curie (1859–1906), the discoverers of radium and polonium. Historically, the curie was chosen as a unit because it is roughly the activity of one gram of pure radium. In terms of becquerels,

$$1 \text{ Ci} = 3.70 \times 10^{10} \text{ Bq}$$

The activity of the radium put into the dial of a watch to make it glow in the dark is about 4×10^4 Bq, and the activity used in radiation therapy for cancer treatment is approximately a billion times greater, or 4×10^{13} Bq.

HALF-LIFE TIME

In the previous section, we have discussed that during decay of a radioactive sample, the amount of radionuclide falls off exponentially with time. Every radioactive sample has a characteristic half-life. Half-life time is defined as the time duration in which half of the total number of nuclei will decay or be left undecayed. Say, for example, at any instant if we look into the quantity of a sample of radioactive element, it is observed that after every 3 hours some half-lives are only a millionth of a second for highly active elements and some less active elements have half-life in billions of years.

In some nuclear power plants, a major problem is disposal of the radioactive wastes since some of the nuclides present in waste have long half-lives.

For a radioactive element in a sample if at $t = 0$, N_0 nuclei are present of active parent element and during observation after

$t = T$, $N_0/2$ are left, then this duration T can be taken as half-life of this element. From radioactive decay equation, we have

$$N = N_0 e^{-\lambda t} \quad \dots(i)$$

Here, at $t = T$, $N = N_0/2$. Thus, we have in Eq. (i)

$$\frac{N_0}{2} = N_0 e^{-\lambda T} \quad \text{or} \quad \ln\left(\frac{1}{2}\right) = -\lambda T$$

$$\text{or } \ln(2) = \lambda T$$

$$\text{or } T = \frac{\ln(2)}{\lambda} = \frac{0.693}{\lambda} \quad \dots(ii)$$

Number of nuclei present after n half-lives, i.e., after a time $t = nT_{1/2}$,

$$N = N_0 e^{-\lambda t} = N_0 e^{-\lambda n T_{1/2}} = N_0 e^{-\lambda n \frac{\ln 2}{\lambda}} \\ = N_0 e^{\ln 2^{(-n)}} = N_0 (2)^{-n} = N_0 (1/2)^n = \frac{N_0}{2^n}$$

$$\left[n = \frac{t}{T_{1/2}}. \text{ It may be a fraction, need not to be an integer.} \right]$$

$$\text{or } N_0 \xrightarrow[\text{half life}]{\text{after 1st}} \frac{N_0}{2} \xrightarrow{2} N_0 \left(\frac{1}{2}\right)^2 \xrightarrow{3} N_0 \left(\frac{1}{2}\right)^3 \\ \dots \xrightarrow{n} N_0 \left(\frac{1}{2}\right)^n$$

ILLUSTRATION 7.25

A radioactive sample has 6.0×10^{18} active nuclei at a certain instant. How many of these nuclei will still be in the same active state after two half-lives?

Sol. In one half-life, the number of active nuclei reduces to half the original number. Thus, in two half-lives the number is reduced to $\left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right)$ of the original number. The number of remaining active nuclei is, therefore,

$$N_{\text{active}} = 6.0 \times 10^{18} \times \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) = 1.5 \times 10^{18}$$

ILLUSTRATION 7.26

The half-life of a radioactive nuclide is 20 h. What fraction of original activity will remain after 40 h?

Sol. 40 h means 2 half-lives. Thus,

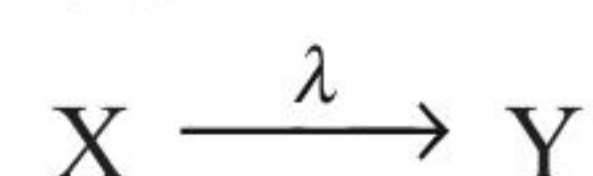
$$A = \frac{A_0}{2^2} = \frac{A_0}{4} \quad \text{or} \quad \frac{A}{A_0} = \frac{1}{4}$$

So, one-fourth of the original activity will remain after 40 h.

Specific activity: The activity per unit mass is called specific activity.

ALTERNATIVE FORM OF DECAY EQUATION IN TERMS OF HALF-LIFE TIME

Let a radioactive element X decays to a daughter nucleus Y with a decay process nuclear reaction written as



If initially N_0 nuclei of element X are present, then after time t number of nuclei present in the sample is given by decay equation given as

$$N = N_0 e^{-\lambda t} \quad \dots(i)$$

If we rearrange the equation, we have $\ln \frac{N}{N_0} = -\lambda T$

We know that half-life of a substance is defined as $T = \frac{\ln(2)}{\lambda}$

or we can write $\lambda = \frac{\ln(2)}{T}$

Therefore, we have $\ln \left(\frac{N}{N_0} \right) = -\frac{\ln(2)}{T} t$ or $\ln \left(\frac{N}{N_0} \right) = \ln(2)^{-t/T}$

Taking antilog on both sides, we get $\frac{N}{N_0} = (2)^{-t/T}$ or $N = N_0 (2)^{-t/T}$

This equation is an alternate form of decay equation useful for numerical applications.

ILLUSTRATION 7.27

Suppose 3.0×10^7 radon atoms are trapped in a basement at a given time. The basement is sealed against further entry of the gas. The half-life of radon is 3.83 days. How many radon atoms remain after 31 days?

Sol. During each half-life, the number of radon atoms is reduced by a factor of two. Thus, we determine the number of half-lives in a period of 31 days and reduce the number of radon atoms by a factor of two for each one.

In a period of 31 days, there are $31 \text{ days} / 3.83 \text{ days} = 8.1$ half-lives. In 8 half-lives, the number of radon atoms is reduced by a factor of $2^8 = 256$. Ignoring the difference between 8 and 8.1 half-lives, we find that the number of atoms remaining is $3.0 \times 10^7 / 256 = 1.2 \times 10^5$.

ILLUSTRATION 7.28

In the above illustration, suppose there are 3.0×10^7 radon atoms ($T_{1/2} = 3.83$ days or 3.31×10^5 s) trapped in a basement.

(a) How many radon atoms remain after 31 days? (b) Find the activity (i) just after the basement is sealed against further entry of radon and (ii) 31 days later.

Sol. The number N of radon atoms remaining after a time t is given by $N = N_0 e^{-\lambda t}$, where $N_0 = 3.0 \times 10^7$ is the original number of atoms when $t = 0$ s and λ is the decay constant. The decay constant is related to the half-life $T_{1/2}$ of the radon atoms by $\lambda = 0.693 / T_{1/2}$. The activity can be obtained from the equation $\Delta N / \Delta t = -\lambda N$.

(a) The decay constant is

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{3.83 \text{ days}} = 0.181 \text{ days}^{-1} = 0.181 \text{ days}^{-1} \dots(i)$$

and the number N of radon atoms remaining after 31 days is

$$N = N_0 e^{-\lambda t} = (3.0 \times 10^7) \exp \{-(0.181 \text{ days}^{-1})(31 \text{ days})\} = 1.1 \times 10^5 \quad \dots(ii)$$

This value is slightly different from that found in Illustration 7.27 because there we ignored the difference between 8.0 and 8.1 half-lives.

(b) (i) The activity can be obtained from equation $\Delta N / \Delta t = -\lambda N$, provided the decay constant is expressed in reciprocal seconds: $\lambda = 0.693 / (3.31 \times 10^5 \text{ s}) = 2.09 \times 10^{-6} \text{ s}^{-1}$. Thus, the number of disintegrations per second is

$$\begin{aligned} \frac{\Delta N}{\Delta t} &= -\lambda N = -(2.09 \times 10^{-6} \text{ s}^{-1})(3.0 \times 10^7) \\ &= -63 \text{ disintegrations s}^{-1} \end{aligned}$$

The activity is the magnitude of $\Delta N / \Delta t$, so initially

$$\text{Activity} = 63 \text{ Bq}$$

(ii) From part (a), the number of radioactive nuclei remaining at the end of 31 days is $N = 1.1 \times 10^5$, and reasoning similar to that in part (b) reveals that

$$\text{Activity} = 0.23 \text{ Bq}$$

SI unit of activity is becquerel (Bq) which is same as 1 dps (disintegrations per second).

The popular unit of activity is curie which is defined as

$$1 \text{ curie} = 3.7 \times 10^{10} \text{ dps} \quad (\text{which is activity of 1 g radium})$$

ILLUSTRATION 7.29

The decay constant for the radioactive nuclide ^{64}Cu is $1.516 \times 10^{-5} \text{ s}^{-1}$. Find the activity of a sample containing 1 mg of ^{64}Cu . Atomic weight of copper = 63.5 g mol^{-1} . Neglect the mass difference between the given radioisotope and normal copper.

Sol. 63.5 g of copper has 6×10^{23} atoms. Thus, the number of atoms in 1 mg of Cu is

$$N = \frac{6 \times 10^{23} \times 1 \mu\text{g}}{63.5 \text{ g}} = 9.45 \times 10^{15}$$

$$\text{Activity} = \lambda N$$

$$= (1.516 \times 10^{-5} \text{ s}^{-1}) \times (9.45 \times 10^{15})$$

$$= 1.43 \times 10^{11} \text{ disintegrations s}^{-1}$$

$$= \frac{1.43 \times 10^{11}}{3.7 \times 10^{10}} \text{ Ci} = 3.86 \text{ Ci}$$

Activity after n half-lives is $A_0 / 2^n$.

AVERAGE LIFE

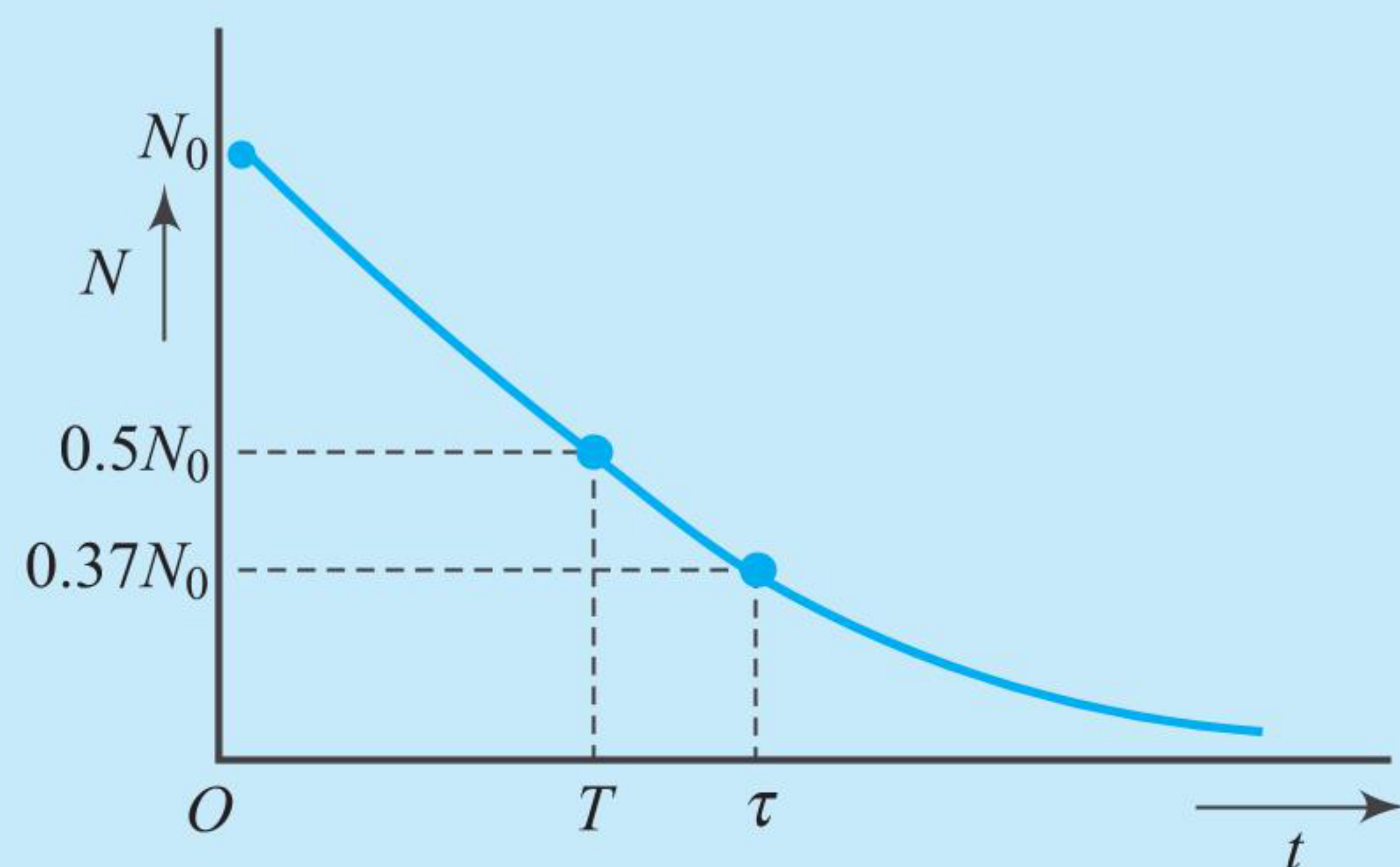
$$T_{\text{avg}} = \frac{\text{Sum of ages of all the nuclei}}{N_0} = \frac{\int_0^\infty N_0 e^{-\lambda t} dt}{N_0} = \frac{1}{\lambda}$$

Important Points:

- Time constant or average life is more than the half-life, i.e.,

$$\tau = T_{\text{av}} > T$$

- The fractional radioactive decay in one mean life (or time constant) is more than that in half life, as shown in figure.



- The number of undecayed nuclei present after n mean life is

$$N = (0.37)^n N_0 = \left(\frac{1}{e}\right)^n N_0$$

- If a nuclide can decay simultaneously by two different processes which have decay constants λ_1 and λ_2 , then the effective decay constant of the nuclide is

$$\lambda = \lambda_1 + \lambda_2; N = N_0 e^{-(\lambda_1 + \lambda_2)t}$$

ILLUSTRATION 7.30

The half-life of ^{198}Au is 2.7 days. Calculate (a) the decay constant, (b) the average-life, and (c) the activity of 1.00 mg of ^{198}Au . Take atomic weight of ^{198}Au to be 198 g mol^{-1} .

Sol.

- (a) The half-life and the decay constant are related as

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

$$\text{or } \lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{2.7 \text{ days}} = \frac{0.693}{2.7 \times 24 \times 3600 \text{ s}} \\ = 2.9 \times 10^{-6} \text{ s}^{-1}$$

- (b) The average-life is $t_{\text{av}} = \frac{1}{\lambda} = 3.9 \text{ days}$.

- (c) The activity is $A = \lambda N$. Now, 198 g of ^{198}Au has 6×10^{23} atoms. The number of atoms in 1.00 mg of ^{198}Au is

$$N = 6 \times 10^{23} \times \frac{1.0 \text{ mg}}{198 \text{ g}} = 3.03 \times 10^{18}$$

$$\text{Thus, } A = \lambda N = (2.9 \times 10^{-6} \text{ s}^{-1})(3.03 \times 10^{18}) \\ = 8.8 \times 10^{12} \text{ disintegrations s}^{-1} \\ = \frac{8.8 \times 10^{12}}{3.7 \times 10^{10}} \text{ Ci} = 240 \text{ Ci}$$

ILLUSTRATION 7.31

A 50.0 g sample of carbon is taken from the pelvis bone of a skeleton and is found to have a ^{14}C decay rate of $200.0 \text{ decays min}^{-1}$. It is known that carbon from a living organism has a decay rate of $\text{decays/min}^{-1} \text{ g}^{-1}$ and that ^{14}C has a half-life of 5730 year $= 3.01 \times 10^9 \text{ min}$. Find the age of the skeleton.

Sol. Let us start with the equation $N = N_0 e^{-\lambda t}$ and multiply both sides by λ to get $\lambda N = \lambda N_0 e^{-\lambda t}$ which is equivalent to

$$R = R_0 e^{-\lambda t} \quad \text{or} \quad \frac{R}{R_0} = e^{-\lambda t}$$

where R is the present activity and R_0 was the activity when the skeleton was a part of a living organism. We can solve for the time by taking the natural log of both sides of this equation.

$$\ln\left(\frac{R}{R_0}\right) = \ln(e^{-\lambda t}) = -\lambda t$$

$$t = -\frac{\ln\left(\frac{R}{R_0}\right)}{\lambda}$$

Because we are given the decay rate and mass of the sample, we can find R_0 as

$$R_0 = \left(15.0 \frac{\text{decays}}{\text{min g}}\right)(50.0 \text{ g}) = 750 \frac{\text{decays}}{\text{min}}$$

The decay constant is found as

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{3.01 \times 10^9 \text{ min}} = 2.30 \times 10^{-10} \text{ min}^{-1}$$

ILLUSTRATION 7.32

- (a) Determine the number of carbon ^{14}C atoms present for every gram of carbon ^{12}C in a living organism. (b) Find (i) the decay constant and (ii) the activity of this sample.

Sol. The total number of carbon ^{12}C atoms in one gram of carbon ^{12}C is equal to the corresponding number of moles times Avogadro's number. Since there is only one ^{14}C atom for every 8.3×10^{11} atoms of ^{12}C , the number of ^{14}C atoms is equal to the total number of ^{12}C atoms divided by 8.3×10^{11} . The decay constant λ for ^{14}C is $\lambda = 0.693/T_{1/2}$, where $T_{1/2}$ is the half-life. The activity is equal to the magnitude of $\Delta N/\Delta t$, which is the decay constant times the number of ^{14}C atoms present.

- (a) One gram of carbon ^{12}C (atomic mass = 12 u) is equivalent to $1.0/12 \text{ mol}$. Since Avogadro's number is $6.02 \times 10^{23} \text{ atoms mol}^{-1}$ and since there is one ^{14}C atom for every 8.3×10^{11} atoms of ^{12}C , the number of ^{14}C atoms for every 1.0 g of carbon ^{12}C is

$$\left(\frac{1.0}{12} \text{ mol}\right) \left(6.02 \times 10^{23} \frac{\text{atoms}}{\text{mol}}\right) \left(\frac{1}{8.3 \times 10^{11}}\right) \\ = 6.0 \times 10^{10} \text{ atoms}$$

- (b) (i) Since the half-life of ^{14}C is 5730 years ($1.81 \times 10^{11} \text{ s}$), the decay constant is

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{1.81 \times 10^{11} \text{ s}} = 3.83 \times 10^{-12} \text{ s}^{-1}$$

- (ii) The activity is the magnitude of $\Delta N/\Delta t$ or λN . Thus, we find activity of ^{14}C for every 1.0 g of carbon ^{12}C in a living organism $= \lambda N = (3.83 \times 10^{-12} \text{ s}^{-1})(6.0 \times 10^{10} \text{ atoms}) = 0.23 \text{ Bq}$.

An organism that lived thousands of years ago presumably had an activity of about 0.23 Bq per gram of carbon. When the organism died, the activity began decreasing. From a sample of the remains, the current activity per gram of carbon can be

measured and compared to the value of 0.23 Bq to determine the time that has transpired since death.

ILLUSTRATION 7.33

At a given instant, there are 25% undecayed radioactive nuclei in a sample. After 10 s, the number of undecayed nuclei reduces to 12.5%. Calculate (a) mean life of the nuclei and (b) the time in which the number of decayed nuclei will further reduce to 6.25% of the reduced number.

Sol. Half-life of radioactive sample, i.e., the time in which number of undecayed nuclei becomes half (T) is 10 s.

(a) Mean life, $\tau = \frac{T}{\log_e 2} = \frac{10}{0.693} \text{ s} = 1.443 \times 10 = 14.43 \text{ s}$

(b) The reduced number further reduces to 6.25% in n half-lives given by

$$\frac{N}{100} = \left(\frac{1}{2}\right)^n \Rightarrow \frac{6.25}{100} = \left(\frac{1}{2}\right)^n$$

$$\Rightarrow \frac{1}{16} = \left(\frac{1}{2}\right)^n \Rightarrow \left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^n \Rightarrow n = 4$$

Time, $t = 4T = 4 \times 10 = 40 \text{ s}$

ILLUSTRATION 7.34

' m_1 ' g of non-radioactive isotopes X^A are mixed with ' m_2 ' g of the radioactive isotope $X^{A'}$. How much will the specific activity decrease? Half-life of $X^{A'} = T$. Take N_A as Avagadro number.

Sol. Before mixing the two, the specific activity a_1 is given by

$$a_1 = \frac{dN}{m_2 dt} = \frac{\lambda N}{m_2} = \frac{\log_e 2}{T} \times \frac{N_A m_2}{A'} \times \frac{1}{m_2} = \frac{\log_e 2}{T} \times \frac{N_A}{A'}$$

After mixing, the activity a_2 is given by

$$a_2 = \frac{dN}{(m_1 + m_2) dt} = \frac{\log_e 2}{T} \times \frac{N_A m_2}{A'(m_1 + m_2)}$$

$$\therefore \text{Change in activity} = a_1 - a_2 = \frac{(\log_e 2) N_A m_1}{T A' (m_1 + m_2)}$$

ILLUSTRATION 7.35

A stable nuclei C is formed from radioactive nuclei A and B with decay constants of λ_1 and λ_2 , respectively. Initially, the number of nuclei of A is N_0 and that of B is zero. Nuclei B are produced at a constant rate of P . Find the number of nuclei of C after time t .

Sol. $\frac{dN_B}{dt} = P - \lambda_2 N_B$

$$\int_0^{N_2} \frac{dN_B}{P - \lambda_2 N_B} = \int_0^t dt$$

$$\ln \left(\frac{P - \lambda_2 N_B}{P} \right) = -\lambda_2 t$$

$$N_B = P(1 - e^{-\lambda_2 t})$$

The number of nuclei of A after time t is $N_A = N_0 e^{-\lambda_1 t}$.

Thus, $\frac{dN_C}{dt} = \lambda_1 N_A + \lambda_2 N_B = +\lambda_1 N_0 e^{-\lambda_1 t} + P(1 - e^{-\lambda_2 t})$

$$N_C = N_0(1 - e^{-\lambda_1 t}) + P \left(t + \frac{e^{-\lambda_2 t} - 1}{\lambda_2} \right)$$

ILLUSTRATION 7.36

Natural uranium is a mixture of three isotopes ${}_{92}\text{U}^{234}$, ${}_{92}\text{U}^{235}$, and ${}_{92}\text{U}^{238}$ with the shares of negligible 0.0006%, 0.71%, and 99.28%, respectively. The half-life of the three isotopes are 2.5×10^5 years, 7.1×10^8 years, and 4.5×10^9 years, respectively. Determine the share of radioactivity of each isotope into the total activity of the natural uranium.

Sol. Let A_1 , A_2 and A_3 be activities of ${}_{92}\text{U}^{234}$, ${}_{92}\text{U}^{235}$ and ${}_{92}\text{U}^{238}$, respectively. Total activity, $A = A_1 + A_2 + A_3$

Share of ${}_{92}\text{U}^{234}$ is $\frac{A_1}{A} = \frac{\lambda_1 N_1}{\lambda_1 N_1 + \lambda_2 N_2 + \lambda_3 N_3}$

Let m be the total mass of natural uranium. Then,

$$m_1 = \frac{0.006}{100} m, m_2 = \frac{0.71}{100} m, m_3 = \frac{99.28}{100} m$$

Now, $N_1 = \frac{m_1}{M_1}, N_2 = \frac{m_2}{M_2}, N_3 = \frac{m_3}{M_3}$

where M_1, M_2 and M_3 are atomic weights.

$$\therefore \frac{A_1}{A} = \frac{\left(\frac{m_1}{M_1}\right) \frac{1}{T_1}}{\frac{m_1}{M_1 T_1} + \frac{m_2}{M_2 T_2} + \frac{m_3}{M_3 T_3}} = 0.99 \approx 9.9\%$$

$$\frac{A_2}{A} = 0.039 = 3.9\%$$

$$\frac{A_3}{A} = 0.864 = 86.4\%$$

RADIOACTIVE DATING

One important application of radioactivity is the determination of the age of archeological or geological samples. If an object contains radioactive nuclei when it is formed, then the decay of these nuclei marks the passage of time like a clock. If the half-life is known, a measurement of the number of nuclei present today relative to the number present initially can give the age of the sample. A more accurate way is to determine the present number of radioactive nuclei with the aid of a mass spectrometer.

The present activity of a sample can be measured, but how is it possible to know what the original activity was, perhaps thousands of years ago? Radioactive dating methods entail certain assumptions that make it possible to estimate the original activity. For instance, the radiocarbon technique utilizes the ${}^{14}_6\text{C}$ isotope of carbon, which undergoes β^- decay with a half-life of 5730 years. This isotope is present in the earth's atmosphere at an equilibrium concentration of about one atom for every 8.3×10^{11} atoms of normal carbon ${}^{12}_6\text{C}$. It is often assumed that this value has remained constant over the years because ${}^{14}_6\text{C}$ is created when cosmic rays interact with the earth's upper atmosphere, a

production method that offsets the loss via β^- decay. Moreover, nearly all living organisms ingest the equilibrium concentration of $^{14}_6\text{C}$. However, once an organism dies, metabolism no longer sustains the input of $^{14}_6\text{C}$, and β^- decay causes half of the $^{14}_6\text{C}$ nuclei to disintegrate every 5730 years.

ILLUSTRATION 7.37

The number of ^{238}U atoms in an ancient rock equals the number of ^{206}Pb atoms. The half-life of decay of ^{238}U is 4.5×10^9 years. Estimate the age of the rock assuming that all the ^{206}Pb atoms are formed from the decay of ^{238}U .

Sol. Since the number of ^{206}Pb atoms equals the number of ^{238}U atoms, half of the original ^{238}U atoms have decayed. It takes one half-life to decay half of the active nuclei. Thus, the sample is 4.5×10^9 years old.

ILLUSTRATION 7.38

A bottle of red wine is thought to have been sealed about 5 years ago. The wine contains a number of different kinds of atoms, including carbon, oxygen, and hydrogen. Each of these has a radioactive isotope. The radio-active isotope of carbon is the familiar $^{14}_6\text{C}$, with a half-life of 5730 years. The radioactive isotope of oxygen is $^{15}_8\text{O}$ and has a half-life of 122.2 s. The radioactive isotope of hydrogen is ^3_1H and is called tritium; its half-life is 12.33 years. The activity of each of these isotopes is known at the time the bottle was sealed. However, only one of the isotopes is useful for determining the age of the wine accurately. Which is it?

Sol. In a dating method that measures the activity of a radioactive isotope, the age of the sample is related to the change in the activity during the time period in question. Here, the expected age is about 5 years. This period is only a small fraction of the 5730-year half-life of $^{14}_6\text{C}$. As a result, relatively few of the $^{14}_6\text{C}$ nuclei would decay during the wine's life, and the measured activity would change little from its initial value. To obtain an accurate age from such a small change would require prohibitively precise measurements. Nor is the $^{15}_8\text{O}$ isotope very useful. The difficulty is its relatively short half-life of 122.2 s. During a 5-year period, so many half-lives of 122.2 s would occur that the activity would decrease to a vanishingly small level. It would not be even possible to measure it. The only remaining option is the tritium isotope of hydrogen. The expected age of 5 years is long enough relative to the half-life of 12.33 years that a measurable change in activity will occur, but not so long that the activity will have completely vanished for all practical purposes.

ILLUSTRATION 7.39

The normal activity of living carbon-containing matter is found to be about 15 decays per minute for every gram of carbon. Suppose a specimen from Mohenjodaro gives an activity of 9 decays per minute per gram of carbon. Estimate the approximate age of the Indus-Valley civilization from the known half-life (= 5730 years) of C^{14} .

Sol. Let t be the age of the Indus-Valley civilization.

The rate of decay, i.e., R is given by

$$R \propto N \quad \text{or} \quad R = \lambda N$$

$$\text{As } N = N_0 e^{-\lambda t}, R = \lambda N_0 e^{-\lambda t} = R_0 e^{-\lambda t}$$

where $R_0 = \lambda N_0$ is the initial rate of decay.

$$\text{Thus, } \frac{R}{R_0} = e^{-\lambda t} \quad \dots(i)$$

$$\text{We are given that } \frac{R}{R_0} = \frac{9}{15} = \frac{3}{5} \quad \dots(ii)$$

$$\text{From Eqs. (i) and (ii), } e^{-\lambda t} = \frac{3}{5} \quad \text{or} \quad e^{\lambda t} = \frac{5}{3}$$

$$\text{or } \lambda t = \ln\left(\frac{5}{3}\right) \quad \text{or} \quad t = \frac{\ln(5/3)}{\lambda}$$

$$\text{or } t = \frac{\ln(5/3)}{\ln 2} \times T_{1/2} = \frac{\log(5/3)}{\log 2} \times 5730 \text{ yr} \quad \left(\text{as } T_{1/2} = \frac{\ln 2}{\lambda} \right)$$

$$\text{or } t = \frac{0.2219}{0.3010} \times 5730 \text{ yr} = 4224 \text{ yr}$$

NATURAL RADIOACTIVITY: RADIOACTIVE DECAY SERIES

Radioactive nuclei are generally classified into two groups: (1) unstable nuclei found in nature, which give rise to what is called natural radioactivity and (2) nuclei produced in the laboratory through nuclear reactions, which exhibit artificial radioactivity.

Three series of naturally occurring radioactive nuclei exist (as shown in the following table). Each series starts with a specific long-lived radioactive isotope whose half-life exceeds that of any of its descendants. The fourth series in the table begins with ^{237}Np , a transuranic element (on having an atomic number greater than that of uranium) not found in nature. This element has a half-life of “only” 2.14×10^6 years.

Four radioactive series			
Series	Starting isotope	Half-life (year)	Stable end product
Uranium	$^{238}_{92}\text{U}$	4.47×10^9	$^{206}_{82}\text{Pb}$
Actinium	$^{235}_{92}\text{U}$	7.04×10^8	$^{207}_{82}\text{Pb}$
Thorium	$^{232}_{90}\text{U}$	1.41×10^{10}	$^{208}_{82}\text{Pb}$
Neptunium	$^{237}_{93}\text{U}$	2.14×10^6	$^{209}_{83}\text{Pb}$

The two uranium series are somewhat more complex than the ^{232}Th series. Also, several other naturally occurring radioactive elements that would otherwise have disappeared long ago. For example, because the solar system is about 5×10^9 years old, the supply of ^{226}Ra (whose half-life is only 1600 years) would have been depleted by radioactive decay long ago if it were not for the decay series that starts with ^{238}U , with a half-life of 4.47×10^9 years. Figure shows the radioactive decay series that begins with uranium $^{238}_{92}\text{U}$ and ends with lead $^{206}_{82}\text{Pb}$. Half-lives are given in seconds (s), minutes (min), hours (hr),

days (d), or years (yr). The inset in the upper left in figure identifies the type of decay that each nucleus undergoes.

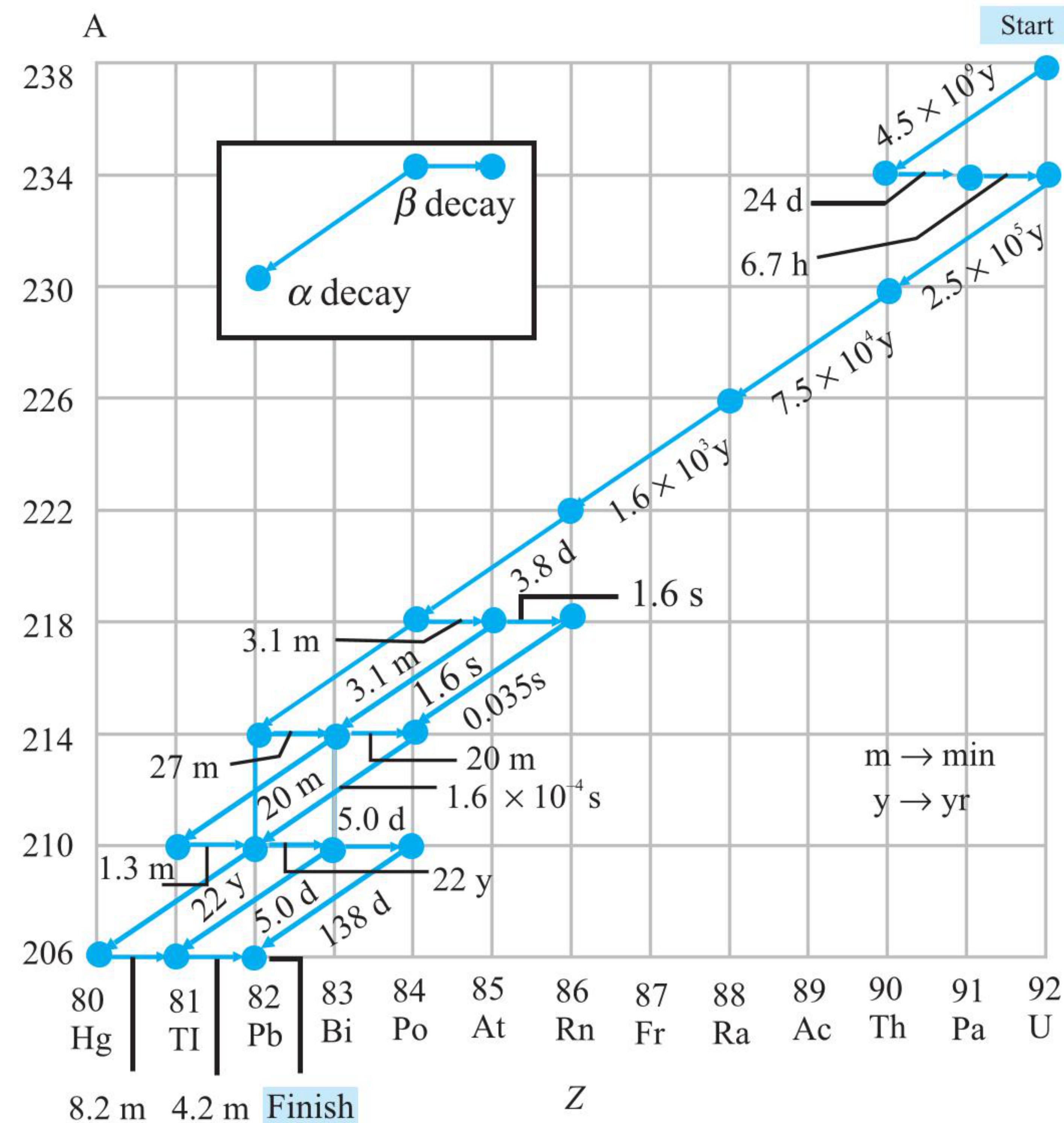


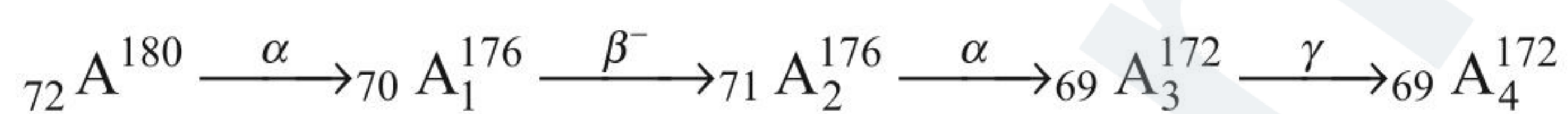
ILLUSTRATION 7.40

A radioactive nucleus undergoes a series of decay according to the scheme



If the mass number and atomic number of A are 180 and 72, respectively, what are these numbers for A_4 ?

Sol. The successive process may be expressed as



RADIOACTIVE EQUILIBRIUM

In a radioactive series, if we talk about an intermediate element, it is produced due to the decay of its previous element and it decays to the next element of the series. In Eq. (i), an intermediate element J decays to K with a decay constant λ_1 and K decays to L with decay constant λ_2 .



If at an instant, N_1 nuclei of J are present, its disintegration rate can be given as

$$R_1 = \lambda_1 N_1 \quad \dots(ii)$$

As J decays to K, the above relation in Eq. (ii) also gives the formation rate of nuclei of K. If at this instant N_2 nuclei of K are present, its decay rate can be given as

$$R_2 = \lambda_2 N_2 \quad \dots(iii)$$

If at some instant the production rate and decay rate of the element K become equal, then the amount of K appears to be a constant as the number of nuclei of K produced per second are equal to the number of nuclei of K disintegrating per second.

This situation for the intermediate element K is called radioactive equilibrium. We can also state that this equilibrium is a dynamic equilibrium in which the amount of K element appears to be a constant along with the process of its continuous formation by decaying element J and its continuous disintegration to element L. Thus, for an element, condition of radioactive equilibrium is

$$\text{Rate of formation} = \text{Rate of disintegration}$$

Here, for element K to be in radioactive equilibrium, we have $\lambda_1 N_1 = \lambda_2 N_2$... (iv)

SIMULTANEOUS DECAY MODES OF RADIOACTIVE ELEMENT

We know that due to radioactive disintegration a radio nuclide transforms into its daughter nucleus. Depending on the nuclear structure and its instability, a parent nucleus may undergo either α or β emission. Sometimes a parent nucleus may undergo both types of emission with the probabilities of α -decay or β -decay. The amount of daughter nuclide produced by α - and β -decays will be in the probability ratio of α - and β -decays.

Now, we analyze the decay of such elements which disintegrate with two or more decay modes simultaneously. If an element decays to different daughter nuclei with different decay constants $\lambda_1, \lambda_2, \lambda_3$, etc., for each decay mode, then the effective decay constant of the parent nuclei can be given as

$$\lambda_{\text{eff}} = \lambda_1 + \lambda_2 + \dots$$

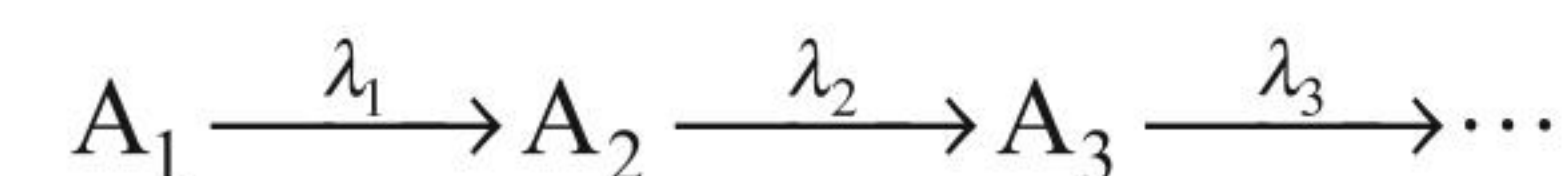
Similarly, for a radioactive element with decay constant λ which decays by both α - and β -decays given that the probability for an α -emission is P_1 and that for β -emission is P_2 , the decay constant of the element can be split for individual decay modes. Like in this case the decay constants for α - and β -decays separately can be given as

$$\lambda_\alpha = P_1 \lambda$$

$$\lambda_\beta = P_2 \lambda$$

ACCUMULATION OF A RADIOACTIVE ELEMENT IN RADIOACTIVE SERIES

In a radioactive series, we have discussed that each element decays into its daughter nuclei until a stable element appears. Consider a radioactive series shown below



To analyze mathematically the above series, we assume initially at $t = 0$, N_0 atoms of parent element A_1 are present which decay to the element A_2 with a decay constant λ_1 . Thus, after time t , number of undecayed nuclei of A_1 present at a time instant t can be given by decay law as

$$N_1 = N_0 e^{-\lambda_1 t} \quad \dots(i)$$

Due to disintegration of A_1 , nuclei of A_2 are formed and these start decaying with a decay constant λ_2 to another element A_3 . Let at an instant t , N_2 undecayed nuclei of A_2 be present. Then, the decay rate of A_2 at this instant can be given as

$$\text{Decay rate of } A_2 = \lambda_2 N_2 \quad \dots(ii)$$

Due to disintegration of A_1 , A_2 is produced. Thus, the production rate of nuclei of A_2 will be the decay rate of nuclei of A_1 . Hence, production rate of A_2 at this instant can be given as

$$\text{Production rate of } A_2 = \lambda_1 N_1 \quad \dots(iii)$$

Now, in a further time dt , if dN_2 nuclei of element A_2 are accumulated, then the accumulation rate of nuclei of element A_2 can be given as

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2$$

$$\text{or } \frac{dN_2}{dt} + \lambda_2 N_2 = \lambda_1 N_0 e^{-\lambda_1 t} \quad \dots(\text{iv})$$

Equation (iv) is a simple linear differential equation which on solving gives the number of nuclei of element A_2 as a function of time t . On solving the equation, we get

$$N_2 = \frac{\lambda_1 N_0}{(\lambda_1 - \lambda_2)} (e^{-\lambda_2 t} - e^{-\lambda_1 t}) \quad \dots(\text{v})$$

Here, we can see that in the beginning as $N_2 = 0$. Due to disintegration of A_1 , A_2 is being formed and as the amount of A_1 is decreased and that of A_2 is increased, decay rate of A_2 increases and that of A_1 decreases. After a time when both decay rates become equal, the element A_2 will be said to be in radioactive equilibrium when the yield of the radionuclide A_2 is maximum.

Let us take some examples to understand the above phenomenon better.

ILLUSTRATION 7.41

Suppose the daughter nucleus in a nuclear decay is itself radioactive. Let λ_p and λ_d be the decay constants of the parent and the daughter nuclei. Also, let N_p and N_d be the number of parent and daughter nuclei at time t . Find the condition for which the number of daughter nuclei becomes constant.

Sol. The number of parent nuclei decaying in a short time interval t to $t + dt$ is $\lambda_p N_p dt$. The number of daughter nuclei decaying during the same time interval is $\lambda_d N_d dt$. The number of daughter nuclei will be constant if

$$\lambda_p N_p dt = \lambda_d N_d dt \quad \text{or} \quad \lambda_p N_p = \lambda_d N_d$$

ILLUSTRATION 7.42

A radioactive nucleus can decay by two different processes. The half-life for the first process is t_1 and that for the second process is t_2 . Show that the effective half-life t of the nucleus is given by

$$\frac{1}{t} = \frac{1}{t_1} + \frac{1}{t_2}$$

Sol. The decay constant for the first process is $\lambda_1 = \frac{\ln 2}{t_1}$ and for the second process is $\lambda_2 = \frac{\ln 2}{t_2}$. The probability that an active

nucleus decays by the first process in a time interval dt is $\lambda_1 dt$. Similarly, the probability that it decays by the second process is $\lambda_2 dt$. The probability that it either decays by the first process or by the second process is $\lambda_1 dt + \lambda_2 dt$. If the effective decay constant is λ , this probability is also equal to λdt . Thus,

$$\lambda dt = \lambda_1 dt + \lambda_2 dt$$

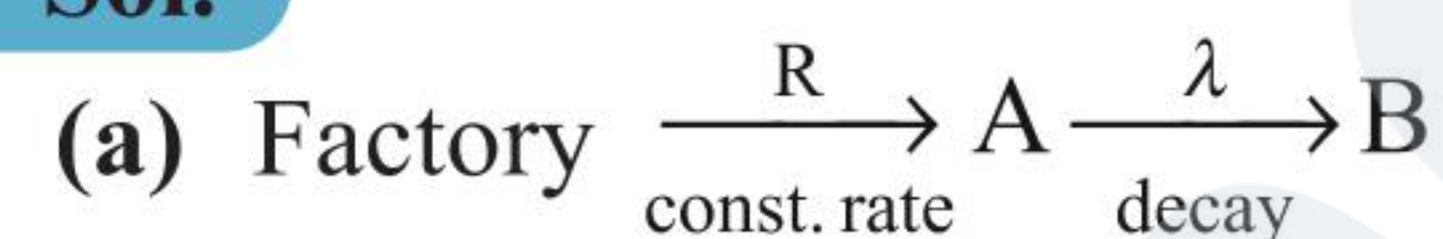
$$\text{or } \lambda = \lambda_1 + \lambda_2$$

$$\text{or } \frac{1}{t} = \frac{1}{t_1} + \frac{1}{t_2}$$

ILLUSTRATION 7.43

A factory produces a radioactive substance A at a constant rate R which decays with a decay constant λ to form a stable substance. Find (a) the number of nuclei of A and (b) number of nuclei of B, at any time t assuming the production of A starts at $t = 0$. (c) Also, find out the maximum number of nuclei of "A" present at any time during its formation.

Sol.



Let N be the number of nuclei of A at any time t .

$$\therefore \frac{dN}{dt} = R - \lambda N \quad \int_0^N \frac{dN}{R - \lambda N} = \int_0^t dt$$

On solving, we will get $N = R/\lambda(1 - e^{-\lambda t})$

(b) Number of nuclei of B at any time t ,

$$N_B = Rt - N_A = Rt - R/\lambda(1 - e^{-\lambda t}) = R/\lambda(\lambda t - 1 + e^{-\lambda t})$$

(c) Maximum number of nuclei of "A" present at any time during its formation = R/λ .

ILLUSTRATION 7.44

Nuclei of radioactive element A are being produced at a constant rate α . The element has a decay constant λ . At time $t = 0$, there are N_0 nuclei of the element.

(a) Calculate the number N of nuclei of A at time t .

(b) If $\alpha = 2N_0\lambda$, calculate the number of nuclei of A after one half-life time of A and also the limiting value of N at $t \rightarrow \infty$.

Sol.

(a) The rate of formation of radioactive nuclei is α .

Rate of decay of radioactive nuclei is λN . Therefore,

$$\frac{dN}{dt} = \alpha - \lambda N$$

$$\Rightarrow \frac{dN}{\alpha - \lambda N} = dt$$

$$\text{Integrating, } \frac{\log_e(\alpha - \lambda N)}{-\lambda} = t + A \quad \dots(\text{i})$$

where A is constant of integration.

At $t = 0$, $N = N_0$

$$\therefore \frac{\log_e(\alpha - \lambda N_0)}{-\lambda} = A$$

Therefore, Eq. (i) gives

$$\frac{\log_e(\alpha - \lambda N)}{-\lambda} = t + \frac{\log_e(\alpha - \lambda N_0)}{-\lambda}$$

$$\Rightarrow \log_e \left(\frac{\alpha - \lambda N}{\alpha - \lambda N_0} \right) = -\lambda t$$

$$\Rightarrow \frac{\alpha - \lambda N}{\alpha - \lambda N_0} = e^{-\lambda t}$$

$$\Rightarrow N = \frac{\alpha}{\lambda}(1 - e^{-\lambda t}) + N_0 e^{-\lambda t} \quad \dots(\text{ii})$$

(b) Given, $a = 2N_0\lambda$

$$t = T_{1/2} = \frac{0.693}{\lambda}$$

$$\therefore N = \frac{2N_0\lambda}{\lambda}(1 - e^{-0.693}) + N_0e^{-0.693}$$

$$= N_0(2 - e^{-0.693}) = N_0(2 - 0.5)$$

$$\therefore N = 1.5 N_0$$

When $t \rightarrow \infty$, Eq. (ii) gives $N = \frac{\alpha}{\lambda}$

ILLUSTRATION 7.45

The mean lives of a radioactive substance are 1620 years and 405 years for α -emission and β -emission, respectively. Find out the time during which three-fourth of a sample will decay if it is decaying both by α -emission and β -emission simultaneously.

Sol. Radioactivity is the statistical phenomenon. In statistics the important theorem is that the probability of a composite event is equal to the product of probabilities of individual and independent events. Let λ_α and λ_β be the decay constant for α - and β -emission, respectively.

According to theorem, the decay constant for composite event is $\lambda_\alpha + \lambda_\beta$.

If T is half-life of composite event and T_α and T_β are the half-lives of α - and β -emissions, then $\lambda = \frac{0.693}{T}$.

$$\frac{1}{T} = \frac{1}{T_\alpha} + \frac{1}{T_\beta} \quad \text{or} \quad \frac{1}{\tau} = \frac{1}{\tau_\alpha} + \frac{1}{\tau_\beta}$$

$$\text{This gives } \tau = \frac{\tau_\alpha \tau_\beta}{\tau_\alpha + \tau_\beta} = \frac{1620 \times 405}{1620 + 405} = 324 \text{ years}$$

Let t be the time in which the given sample decays three-fourth. Therefore, the fraction of sample undecayed in time t is $1/4$. That is,

$$\frac{N}{N_0} = \frac{1}{4}$$

From the relation $N = N_0 e^{-\lambda t}$, we have $\frac{N}{N_0} = e^{-\lambda t}$

$$\text{or } \log_e \frac{N}{N_0} = -\lambda t$$

$$\Rightarrow t = \frac{1}{\lambda} \log_e \frac{N_0}{N} = \tau \log_e \frac{N_0}{N} \quad \left(\because \lambda = \frac{1}{\tau} \right)$$

$$= 2.3026 \tau \log_{10} \frac{N_0}{N}$$

$$= 2.3026 \times 324 \log_{10} 4$$

$$= 2.3026 \times 324 \times 0.6031$$

Required time, $t = 449.94$ years

ILLUSTRATION 7.46

Find the half-life of uranium, given that 3.32×10^{-7} g of radium is found per gram of uranium in old minerals. The atomic weights of uranium and radium are 238 and 226 and half-life of radium is 1600 years (Avogadro number is 6.023×10^{23} /g-atom).

Sol. In very old minerals, the amount of an element is constant. This implies that the element exists in radioactive equilibrium. Thus, here we can use

$$\lambda_U N_U = \lambda_R N_R$$

$$\text{or } \frac{N_U}{T_U} = \frac{N_R}{T_R}$$

$$\text{or } T_U = \frac{N_U}{N_R} \times T_R$$

$$\text{or } T_U = \frac{m_U A_R}{m_R A_U} \times T_R$$

$$\text{or } T_U = \frac{1 \times 226}{3.32 \times 10^{-7} \times 238} \times 1600 \text{ years} = 4.7 \times 10^9 \text{ years}$$

CONCEPT APPLICATION EXERCISE 7.3

- The activity of a sample is R_1 at time t_1 and R_2 at time t_2 . The mean life of the sample is τ . What is the number of nuclei that have disintegrated in time interval $(t_1 - t_2)$?
- The half-life of cobalt-60 is 5.25 yrs. After how long does its activity reduce to about one eight of its original value?
- A nucleus of U_{x_1} has a half-life of 24.1 days. How long a sample of U_{x_1} take to change to 90% of it to U_{x_2} ?
- Small amount of a radioactive substance (half-life = 10 days) is spread inside a room and consequently the level of radiation becomes 50 times the level for normal occupancy of the room. After how many days will the room be safe for occupation?
- Find the half-life of ^{238}U if 1 g of it emits 1.24×10^4 α -particles per second. Avogadro's number = 6.023×10^{23} .
- The half-life of radon is 3.8 days. After how many days will only one-twentieth of radon sample be left over?
- A radioactive sample has mass m , decay constant λ , and molecular weight M . If the Avogadro number is N_A , then
 - find the initial number of nuclei present;
 - find the number of decayed nuclei after a time t ;
 - find the activity of the sample after a time t .
- Calculate the time taken to decay 100 percent of a radioactive sample in terms of (a) half-life T and (b) mean-life T_{av} .
- The activity of a sample of radioactive material is A_1 at time t_1 and A_2 at time t_2 ($t_2 > t_1$). Obtain an expression for its mean-life.
- Determine the average ^{14}C activity in decays per minute per gram of natural carbon found in living organisms if the concentration of ^{14}C relative to that of ^{12}C is 1.4×10^{-12} and half-life of ^{14}C is $T_{1/2} = 57.30$ years.
- A radio nuclide A_1 with decay constant λ_1 transforms into a radio nuclide A_2 with decay constant λ_2 . Assuming that at the initial moment, the preparation contained only the radio nuclide A_1 .

(a) Find the equation describing accumulation of radio nuclide A_2 with time.

(b) Find the time interval after which the activity of radio nuclide A_2 reaches its maximum value.

12. The isotope $^{14}_6\text{C}$ is radioactive and has a half-life of 5730 years. If you start with a sample of 1000 carbon-14 nuclei, how many will still be around in 17,190 years?

13. The half-life of the radioactive nucleus $^{226}_{86}\text{Ra}$ is 1.6×10^3 yr. If a sample contains 3.0×10^{16} such nuclei, determine the activity at this time.

ANSWERS

1. $(R_1 - R_2)\tau$

2. 15.75 yr

3. 80 days

4. 56.45 days

5. 4.5×10^9 yr

6. 16.45 days

7. (a) $N_A \frac{m}{M}$ (b) $\frac{mN_A(1 - e^{-\lambda t})}{M}$ (c) $\frac{mN_A \lambda e^{-\lambda t}}{M}$

8. (a) $\frac{T}{0.693} \ln \left| \frac{1}{1 - \eta} \right|$ (b) $T_{av} \ln \left| \frac{1}{1 - \eta} \right|$ 9. $T = \frac{t_2 - t_1}{\ln |A_1 / A_2|}$

10. 16 decay $\text{g}^{-1} \text{min}^{-1}$

11. (a) $N_2(t) = \frac{\lambda_1 N_0}{\lambda_2 - \lambda_1} (e^{-\lambda_1 t} - e^{-\lambda_2 t})$ (b) $\frac{\ln(\lambda_2 / \lambda_1)}{\lambda_2 - \lambda_1}$

12. ~500 nuclei

13. 11.1 μCi

NUCLEAR REACTIONS

It is possible to change the structure of nuclei by bombarding them with energetic particles. Such changes are called nuclear reactions. Rutherford was the first to observe nuclear reactions, using naturally occurring radioactive source for the bombarding particles. He found that protons were released when alpha particles were allowed to collide with nitrogen atoms. The process can be represented symbolically as



This equation says that an alpha particle (^4_2He) strikes a nitrogen nucleus and produces an unknown product nucleus (X) and a proton (^1_1H). Balancing atomic numbers and mass numbers, as we did for radioactive decay, enables us to conclude that the unknown is characterized as $^{17}_8\text{X}$. Because the element with atomic number 8 is oxygen, we see that the reaction is



This nuclear reaction starts with two stable isotopes, helium and nitrogen, and produces two different stable isotopes, hydrogen and oxygen.

KINEMATICS OF NUCLEAR REACTIONS

When two particles approach each other, their mutual interaction changes their motion. Exchange of momentum and energy takes place between particles.

When the final particles or system are the same as initial ones, the term scattering is used. The collision is said to be a reaction if final particles are not identical to initial ones, e.g., in a collision between an electron and a proton, the final product may be a hydrogen atom.

Since only internal force is involved in a collision, both the momentum and total energy are conserved. From conservation of momentum,

$$\underbrace{P'_1 + P'_2}_{\text{after}} = \underbrace{P_1 + P_2}_{\text{before}} \quad \dots(\text{i})$$

The kinetic energies of particles before and after collision are given by

$$E_K = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} \quad (\text{before}) \quad \dots(\text{ii})$$

$$E'_K = \frac{1}{2} m'_1 v'^2_1 + \frac{1}{2} m'_2 v'^2_2 = \frac{p'^2_1}{2m'_1} + \frac{p'^2_2}{2m'_2} \quad (\text{after}) \quad \dots(\text{iii})$$

The internal energy of the particles may be different after collision. Hence,

$$\underbrace{E'_K + E'_{\text{int}}}_{\text{after}} = \underbrace{E_K + E_{\text{int}}}_{\text{before}}$$

If the internal energy changes in the collision, the kinetic energy also changes.

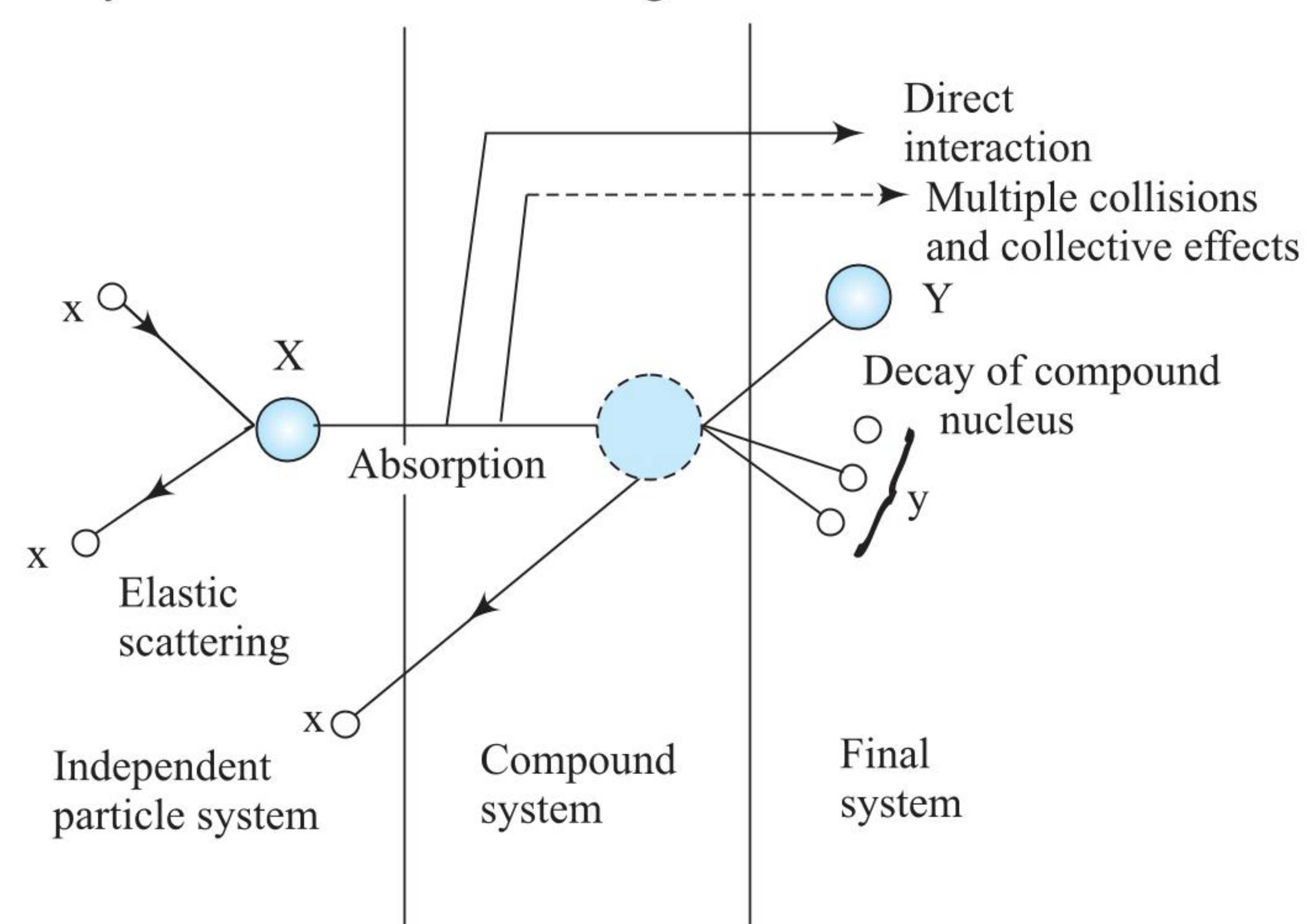
The Q of the reaction or collision is defined by

$$\begin{aligned} Q &= E'_K - E_K = U_{\text{int}} - U'_{\text{int}} \\ &= (U_{\text{int}})_{\text{before}} - (U_{\text{int}})_{\text{after}} = -\Delta U_{\text{int}} \\ &= \left(\frac{p'^2_1}{2m'_1} + \frac{p'^2_2}{2m'_2} \right) - \left(\frac{p^2_1}{2m_1} + \frac{p^2_2}{2m_2} \right) \quad \dots(\text{iv}) \end{aligned}$$

When $Q = 0$, there is no change in kinetic energy or internal potential energy and the collision is elastic.

When $Q < 0$, there is a decrease in kinetic energy with a corresponding increase in internal energy of the particles. Such collision is called endothermic. When $Q > 0$, there is an increase in kinetic energy at the expense of internal energy. This is an exothermic reaction.

Figure shows the possible stages of nuclear reactions. Reflection of the incident particles as described by Rutherford's theory is called elastic scattering.



In a direct interaction, the incident particles (which have high energy and can penetrate deeper into the nucleus) interact with a single nucleon in the nucleus so that the nucleon leaves the nucleus. If the nucleon does not leave the nucleus but interacts with several other nucleons, then the nucleus is raised to an excited state called a compound nucleus. The energy carried by

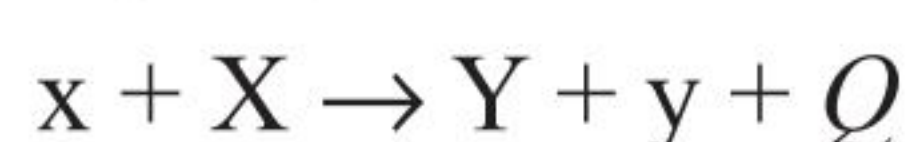
the incident particle is shared by many nucleons. The compound nucleus can emit a particle identical to the incident particle and with the same kinetic energy or emit photons or other particles. The decay of the compound nucleus is a statistical process. It does not depend on the process of formation or external conditions applied, similar to radioactivity. The emission of particles or photons cannot be predicted.

A nuclear reaction can occur only if certain conservation laws are followed. These are:

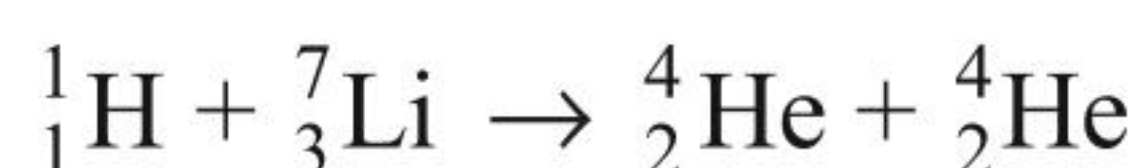
1. Conservation of mass number A .
2. Conservation of charge.
3. Conservation of energy, linear momentum, and angular momentum.

ENERGY CONSERVATION IN NUCLEAR REACTIONS

Consider a general reaction of particle x that is incident on nucleus X resulting in nucleus Y and particle y . The reaction may be written as



Sometimes the reaction is written as $X(x, y)Y$, e.g., consider the reaction ${}^7\text{Li} (p, \alpha) {}^4\text{He}$ which can be written as



We apply conservation of energy to a reaction to compute total kinetic energy released or absorbed in the reaction. Here, we assume that target particle is at rest, K_x, K_y, K_Y represent kinetic energies of bombarding particle x and reaction products y and Y , respectively.

From conservation of energy,

$$m_x c^2 + K_x + m_X c^2 = m_y c^2 + K_y + m_Y c^2 + K_Y$$

The total energy released in the reaction (Q) is equal to the difference in kinetic energies of the final particles and of initial particles. So,

$$Q = K_Y + K_y - K_x = (m_x + m_X - m_y - m_Y)c^2$$

which is the energy released in the reaction and is called the Q value of the reaction.

In exothermic reactions, energy is released by a nuclear reaction. In an exothermic reaction, the total mass of the initial particles is greater than that of the final particles and the Q value is positive. In these reactions, some mass is converted to energy.

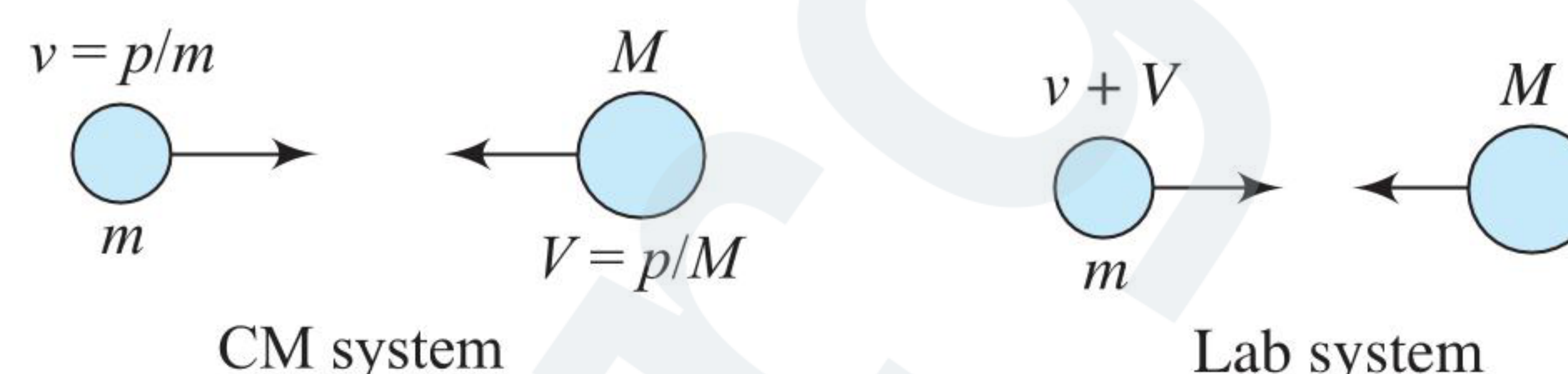
In endothermic reactions, energy is absorbed in a nuclear reaction. If the total mass of the initial particles is less than that of the final particles, the Q value is negative and some minimum input kinetic energy from the bombarding particles is required. For example,



The minimum kinetic energy needed to initiate a nuclear reaction is called threshold energy. An endothermic reaction cannot take place unless certain threshold energy is supplied to the system. The threshold energy K_{th} must not only supply $|Q|$ but also some kinetic energy to the products to conserve momentum. In the center of mass frame, the total momentum is zero; the threshold energy is just $|Q|$.

But for reactions occurring with nucleus X at rest relative to the laboratory (called the laboratory frame), the incident particle x must have energy greater than $|Q|$ because by conservation of momentum, the kinetic energies of y and Y cannot be zero. Consider a particle x , of mass m , incident on X , of mass M . Both particles have momenta of equal magnitude in the center of mass frame.

$$\text{The total kinetic energy is } E_{CM} = \frac{p^2}{2m} + \frac{p^2}{2M} = \frac{1}{2} p^2 \left(\frac{m+M}{mM} \right)$$



We can transform the velocity of incident and target particles to the lab frame by adding V to each velocity so that M is at rest and m has velocity $v + V$. The momentum of m in the lab frame is then

$$P_{lab} = m(v + V) = mv \left(1 + \frac{m}{M} \right) = p \left(\frac{m+M}{M} \right)$$

And its energy is

$$E_{lab} = \frac{P_{lab}^2}{2m} = \frac{p^2}{2m} \left(\frac{M+m}{M} \right)^2 = \frac{M+m}{M} E_{CM}$$

Hence, the threshold energy for an endothermic reaction in the lab frame is

$$E = \frac{m+M}{M} |Q|$$

ILLUSTRATION 7.47

Find the Q value of the reaction $p + {}^7\text{Li} \rightarrow {}^4\text{He} + {}^4\text{He}$. Determine whether the reaction is exothermic or endothermic. The atomic masses of ${}^1\text{H}$, ${}^4\text{He}$, and ${}^7\text{Li}$ are 1.007825 u, 4.002603 u, and 7.016004 u, respectively.

Sol. The total mass of the initial particles,

$$m_i = 1.007825 + 7.016004 = 8.023829 \text{ u}$$

and the total mass of the final particles

$$m_f = 2 \times 4.002603 = 8.005206 \text{ u}$$

Difference between initial and final mass of particles

$$\Delta m = m_i - m_f = 8.023829 - 8.005206 = 0.018623 \text{ u}$$

This mass is converted into energy and the reaction is exothermic.

The Q value is positive and given by

$$Q = (\Delta m)c^2 = 0.018623 \times 931.5 = 17.35 \text{ MeV}$$

ILLUSTRATION 7.48

Consider a collision between two particles one of which is at rest and the other strikes it head on with momentum p_1 . Calculate the energy of reaction Q in terms of the kinetic energy of the particles before and after they collide.

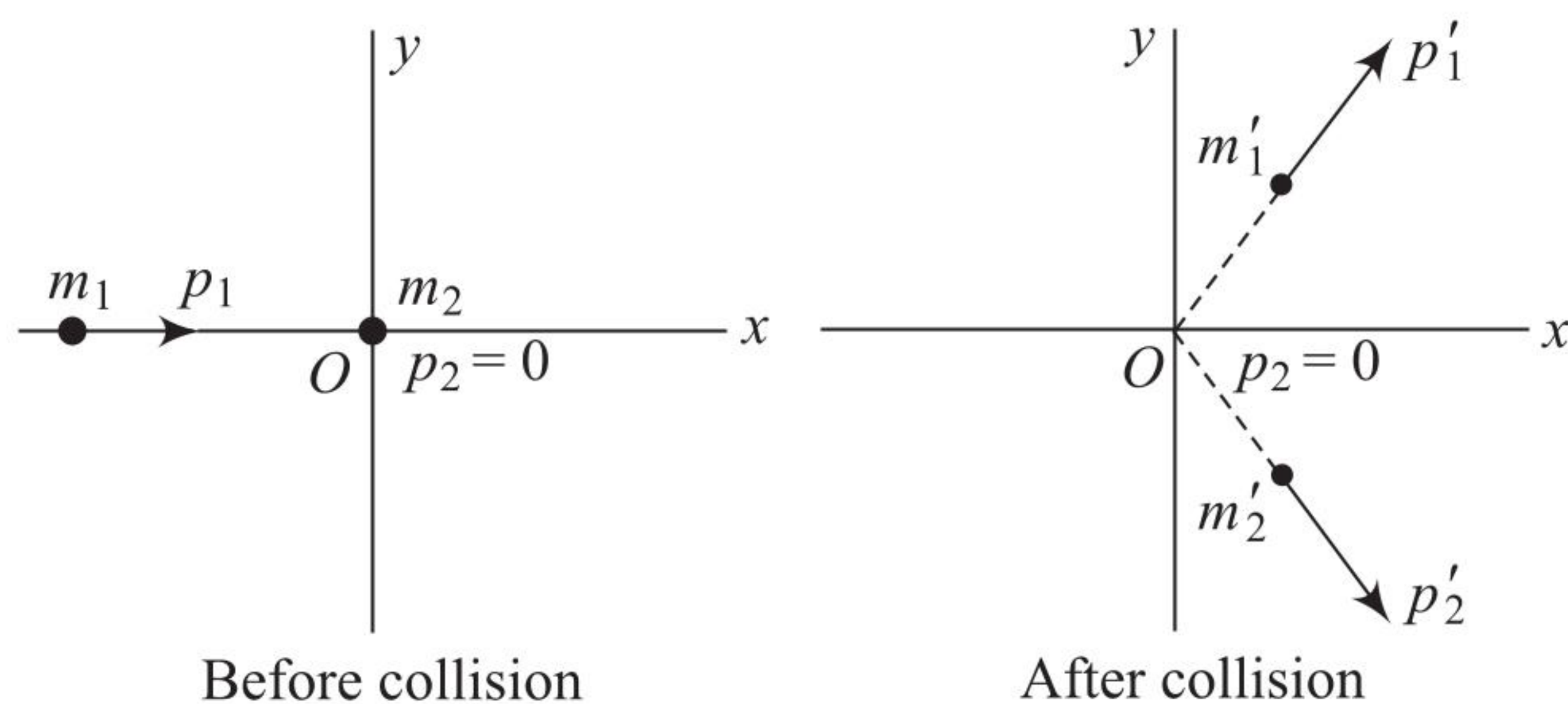
Sol. Initially, m_1 has a momentum p_1 and m_2 is at rest ($p_2 = 0$) in the lab frame. The masses of the particles after the collision are m'_1 and m'_2 .

The conservation of momentum gives

$$p'_1 + p'_2 = p_1 \text{ or } p'_2 = p_1 - p'_1 \quad \dots(i)$$

Squaring this equation, we have

$$\begin{aligned} p'^2_2 &= (p_1 - p'_1)^2 = p_1^2 + p'^2_1 - 2p_1 p'_1 \\ &= p_1^2 + p'^2_1 - 2p_1 p'_1 \cos \theta \end{aligned}$$



We have

$$\begin{aligned} Q &= \frac{p'^2_1}{2m'_1} + \frac{p'^2_2}{2m'_2} - \frac{p_1^2}{2m_1} + \frac{p'^2_1}{2m'_1} \\ &\quad + \frac{1}{2m'_2} (p_1^2 + p'^2_1 - 2p_1 p'_1 \cos \theta) - \frac{p_1^2}{2m_1} \end{aligned}$$

$$\text{or } Q = \frac{1}{2} \left(\frac{1}{m'_1} + \frac{1}{m'_2} \right) p'^2_1 + \frac{1}{2} \left(\frac{1}{m'_2} - \frac{1}{m_1} \right) p_1^2 - \frac{p_1 p'_1}{m'_2} \cos \theta \quad \dots(ii)$$

Note that the kinetic energy of a particle can be expressed in terms of momentum of particles as $E_k = \frac{p^2}{2m}$.

Now, we can express the above results as

$$Q = E'_{k,1} \left(1 - \frac{m_1}{m'_2} \right) = - \frac{2(m_1 m'_1 E_{k,1} E'_{k,1})^{1/2}}{m'_2} \cos \theta \quad \dots(iii)$$

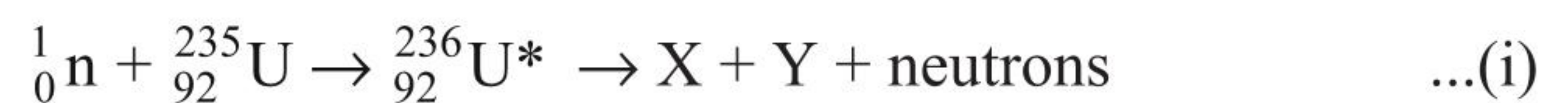
The equation derived above is called Q equation. It is applied in analysis of nuclear collisions.

NUCLEAR FISSION

Nuclear fission occurs when a heavy nucleus, such as ^{235}U , splits, or fissions, into two smaller nuclei. In such a reaction, the total mass of the products is less than the original mass of the heavy nucleus.

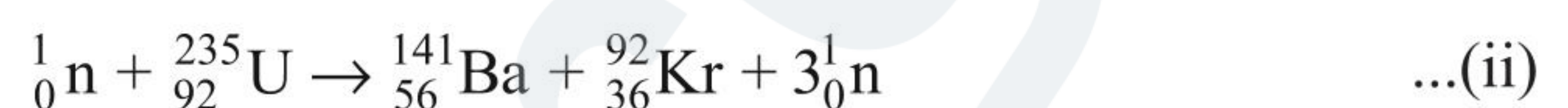
Nuclear fission was first observed in 1939 by Otto Hahn and Fritz Strassman, following some basic studies by Fermi. After bombarding uranium ($Z = 92$) with neutrons, Hahn and Strassman discovered among the reaction products two medium-mass elements, barium and lanthanum. Shortly thereafter, Lisa Meitner and Otto Frisch explained what had happened. The uranium nucleus had split into two nearly equal fragments after absorbing a neutron. Such an occurrence was of considerable interest to physicists attempting to understand the nucleus. But it was to have even more far-reaching consequences. Measurements showed that about 200 MeV of energy is released in each fission event, and this fact was to affect the course of human history.

The fission of ^{235}U by slow (low-energy) neutrons can be represented by the equation



where $^{236}\text{U}^*$ is an intermediate state that lasts only for about 10^{-12} s before splitting into X and Y. The resulting nuclei, X and Y, are called fission fragments. Many combinations of X and Y satisfy the requirements of conservation of energy and charge. In the fission of uranium, there are about 90 different daughter nuclei that can be formed. The process also results in the production of several (typically two or three) neutrons per fission event. On the average, 2.47 neutrons are released per event.

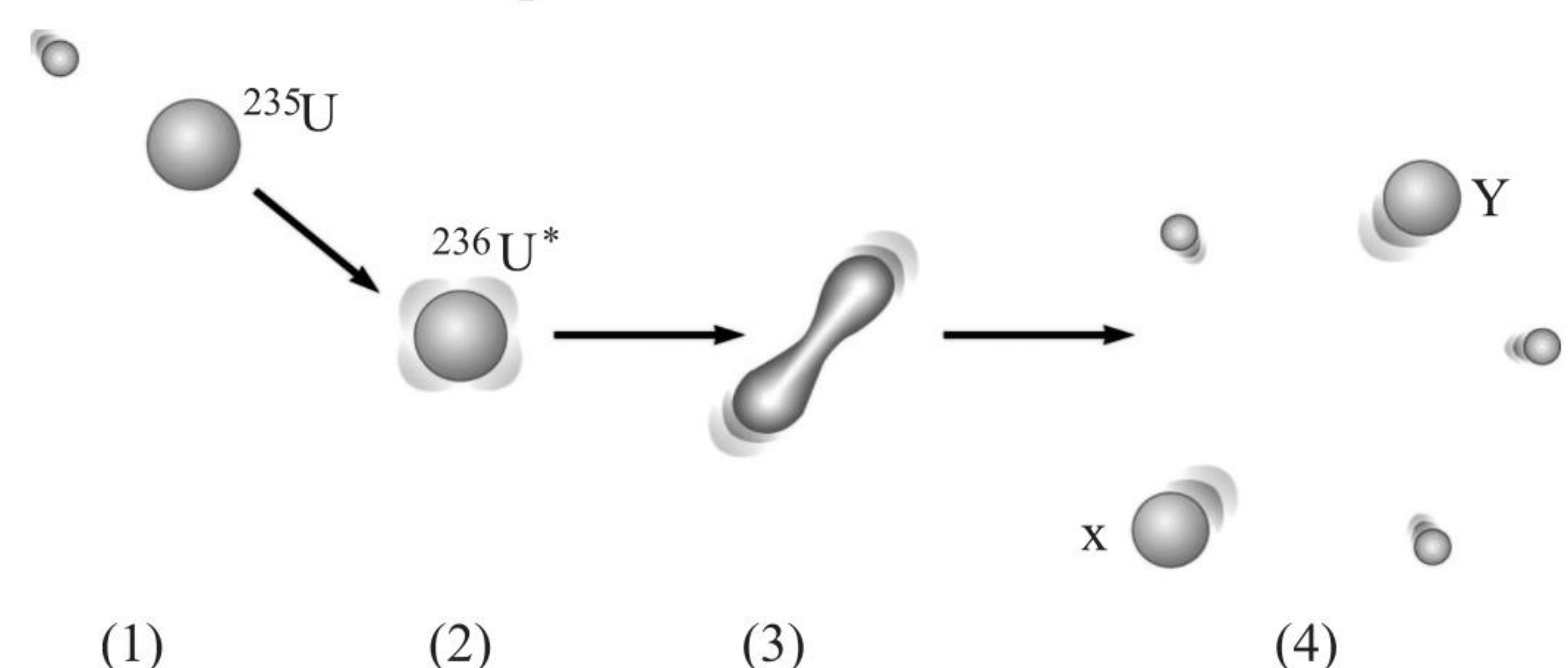
A typical reaction of this type is



The fission fragments, barium and krypton, and the released neutrons have a great deal of kinetic energy following the fission event.

The breakup of the uranium nucleus can be equated to the case of a drop of water when excess energy is added to it. All of the atoms in the drop have energy, but not enough to break up the drop. However, if enough energy is added to set the drop vibrating, it will undergo elongation and compression until the amplitude of vibration becomes large enough to cause the drop to break apart. In the uranium nucleus, a similar process occurs (see figure). The sequence of events is as follows:

1. ^{235}U nucleus captures a thermal (slow-moving) neutron.
2. This capture results in the formation of $^{236}\text{U}^*$, and the excess energy of this nucleus causes it to undergo violent oscillations.
3. The $^{236}\text{U}^*$ nucleus becomes highly elongated, and the force of repulsion between protons in the two halves of the dumbbell shape tends to increase the distortion.
4. The nucleus splits into two fragments, emitting several neutrons in the process.

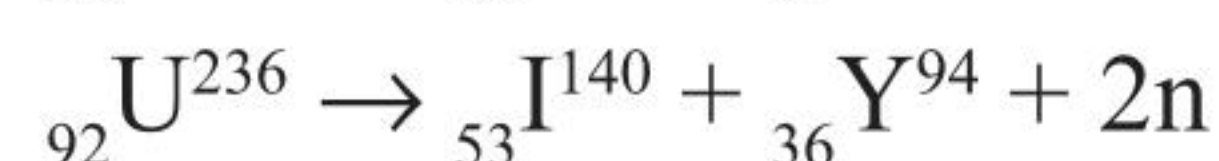
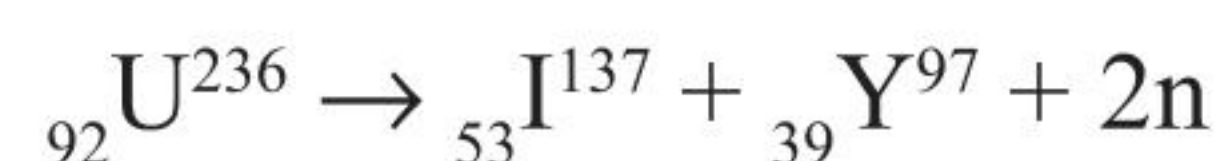


Let us estimate the disintegration energy, Q , released in a typical fission process. From figure, we see that the binding energy per nucleon is about 7.2 MeV for heavy nuclei (those having a mass number of approximately 240) and about 8.2 MeV for nuclei of intermediate mass. This means that the nucleons in the fission fragments are more tightly bound and, therefore, have less mass than the nucleons in the original heavy nucleus. This decrease in mass per nucleon appears as released energy when fission occurs. The amount of energy released is $(8.2 - 7.2)$ MeV per nucleon. Assuming a total of 240 nucleons, we find that the energy released per fission event is

$$\begin{aligned} Q &= (240 \text{ nucleons}) (8.2 \text{ MeV/nucleon} - 7.2 \text{ MeV/nucleon}) \\ &= 240 \text{ MeV} \end{aligned}$$

This is a very large amount of energy relative to the amount released in chemical processes.

In nuclear fission, heavy nuclei of A above 200 break up into two or more fragments of comparable masses. The most attractive bid, from a practical point of view, to achieve energy from nuclear fission is to use ${}_{92}\text{U}^{236}$ as the fission material. The technique is to hit a uranium sample by slow-moving neutrons (kinetic energy ≈ 0.04 eV, also called thermal neutrons). A ${}_{92}\text{U}^{235}$ nucleus has large probability of absorbing a slow neutron and forming ${}_{92}\text{U}^{236}$ nucleus. This nucleus then fissions into two parts. A variety of combinations of the middle-weight nuclei may be formed due to the fission. For example, one may have



and a number of other combinations.

1. On an average, 2.5 neutrons are emitted in each fission event.
2. Mass lost per reaction ≈ 0.2 amu
3. In nuclear fission, the total BE increases and excess energy is released.
4. In each fission event, about 200 MeV of energy is released, a large part of which appears in the form of kinetic energies of the two fragments. Neutrons take away about 5 MeV. For example,



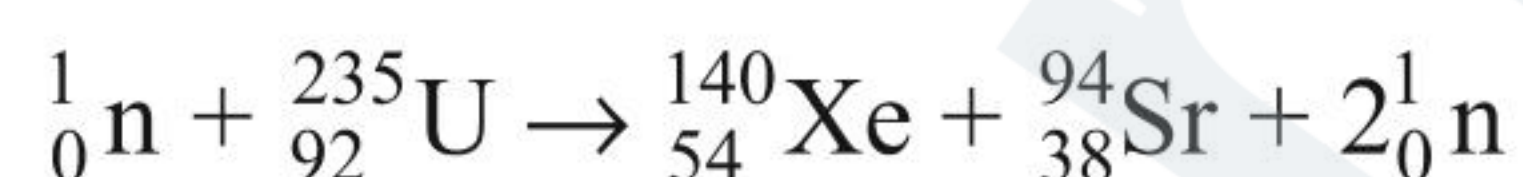
$$\begin{aligned} Q \text{ value} &= [(M_{\text{U}} - 92m_{\text{e}} + m_{\text{n}}) - \{(M_{\text{Ba}} - 56m_{\text{e}}) \\ &\quad + (M_{\text{Kr}} - 36m_{\text{e}}) + 3m_{\text{n}}\}]c^2 \\ &= [(M_{\text{U}} + m_{\text{n}}) - (M_{\text{Ba}} + M_{\text{Kr}} + 3m_{\text{n}})]c^2 \end{aligned}$$

5. A very important and interesting feature of neutron-induced fission is the chain reaction. For working of nuclear reactor, refer your text book.

ILLUSTRATION 7.49

Two other possible ways by which ${}^{235}\text{U}$ can undergo fission when bombarded with a neutron are (1) by the release of ${}^{140}\text{Xe}$ and ${}^{94}\text{Sr}$ as fission fragments and (2) by the release of ${}^{132}\text{Sn}$ and ${}^{101}\text{Mo}$ as fission fragments. In each case, neutrons are also released. Find the number of neutrons released in each of these events.

Sol. By balancing mass numbers and atomic numbers, we find that these reactions can be written as



Thus, two neutrons are released in the first event and three in the second.

ILLUSTRATION 7.50

Calculate the total energy released if 1.0 kg of ${}^{235}\text{U}$ undergoes fission, taking the disintegration energy per event to be $Q = 208$ MeV (a more accurate value than the estimate given previously).

Sol. We need to know the number of nuclei in 1.00 kg of uranium. Because $A = 235$, the number of nuclei is

$$\begin{aligned} N &= \left(\frac{6.02 \times 10^{23} \text{ nuclei mol}^{-1}}{235 \text{ g mol}^{-1}} \right) (1.00 \times 10^3 \text{ g}) \\ &= 2.56 \times 10^{24} \text{ nuclei} \end{aligned}$$

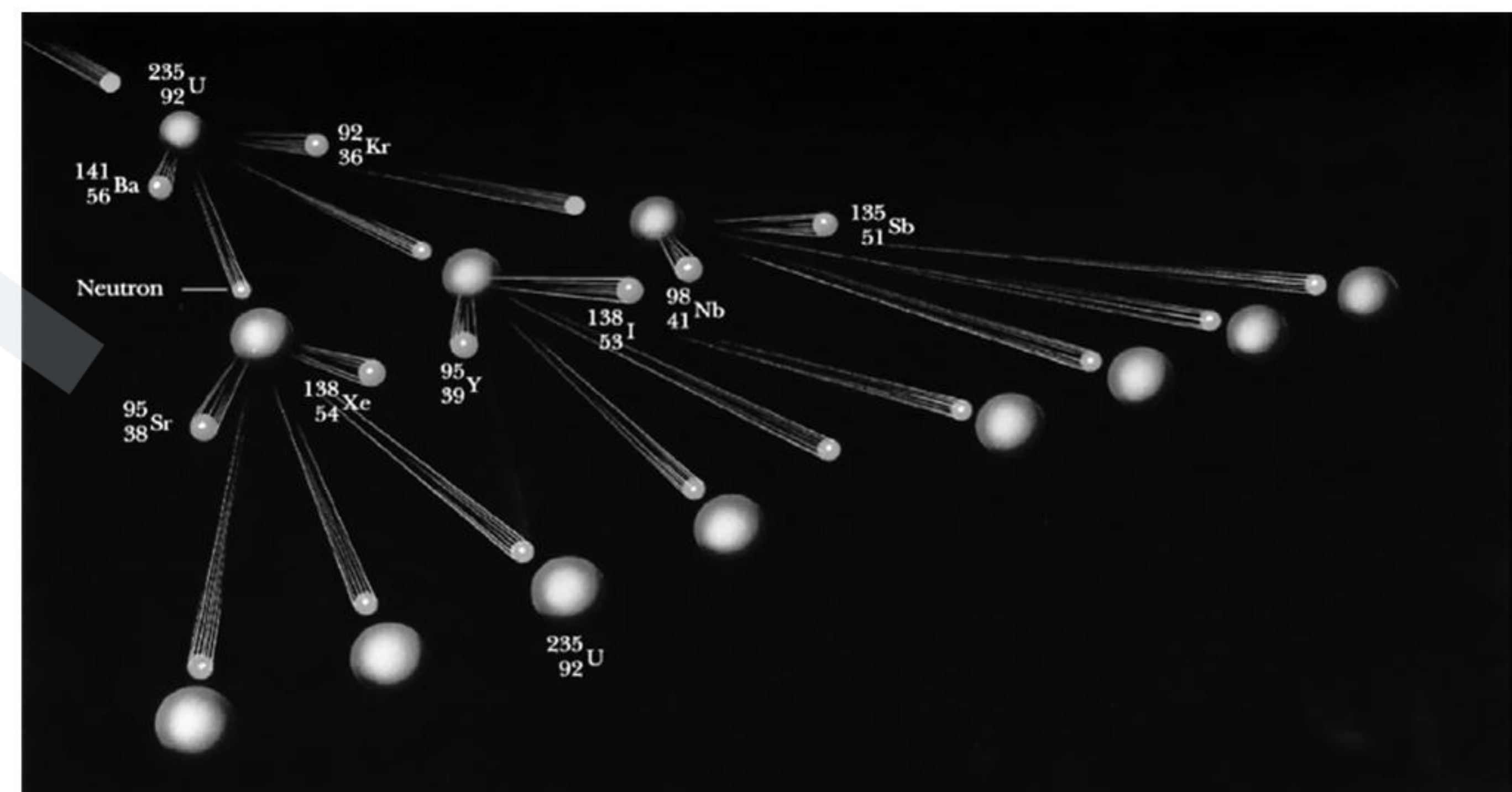
Hence, the disintegration energy is

$$\begin{aligned} E &= nQ = (2.56 \times 10^{24} \text{ nuclei}) \left(208 \frac{\text{MeV}}{\text{nucleus}} \right) \\ &= 5.32 \times 10^{26} \text{ MeV} \end{aligned}$$

Because 1 MeV is equivalent to 4.45×10^{-20} kWh, $E = 2.37 \times 10^7$ kWh. This is enough energy to keep a 100 W light bulb burning for about 30000 years.

NUCLEAR REACTORS

We have seen that neutrons are emitted when ${}^{235}\text{U}$ undergoes fission. These neutrons can in turn trigger other nuclei to undergo fission, with the possibility of a chain reaction (see figure). Calculations show that if the chain reaction is not controlled (that is, if it does not proceed slowly), it can result in a violent explosion, with the release of an enormous amount of energy. This, of course, was the principle behind the first nuclear bomb, an uncontrolled fission reaction.



A nuclear reactor is a system designed to maintain what is called a self-sustained chain reaction. This important process was first achieved in 1942 by a group led by Fermi at the University of Chicago, with natural uranium as the fuel. Most reactors in operation today also use uranium as fuel. Natural uranium contains only about 0.7% of the ${}^{235}\text{U}$ isotope, with the remaining 99.3% being the ${}^{238}\text{U}$ isotope. This is important to the operation of a reactor because ${}^{238}\text{U}$ almost never undergoes fission. Instead, it tends to absorb neutrons, producing neptunium and plutonium. For this reason, reactor fuels must be artificially enriched so that percentage of the ${}^{235}\text{U}$ isotope can be increased.

Earlier, we mentioned that an average of about 2.5 neutrons are emitted in each fission event of ${}^{235}\text{U}$. In order to achieve a self-sustained chain reaction, one of these neutrons must be captured by another ${}^{235}\text{U}$ nucleus and cause it to undergo fission. A useful parameter for describing the level of reactor operation is the reproduction constant K , defined as the average number of neutrons from each fission event that will cause another event. As we have seen, K can have a maximum value of 2.5 in the fission of uranium. However, in practice K is less than this because of several factors, which we shall discuss in a moment.

In a self-sustained chain reaction, $K = 1$ and the reactor is said to be critical. When K is less than unity, the reactor is subcritical and the reaction dies out. When K is greater than unity, the reactor is said to be supercritical, and a runaway reaction occurs. In a nuclear reactor used to furnish power to a utility company, it is necessary to maintain a K value close to unity.

NEUTRON LEAKAGE

In any reactor, a fraction of the neutrons produced in fission will leak out of the core before inducing other fission events. If the fraction leaking out is too large, the reactor will not operate. The percentage lost is large if the reactor is very small because leakage is a function of the ratio of surface area to volume. Therefore, a critical requirement of reactor design is choosing the correct surface area to volume ratio so that a sustained reaction can be achieved.

REGULATING NEUTRON ENERGIES

The neutrons released in fission events are very energetic, with kinetic energies of about 2 MeV. It is found that slow neutrons are far more likely than fast neutrons to produce fission events in ^{235}U . ^{238}U does not absorb slow neutrons. Therefore, in order for the chain reaction to continue, the neutrons must be slowed down. This is accomplished by surrounding the fuel with a moderator substance.

To understand how neutrons are slowed down, consider a collision between a light object and a very massive one. In such an event, the light object rebounds from the collision with most of its original kinetic energy. However, if the collision is between objects with nearly the same masses, the incoming projectile transfers a large percentage of its kinetic energy to the target. In the first nuclear reactor ever constructed, Fermi placed bricks of graphite (carbon) between the fuel elements. Carbon nuclei are about 12 times more massive than neutrons, but after about 100 collisions with carbon nuclei, a neutron is slowed sufficiently to increase its likelihood of fission with ^{235}U . In this design, the carbon is the moderator; most modern reactors use heavy water (D_2O) as the moderator.

NEUTRON CAPTURE

In the process of being slowed down, the neutrons may be captured by nuclei that do not undergo fission. The most common event of this type is neutron capture by ^{238}U . The probability of neutron capture by ^{238}U is very high when the neutrons have high kinetic energies and very low when they have low kinetic energies. Thus, the slowing down of the neutrons by the moderator serves the dual purpose of making them available for reaction with ^{235}U and decreasing their chances of being captured by ^{238}U .

ILLUSTRATION 7.51

Suppose, we think of fission of a $^{56}_{26}\text{Fe}$ nucleus into two equal fragments, $^{28}_{13}\text{Al}$. Is the fission energetically possible? Argue by working out Q of the process. Given $m(^{56}_{26}\text{Fe}) = 55.93494\text{u}$ and $m(^{28}_{13}\text{Al}) = 27.98191\text{u}$.

Sol. The fission of Fe-56 into two fragments of $^{28}_{13}\text{Al}$ with energy released Q can be written as $^{56}_{26}\text{Fe} \rightarrow ^{28}_{13}\text{Al} + ^{28}_{13}\text{Al} + Q$

$$\begin{aligned} Q &= [m(^{56}_{26}\text{Fe}) - 2m(^{28}_{13}\text{Al})]c^2 \\ &= [55.93494 - 2 \times 27.98191] \times 931.5 \text{ MeV} \\ &= -0.02888 \times 931.5 = -26.90 \text{ MeV} \end{aligned}$$

As the Q -value is negative, the fission is not possible energetically.

ILLUSTRATION 7.52

The fission properties of $^{239}_{94}\text{Pu}$ are very similar to those of $^{235}_{92}\text{U}$. The average energy released per fission is 180 MeV. How much energy, in MeV, is released if all the atoms in 1 kg of pure $^{239}_{94}\text{Pu}$ undergo fission?

Sol. Number of atoms in 1 kg of pure $^{239}_{94}\text{Pu}$

$$= \left(\frac{6.023 \times 10^{23}}{239} \right) \times 1000 = 2.52 \times 10^{24}$$

$$\begin{aligned} \text{Energy released in the fission} &= (2.52 \times 10^{24}) \times 180 \\ &= 4.536 \times 10^{26} \text{ MeV} \end{aligned}$$

ILLUSTRATION 7.53

A 1000 MW fission reactor consumes half of its fuel in 5.00 yr. How much $^{235}_{92}\text{U}$ did it contain initially? Assume that the reactor operates 80% of the time, that all the energy generated arises from the fission of $^{235}_{92}\text{U}$ and this nuclei is consumed only by the fission process.

Sol. In the fission of one nucleus of 1 kg of $^{235}_{92}\text{U}$, energy generated is 200 MeV.

Energy generated in the fission of 1 kg of $^{235}_{92}\text{U}$

$$\begin{aligned} &= (6.023 \times 10^{23}) \left(\frac{1000 \text{ g}}{235 \text{ g}} \right) (200 \times 1.6 \times 10^{-13} \text{ J}) \\ &= 8.2 \times 10^{13} \text{ J/kg} \end{aligned}$$

Time for which the reactor actually operates

$$= \left(\frac{8}{100} \right) \times 5 \text{ yr} = 4 \text{ yr} = 4 \times (3.154 \times 10^7 \text{ s}) = 12.62 \times 10^7 \text{ s}$$

Energy generated by the reactor in 5y = power $\times$ time

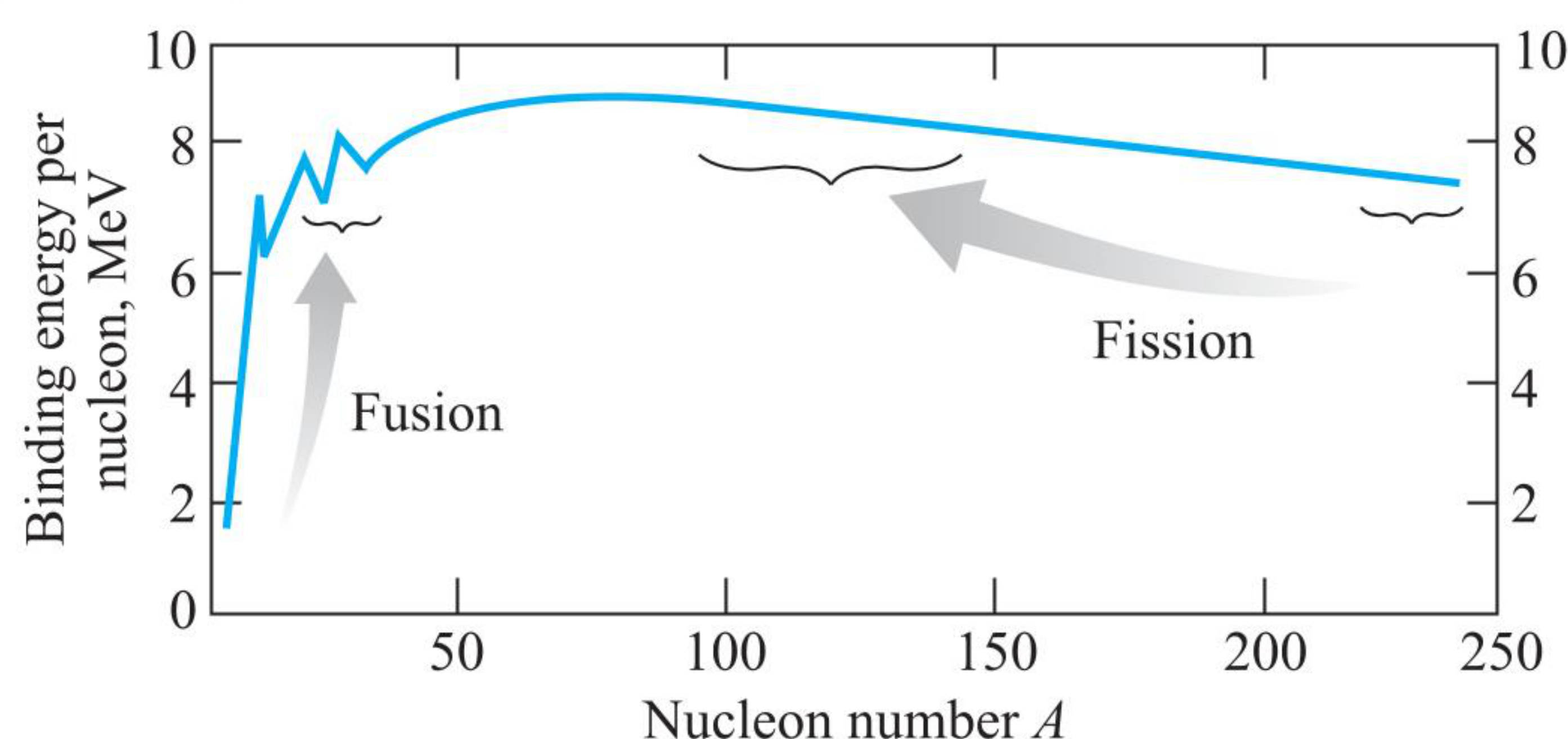
$$= (1000 \times 10^6 \text{ W}) (12.62 \times 10^7 \text{ s}) = 12.62 \times 10^{16} \text{ J}$$

$$\text{Fuel } (\text{U}^{235}) \text{ consumed in 5 years} = \frac{12.62 \times 10^{16} \text{ J}}{8.2 \times 10^{13} \text{ J/kg}} = 1.5 \times 10^3 \text{ kg}$$

NUCLEAR FUSION

Figure shows that the binding energy for lighter nuclei (those having a mass number lower than 20) is much smaller than the binding energy for heavier nuclei. This suggests a possible process that is the reverse of fission. When two light nuclei combine to form a heavier nucleus, the process is called nuclear fusion. Because the mass of the final nucleus is less than the masses of the original nuclei, there is a loss of mass accompanied

by a release of energy. Although fusion power plants have not yet been developed, a worldwide effort is under way to harness the energy from fusion reactions in the laboratory. Later, we shall discuss the possibilities and advantages of this process for generating electric power.



FUSION IN THE SUN

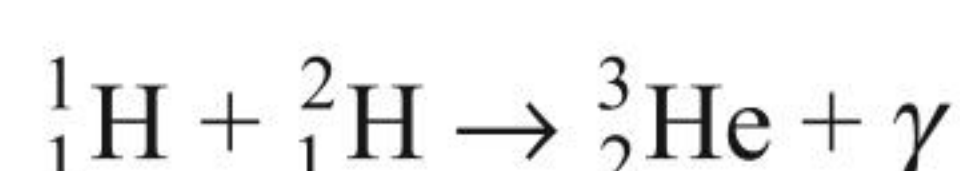
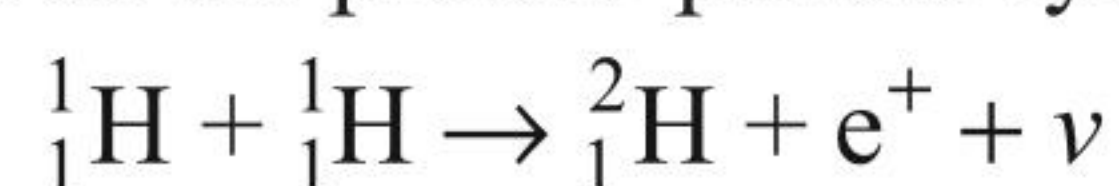
All stars generate their energy through fusion process. About 90% of the stars, including the Sun, fuse hydrogen, whereas some older stars fuse helium or other heavier elements. Stars are born in regions of space containing vast clouds of dust and gas. Recent mathematical models of these clouds indicate that star formation is triggered by shock waves passing through a cloud. These shock waves are similar to sonic booms and are produced by events such as the explosion of a nearby star, called a supernova explosion. The shock wave compresses certain regions of the cloud, causing these regions to collapse under their own gravity. As the gas falls inward toward the center, the atoms gain speed, which causes the temperature of the gas to rise. Two conditions must be met before fusion reactions in the star can sustain its energy needs.

1. The temperature must be high enough (about 10^7 K for hydrogen) to allow the kinetic energy of the positively charged hydrogen nuclei to overcome their mutual Coulomb repulsion as they collide.
2. The density of nuclei must be high enough to ensure a high rate of collision.

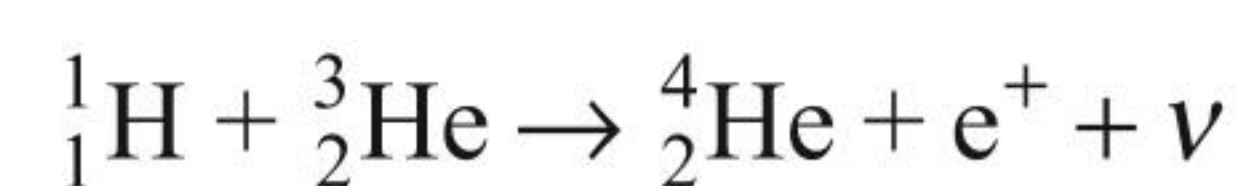
When fusion reactions occur at the core of a star, the energy liberated eventually becomes sufficient to prevent further collapse of the star under its own gravity. The star then continues to live out the remainder of its life under a balance between the inward force of gravity pulling it toward collapse and the outward force due to thermal effects and radiation pressure.

The proton–proton cycle is a series of three nuclear reactions that are believed to be the stages in the liberation of energy in the Sun and other stars rich in hydrogen. An overall view of the proton–proton cycle is that four protons combine to form an alpha particle and two positrons, with the release of 25 MeV of energy in the process.

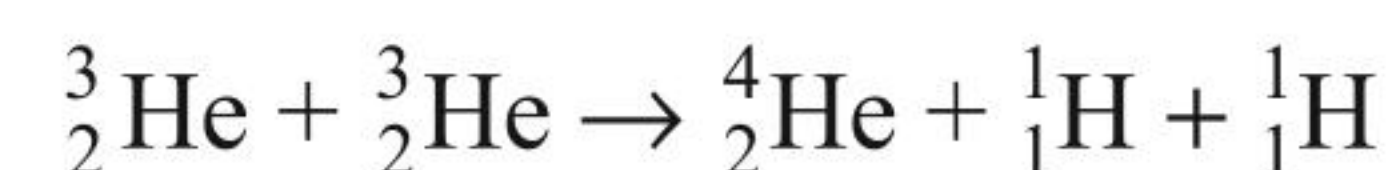
The specific steps in the proton–proton cycle are



This second reaction is followed by either hydrogen–helium fusion or helium–helium fusion:



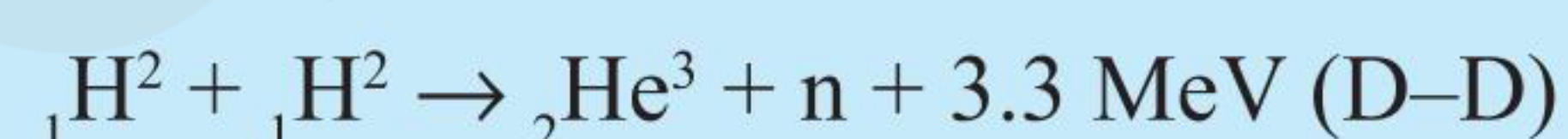
or



The energy liberated is carried primarily by gamma rays, positrons, and neutrinos, as can be seen from the reactions. The gamma rays are soon absorbed by the dense gas, thus, raising its temperature. The positrons combine with electrons to produce gamma rays, which in turn are also absorbed by the gas within a few centimeters. The neutrinos, however, almost never interact with matter; hence, they escape from the star, carrying about 2% of the generated energy with them. These energy-liberating fusion reactions are called thermonuclear fusion reactions. The hydrogen (fusion) bomb, first exploded in 1952, is an example of an uncontrolled thermonuclear fusion reaction.

Note:

- Some unstable light nuclei of A below 20, fuse together, the BE per nucleon increases and hence the excess energy is released. The easiest thermonuclear reaction that can be handled on earth is the fusion of two deuterons (D–D reaction) or fusion of a deuteron with a triton (D–T reaction).



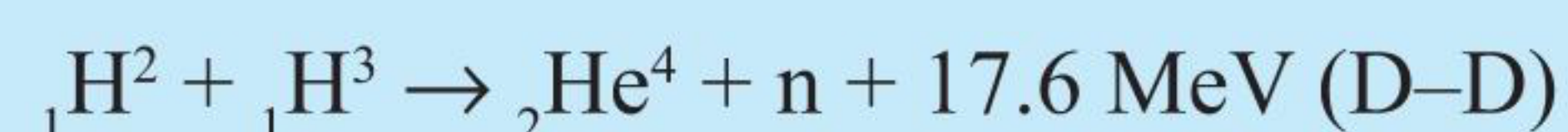
$$Q\text{-value} = [2(M_{\text{D}} - m_{\text{e}}) - \{(M_{\text{He-3}} - 2m_{\text{e}}) + m_{\text{n}}\}]c^2$$

$$= [2M_{\text{D}} - (M_{\text{He-3}} + m_{\text{n}})]c^2$$



$$Q\text{-value} = [2(M_{\text{D}} - m_{\text{e}}) - \{(M_{\text{T}} - m_{\text{e}}) + (M_{\text{H}} - m_{\text{e}})\}]c^2$$

$$= [2M_{\text{D}} - (M_{\text{T}} + M_{\text{H}})]c^2$$



$$Q\text{-value} = [\{(M_{\text{D}} - m_{\text{e}}) + (M_{\text{T}} - m_{\text{e}})\} - \{(M_{\text{He-4}} - 2m_{\text{e}}) + m_{\text{n}}\}]c^2$$

$$= [(M_{\text{D}} + M_{\text{T}}) - (M_{\text{He-4}} + m_{\text{n}})]c^2$$

In case of fission and fusion, $\Delta m = \Delta m_{\text{atom}} = \Delta m_{\text{nucleus}}$.

- These reactions take place at ultra-high temperature ($\cong 10^7$ – 10^9). At high pressure, it can take place at low temperature also. For these reactions to take place, nuclei should be brought up to 1 fermi distance which requires very high kinetic energy.
- Energy released in fusion exceeds the energy liberated in the fission of heavy nuclei.

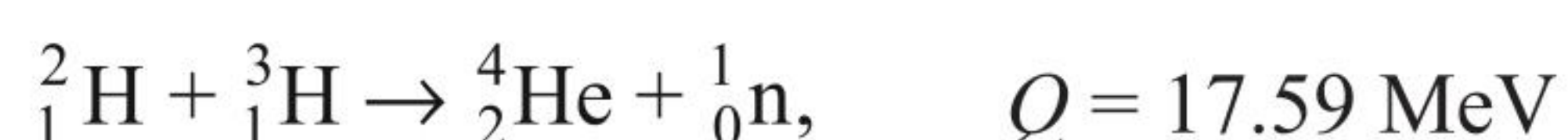
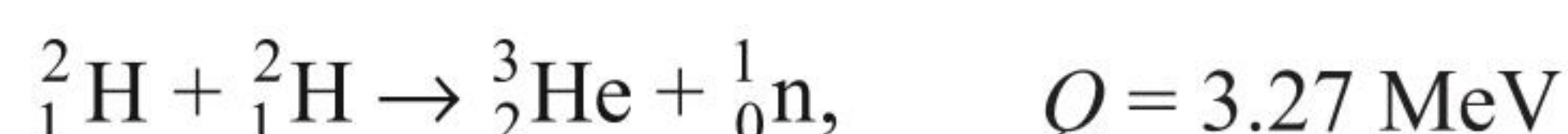
FUSION REACTORS

The enormous amount of energy released in fusion reactions suggests the possibility of harnessing this energy for useful purposes on earth. A great deal of effort is focused on developing a sustained and controllable thermonuclear reactor—a fusion power reactor. Controlled fusion is often called the ultimate energy source because of the vast availability of its fuel source, i.e., water. An additional advantage of fusion reactors is that comparatively few radioactive by products are formed. As noted in Eq. (iii), the end product of the fusion of hydrogen nuclei is safe, nonradioactive helium. Unfortunately, a thermonuclear

reactor that can deliver a net power output over a reasonable time interval is not yet a reality, and many difficulties must be solved before a successful device is constructed.

We have seen that the Sun's energy is based, in part, on a set of reactions in which ordinary hydrogen is converted to helium. Unfortunately, the proton-proton interaction is not suitable for use in a fusion reactor because the event requires very high pressures and densities. The process works in the Sun only because of the extremely high density of protons in the Sun's interior. In fact, even at the densities and temperature that exist at the center of the Sun, the average proton takes 14 billion years to react!

The fusion reactions that appear most promising in the construction of a fusion power reactor involve deuterium and tritium, which are isotopes of hydrogen. These reactions are



where the Q values refer to the amount of energy released per reaction. As noted earlier, deuterium is available in almost unlimited quantities from our lakes and oceans and is very inexpensive to extract. Tritium, however, is radioactive ($T_{1/2} = 12.3 \text{ yr}$) and undergoes beta decay to ${}^3\text{He}$. For this reason, tritium does not occur naturally to any great extent and must be artificially produced.

One of the major problems in obtaining energy from nuclear fusion is the fact that the Coulomb repulsion force between two charged nuclei must be overcome before they can fuse. The fundamental challenge is to give the two nuclei enough kinetic energy to overcome this repulsive force. This can be accomplished by heating the fuel to extremely high temperatures (about 10^8 K , far greater than the interior temperature of the Sun). As you might expect, such high temperatures are not easy to obtain in a laboratory or a power plant. At these high temperatures, the atoms are ionized and the system consists of a collection of electrons and nuclei, commonly referred to as a plasma.

ILLUSTRATION 7.54

On disintegration of one atom of ${}^{235}\text{U}$, the amount of energy obtained is 200 MeV. The power obtained in a reactor is 1000 kilo watt. How many atoms are disintegrated per second in the reactor? What is the decay in mass per hour?

Sol. Power produced in the reactor is

$$P = 1000 \text{ kW} = 1000 \times 10^3 \text{ W} = 10^6 \text{ J s}^{-1}$$

$$= \frac{10^6}{1.6 \times 10^{-19}} \text{ eV s}^{-1} = 6.25 \times 10^{18} \text{ MeV s}^{-1}$$

As in each disintegration 200 MeV energy is released, number of atoms disintegrated per second are

$$N = \frac{6.25 \times 10^{18}}{200} = 3.125 \times 10^{16} \text{ s}^{-1}$$

Energy released per second is 10^6 J .

Energy released per hour = $10^6 \times 60 \times 60 \text{ J}$

Thus, mass decay per hour can be given as

$$\Delta m = \frac{\Delta E}{c^2} \quad (\text{Einstein's mass-energy formula})$$

$$= \frac{10^6 \times 60 \times 60 \text{ J}}{(3 \times 10^8 \text{ m s}^{-1})^2} = 4 \times 10^{-8} \text{ kg} = 4 \times 10^{-5} \text{ g}$$

ILLUSTRATION 7.55

A deuterium reaction that occurs in an experimental fusion reactor is in two stages:

- Two deuterium (${}_1^2\text{D}$) nuclei fuse together to form a tritium nucleus, with a proton as a by product written as $\text{D}(\text{D}, \text{p})\text{T}$.
- A tritium nucleus fuses with another deuterium nucleus to form a helium ${}_2^4\text{He}$ nucleus with neutron as a by-product, written as $\text{T}(\text{D}, \text{n}){}_2^4\text{He}$.

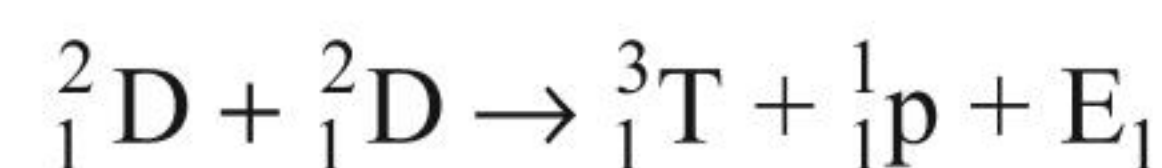
Compute: (a) the energy released in each of the two stages, (b) the energy released in the combined reaction per deuterium. (c) What percentage of the mass energy of the initial deuterium is released. Given,

$${}_1^2\text{D} = 2.014102 \text{ amu}; {}_1^3\text{T} = 3.016049; {}_2^4\text{He} = 4.002603 \text{ amu};$$

$${}_1^1\text{H} = 1.007825 \text{ amu}; {}_0^1\text{n} = 1.00665 \text{ amu}$$

Sol.

- (a) (i) The reaction involved in the process is



Mass defect of the above reaction is

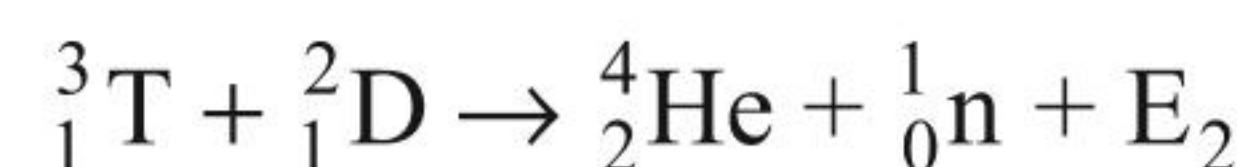
$$\Delta m = [2(2.014102) - (3.016049 + 1.007825)]$$

$$= 0.00433 \text{ amu}$$

Energy released in the process is

$$E_1 = 0.00433 \times 931.2 \text{ MeV} = 4.033 \text{ MeV}$$

- (ii) The reaction involved in the process is



Mass defect of above reaction is

$$\Delta m = [(3.016049 + 2.014102) - (4.002603 + 1.008665)]$$

$$= 0.01888 \text{ amu}$$

Energy released in the process is

$$E_2 = 0.01888 \times 931 \text{ MeV} = 17.585 \text{ MeV}$$

Total energy released in both processes is

$$17.585 + 4.033 = 21.618 \text{ MeV}$$

- (b) Energy released per deuterium atom is

$$E_1 = \frac{21.618}{3} = 7.206 \text{ MeV}$$

- (c) % of rest mass of ${}_1^2\text{D}$ released

$$f = \frac{7.206}{2.014102 \times 931} = 0.385\%$$

ILLUSTRATION 7.56

In the process of nuclear fission of 1 g uranium, the mass lost is 0.92 mg. The efficiency of power house run by the fission reactor is 10%. To obtain 400 megawatt power from the power house, how much uranium will be required per hour? ($c = 3 \times 10^8 \text{ m s}^{-1}$)

Sol. Power to be obtained from power house = 400 megawatt
 In this case, energy obtained per hour
 $= 400 \text{ megawatt} \times 1 \text{ hour}$
 $= (400 \times 10^6 \text{ W}) \times 3600 \text{ s}$
 $= 144 \times 10^{10} \text{ J}$

Here, only 10% of output is utilized. In order to obtain 144×10^{10} joule of useful energy, the output energy from the power house is given by

$$E = \frac{(144 \times 10^{10}) \times 100}{10} = 144 \times 10^{11} \text{ J}$$

Let this energy is obtained from a mass loss of Δm kg. Then,

$$(\Delta m)c^2 = 144 \times 10^{11} \text{ J}$$

$$\text{or } \Delta m = \frac{144 \times 10^{11}}{(3 \times 10^8)^2} = 16 \times 10^{-5} \text{ kg} = 0.16 \text{ g}$$

Since 0.92 mg ($= 0.92 \times 10^{-3} \text{ g}$) mass is lost in 1 g uranium, hence, for a mass loss of 0.16 g, the uranium required is given by

$$\Delta m' = \frac{1 \times 0.16}{0.92 \times 10^{-3}} = 174 \text{ g}$$

Thus, to run the power house, 174 g uranium is required per hour.

ILLUSTRATION 7.57

A nuclear reactor using ^{235}U generates 250 MW of electrical power. The efficiency of the reactor (i.e., efficiency of conversion of thermal energy into electrical energy) is 25%. What is the amount of ^{235}U used in the reactor per year? The thermal energy released per fission of ^{235}U is 200 MeV.

Sol. Rate of electrical energy generation is 250 MW = $250 \times 10^6 \text{ W}$ (or J s^{-1}). So, electrical energy generation is 250 MW = $250 \times 10^6 \text{ W}$ (or J s^{-1}). Therefore, electrical energy generated in 1 year is

$$(250 \times 10^6 \text{ J s}^{-1}) \times (365 \times 24 \times 60 \times 60 \text{ s}) = 7.884 \times 10^{15} \text{ J}$$

Thermal energy from fission of one ^{235}U nucleus is 200 MeV = $200 \times 1.6 \times 10^{-13} = 3.2 \times 10^{-11} \text{ J}$. Since the efficiency is 25%, the electrical energy obtained from the fission of one ^{235}U nucleus is

$$E_1 = 3.2 \times 10^{-11} \times \frac{25}{100} = 8.0 \times 10^{-12} \text{ J}$$

Therefore, the number of fissions of ^{235}U required in one year will be

$$N = \frac{7.884 \times 10^{15}}{8.0 \times 10^{-12}} = 9.855 \times 10^{26}$$

Number of moles of ^{235}U required per year is

$$n = \frac{9.855 \times 10^{26}}{6.023 \times 10^{23}} = 1.636 \times 10^3$$

Therefore, mass of ^{235}U required per year is

$$m = 1.636 \times 10^3 \times 235 = 3.844 \times 10^5 \text{ g} = 384.4 \text{ kg}$$

ILLUSTRATION 7.58

What is the power output of a ^{235}U reactor if it takes 30 days to use up 2 kg of fuel, and if each fission gives 185 MeV of usable energy?

Sol. 235 amu of uranium gives 185 MeV energy. Therefore, the energy given by 1 amu of ^{235}U

$$= \frac{185}{235} \text{ MeV} = \frac{185}{235} \times 1.6 \times 10^{-13} \text{ J}$$

But 1 amu = $1.66 \times 10^{-27} \text{ kg}$. Therefore, energy released by $1.66 \times 10^{-27} \text{ kg}$ of ^{235}U

$$= \frac{185 \times 1.6 \times 10^{-13}}{235}$$

Hence, energy released by 2 kg of ^{235}U ,

$$W = \frac{185 \times 1.6 \times 10^{-13} \times 2}{235 \times 1.66 \times 10^{-27}} = 1.517 \times 10^{14} \text{ J}$$

Therefore, power output of reactor = $\frac{W}{t} = \frac{1.517 \times 10^{14}}{(30 \text{ days})}$

$$P = \frac{1.517 \times 10^{14}}{30 \times 24 \times 60 \times 60} = 5.85 \times 10^7 \text{ W}$$

ILLUSTRATION 7.59

(a) A nuclear explosion is designed to deliver 1 MW of heat energy, how many fission events must be required in a second to attain this power level. (b) If this explosion is designed with a nuclear fuel consisting of uranium 235 to run a reactor at this power level for one year, then calculate the amount of fuel needed. You can assume that the amount of energy released per fission event is 200 MeV.

Sol.

(a) Energy per fission, $\epsilon = 200 \text{ MeV}$

$$= 200 \times 1.6 \times 10^{-13} \text{ J} = 3.2 \times 10^{-11} \text{ J}$$

Power delivered, $P = 1 \text{ MW} = 10^6 \text{ J s}^{-1}$

If t is the time, then energy delivered = Pt

$$\therefore \text{Number of fissions} = \frac{Pt}{\epsilon} = \frac{10^6 \times t}{3.2 \times 10^{-11}}$$

If $t = 1 \text{ s}$, number of fissions per second is

$$\frac{10^6}{3.2 \times 10^{-11}} = 3.125 \times 10^{16}$$

(b) Total energy required for 1 year = Pt

$$= 10^6 \times 3.15 \times 10^7 \text{ J} = 3.15 \times 10^{13} \text{ J}$$

Hence, number of fissions required is

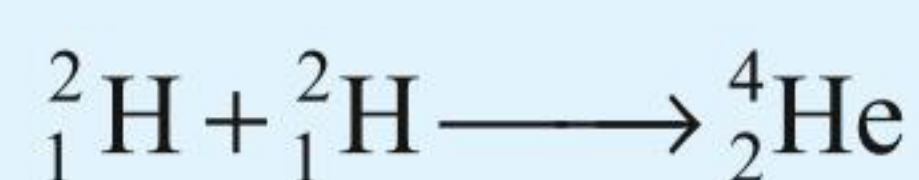
$$\frac{3.15 \times 10^{13}}{3.2 \times 10^{-11}} = 9.85 \times 10^{23}$$

Mass of uranium required is

$$9.85 \times 10^{23} \times \frac{235}{6.02 \times 10^{23}} \text{ g} = 384.5 \text{ g}$$

CONCEPT APPLICATION EXERCISE 7.4

1. If 200 MeV energy is released in the fission of a single nucleus ${}_{92}^{235}\text{U}$, how many fissions must occur per second to produce a power of 1 kW?
2. It is proposed to use nuclear fission reaction:



in a nuclear reactor of 200 MW rating. If the energy from the above reaction is used with a 25% efficiency in the reactor, how many grams of deuterium will be needed per day? The masses of ${}^2_1\text{H}$ and ${}^4_2\text{He}$ are 2.0141 u and 4.0026 u respectively.

3. Fission of one atom of ${}_{92}^{235}\text{U}$ releases 200 MeV of energy. What mass of ${}_{92}^{235}\text{U}$ would be released in a reactor to supply one million kW power per year?
4. What is the power output of a ${}_{92}^{235}\text{U}$ reactor if it takes 30 days to use up 2 kg of fuel and if each fission gives 185 MeV of usable energy?
5. Assuming that in a star, three alpha particles join in a single reaction to form ${}^{12}_6\text{C}$ nucleus, calculate the energy released in this reaction. Given mass of ${}^4_2\text{He} = 4.002604\text{u}$ and that of ${}^{12}_6\text{C} = 12.000000\text{u}$.
6. In a nuclear reactor, fission is produced in 1 g for U^{235} (235.0439 u) in 24 hours by slow neutrons (1.0087 u). Assume that ${}_{35}^{92}\text{Kr}$ (91.8973 u) and ${}_{56}^{141}\text{Ba}$ (140.9139 amu) are produced in all reactions and no energy is lost.
(i) Write the complete reaction (ii) Calculate the total energy produced in kilowatt hour. Given $1\text{u} = 931\text{ MeV}$.

7. It is proposed to use the nuclear fusion reaction : ${}_1\text{H}^2 + {}_1\text{H}^2 \rightarrow {}_2\text{He}^4$ in a nuclear reactor of 200 MW rating. If the energy from above reaction is used with at 25% efficiency in the reactor, how many grams of deuterium will be needed per day. (Mass of ${}_1\text{H}^2$ is 2.0141 u and mass of ${}_2\text{He}^4$ is 4.0026 u)

8. How long can an electric lamp of 100 W be kept glowing by fusion of 2.0 kg of deuterium? Take the fusion reaction as



9. Estimate the minimum amount of ${}_{92}^{235}\text{U}$ that needs to undergo fission in order to run a 1000 MW power reactor per year of continuous operation. Assume an efficiency of about 33 percent.
10. The ${}^{226}_{88}\text{Ra}$ nucleus undergoes α -decay to ${}^{226}_{88}\text{Rn}$. Calculate the amount of energy liberated in this decay. Take the mass of ${}^{226}_{88}\text{Ra}$ to be 226.025 402 u, that of ${}^{226}_{88}\text{Rn}$ to be 222.017571 u, and that of ${}^4_2\text{He}$ to be 4.002602 u.
11. Calculate the energy released when three alpha particles combine to form a ${}^{12}_6\text{C}$ nucleus. The atomic mass of ${}^4_2\text{He}$ is 4.002603 u.

ANSWERS

- | | | |
|--|------------------------------|--------------|
| 1. 3.125×10^{13} | 2. 120.3 g | 3. 384.5 kg |
| 4. 58.54 MW | | 5. 7.277 MeV |
| 6. (i) ${}_{92}\text{U}^{235} + {}_0\text{n}^1 \rightarrow {}_{56}\text{Ba}^{141} + {}_{36}\text{Kr}^{92} + 3{}_0\text{n}^1$ | | |
| (ii) $2.28 \times 10^4\text{ kWh}$ | | |
| 7. 120.83 g/day | 8. $5 \times 10^4\text{ yr}$ | |
| 9. 10 tons | 10. 4.87 MeV | 11. 7.27 MeV |

Exercises

Single Correct Answer Type

1. Consider two arbitrary decay equations and mark the correct alternative(s) given below.



Given: $M({}_{92}^{230}\text{U}) = 230.033927 \text{ u}$,

$M({}_{92}^{229}\text{U}) = 229.03349 \text{ u}$, $m_n = 1.008665 \text{ u}$,

$M({}_{91}^{229}\text{Pa}) = 229.032089 \text{ u}$, $m_p = 1.007825 \text{ u}$, $1 \text{ amu} = 931.5 \text{ MeV}$.

- (1) Only decay (i) is possible.
 (2) Only decay (ii) is possible.
 (3) Both the decays are possible.
 (4) Neither of the two decays is possible.
2. In a sample of rock, the ratio of ${}^{206}\text{Pb}$ to ${}^{238}\text{U}$ nuclei is found to be 0.5. The age of the rock is (given half-life of U^{238} is 4.5×10^9 years)
- (1) 2.25×10^9 year (2) $4.5 \times 10^9 \ln 3$ year
 (3) $4.5 \times 10^9 \frac{\ln\left(\frac{3}{2}\right)}{\ln 2}$ year (4) $2.25 \times 10^9 \ln\left(\frac{3}{2}\right)$ year
3. A radioactive sample decays by 63% of its initial value in 10 s. It would have decayed by 50% of its initial value in
- (1) 7 s (2) 14 s
 (3) 0.7 s (4) 1.4 s
4. A nucleus moving with velocity $\vec{v}$ emits an α -particle. Let the velocities of the α -particle and the remaining nucleus be $\vec{v}_1$ and $\vec{v}_2$ and their masses be m_1 and m_2 , then
- (1) $\vec{v}$, $\vec{v}_1$, and $\vec{v}_2$ must be parallel to each other
 (2) none of the two of $\vec{v}$, $\vec{v}_1$, and $\vec{v}_2$ should be parallel to each other
 (3) $\vec{v}_1 + \vec{v}_2$ must be parallel to $\vec{v}$
 (4) $m_1 \vec{v}_1 + m_2 \vec{v}_2$ must be parallel to $\vec{v}$
5. A certain radioactive material can undergo three different types of decay, each with a different decay constant λ , 2λ , and 3λ . Then, the effective decay constant λ_{eff} is
- (1) 6λ (2) 4λ
 (3) 2λ (4) 3λ
6. In an α -decay, the kinetic energy of α -particle is 48 MeV and Q value of the reaction is 50 MeV. The mass number of the mother nucleus is (assume that daughter nucleus is in ground state)
- (1) 96 (2) 100
 (3) 104 (4) none of these
7. A sample of radioactive material decays simultaneously by two processes A and B with half-lives $\frac{1}{2}$ and $\frac{1}{4}$ h, respectively. For the first half hour it decays with the process A, next one hour with the process B, and for further half an

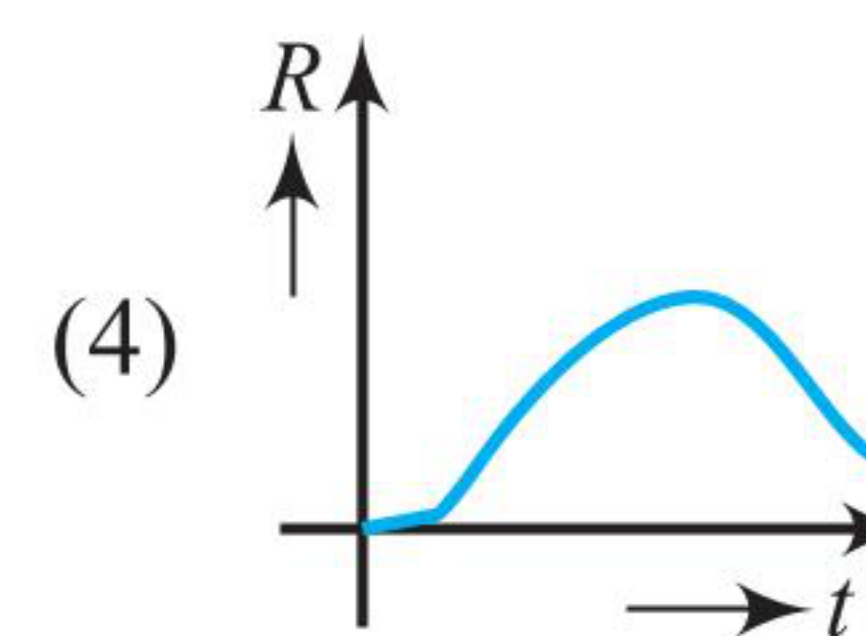
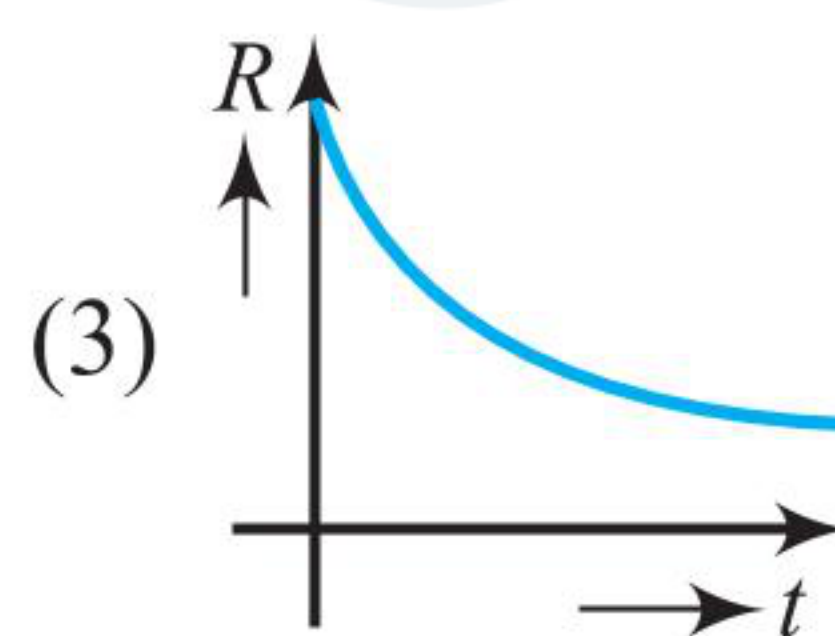
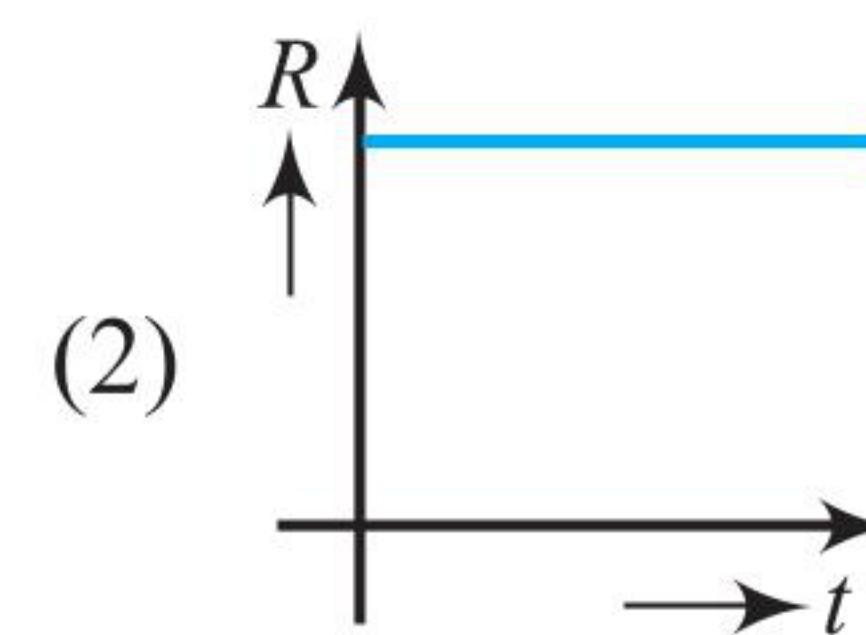
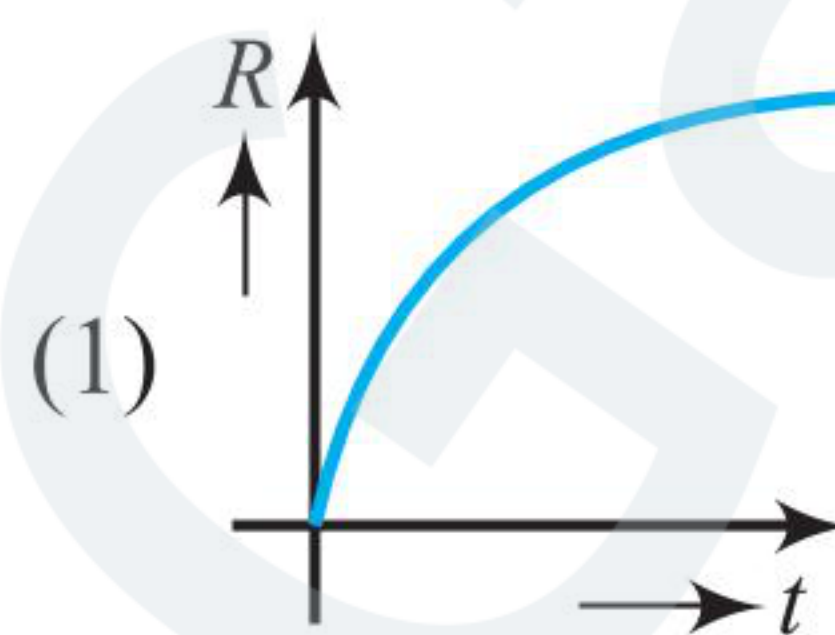
hour with both A and B. If, originally, there were N_0 nuclei, find the number of nuclei after 2 h of such decay.

- (1) $\frac{N_0}{(2)^8}$ (2) $\frac{N_0}{(2)^4}$
 (3) $\frac{N_0}{(2)^6}$ (4) $\frac{N_0}{(2)^5}$

8. In which of the following processes, the number of protons in the nucleus increase?

- (1) α -decay (2) β -decay
 (3) β^+ -decay (4) k-capture

9. A radioactive nucleus X decays to a stable nucleus Y. Then, time graph of rate of formation of Y against time t will be



10. 90% of a radioactive sample is left undecayed after time t has elapsed. What percentage of the initial sample will decay in a total time $2t$?

- (1) 20% (2) 19%
 (3) 40% (4) 38%

11. A radioactive element X converts into another stable element Y. Half-life of X is 2 h. Initially, only X is present. After time t , the ratio of atoms of X and Y is found to be 1:4. Then t in hours is

- (1) 2 (2) 4
 (3) between 4 and 6 (4) 6

12. $A \xrightarrow{\lambda} B \xrightarrow{2\lambda} C$

$T=0$ N_0 0 0
 T N_1 N_2 N_3

The ratio of N_1 to N_2 when N_2 is maximum is

- (1) at no time this is possible (2) 2
 (3) $1/2$ (4) $\frac{\ln 2}{2}$

13. The binding energy of an electron in the ground state of He-atom is $E_0 = 24.6 \text{ eV}$. The energy required to remove both the electrons from the atom is

- (1) 24.6 eV (2) 79.0 eV
 (3) 54.4 eV (4) none of these

14. The mean life time of a radionuclide, if its activity decreases by 4% for every 1 h, would be (product is non-radioactive, i.e., stable)

- (1) 25 h (2) 1.042 h
 (3) 2 h (4) 30 h

15. On an average, a neutron loses half of its energy per collision with a quasi-free proton. To reduce a 2 MeV neutron to a thermal neutron having energy 0.04 eV, the number of collisions required is nearly
 (1) 50 (2) 52
 (3) 26 (4) 15
16. Atomic masses of two isobars ${}^{64}_{29}\text{Cu}$ and ${}^{64}_{30}\text{Zn}$ are 63.9298 u and 63.9292 u, respectively. It can be concluded from this data that
 (1) both the isobars are stable
 (2) ${}^{64}\text{Zn}$ is radioactive, decaying to ${}^{64}\text{Cu}$ through β -decay
 (3) ${}^{64}\text{Cu}$ is radioactive, decaying to ${}^{64}\text{Zn}$ through β -decay
 (4) ${}^{64}\text{Cu}$ is radioactive, decaying to ${}^{64}\text{Zn}$ through γ -decay
17. If a nucleus such as ${}^{226}\text{Ra}$ that is initially at rest undergoes α -decay, then which of the following statements is true?
 (1) The alpha particle has more kinetic energy than the daughter nucleus.
 (2) The alpha particle has less kinetic energy than the daughter nucleus.
 (3) The alpha particle and daughter nucleus both have same kinetic energy
 (4) We cannot say anything about kinetic energy of alpha particle and daughter nucleus.
18. If the Q value of an endothermic reaction is 11.32 MeV, then the minimum energy of the reactant nuclei to carry out the reaction is (in laboratory frame of reference)
 (1) 11.32 MeV (2) less than 11.32 MeV
 (3) greater than 11.32 MeV (4) Data is insufficient
19. 1.00 kg of ${}^{235}\text{U}$ undergoes fission process. If energy released per event is 200 MeV, then the total energy released is
 (1) 5.12×10^{24} MeV (2) 6.02×10^{23} MeV
 (3) 5.12×10^{26} MeV (4) 6.02×10^{26} MeV
20. Mark out the incorrect statement.
 (1) A free neutron can transform itself into photon.
 (2) A free proton can transform itself into neutron.
 (3) In beta minus decay, the electron originates from nucleus.
 (4) All of the above.
21. U-235 can decay by many ways, let us here consider only two ways, A and B. In decay of U-235 by means of A, the energy released per fission is 210 MeV while in B it is 186 MeV. Then, the uranium 235 sample is more likely to decay by
 (1) scheme A
 (2) scheme B
 (3) equally likely for both schemes
 (4) it depends on half-life of schemes A and B
22. ${}^{49}_{19}\text{K}$ isotope of potassium has a half-life of 1.4×10^9 yr and decays to form stable argon, ${}^{40}_{18}\text{Ar}$. A sample of rock has been taken which contains both potassium and argon in the ratio 1:7, i.e.,

$$\frac{\text{Number of potassium-40 atoms}}{\text{Number of argon-40 atoms}} = \frac{1}{7}$$
- Assuming that when the rock was formed no argon-40 was present in the sample and none has escaped subsequently, determine the age of the rock.
 (1) 4.2×10^9 years (2) 9.8×10^9 years
 (3) 1.4×10^9 years (4) 10×10^9 years
23. The probability of survival of a radioactive nucleus for one mean life is
 (1) $\frac{1}{e}$ (2) $1 - \frac{1}{e}$
 (3) $\frac{\ln 2}{e}$ (4) $1 - \frac{\ln 2}{e}$
24. Consider one of fission reactions of ${}^{235}\text{U}$ by thermal neutrons
 ${}^{235}_{92}\text{U} + n \rightarrow {}^{94}_{38}\text{Sr} + {}^{140}_{54}\text{Xe} + 2n$. The fission fragments are however unstable and they undergo successive β -decay until ${}^{94}_{38}\text{Sr}$ becomes ${}^{94}_{40}\text{Zr}$ and ${}^{140}_{54}\text{Xe}$ becomes ${}^{140}_{58}\text{Ce}$. The energy released in this process is
 [Given: $m({}^{235}\text{U}) = 235.439$ u, $m(n) = 1.00866$ u, $m({}^{94}\text{Zr}) = 93.9064$ u, $m({}^{140}\text{Ce}) = 139.9055$ u, $1 \text{ u} = 931 \text{ MeV}$]
 (1) 156 MeV (2) 208 MeV
 (3) 456 MeV (4) cannot be computed
25. A star initially has 10^{40} deuterons. It produces energy via the processes ${}^2_1\text{H} + {}^2_1\text{H} \rightarrow {}^3_1\text{He} + p$ and ${}^2_1\text{H} + {}^3_1\text{H} \rightarrow {}^4_2\text{He} + n$. If the average power radiated by the star is 10^{16} W, the deuteron supply of the star is exhausted in a time of the order of
 [Given: $M({}^2\text{H}) = 2.014$ u, $M(n) = 1.008$ u, $M(p) = 1.008$ u, and $M({}^4\text{He}) = 4.001$ u]
 (1) 10^6 s (2) 10^8 s
 (3) 10^{12} s (4) 10^{16} s
26. Two radioactive materials X_1 and X_2 have decay constants 10λ and λ , respectively. If initially they have the same number of nuclei, the ratio of the number of nuclei of X_1 to that of X_2 will be $1/e$ after a time
 (1) $\frac{1}{10\lambda}$ (2) $\frac{1}{11\lambda}$
 (3) $\frac{11}{10\lambda}$ (4) $\frac{1}{9\lambda}$
27. A radioactive substance is being consumed at a constant rate of 1 s^{-1} . After what time will the number of radioactive nuclei become 100. Initially, there were 200 nuclei present.
 (1) 1 s (2) $\frac{1}{\ln(2)} \text{ s}$
 (3) $\ln(2) \text{ s}$ (4) 2 s
28. A radioactive isotope is being produced at a constant rate X . Half-life of the radioactive substance is Y . After some time, the number of radioactive nuclei become constant. The value of this constant is
 (1) $\frac{XY}{\ln(2)}$ (2) XY
 (3) $(XY) \ln(2)$ (4) $\frac{X}{Y}$

29. A radioactive substance X decays into another radioactive substance Y. Initially, only X was present. λ_x and λ_y are the disintegration constants of X and Y. N_y will be maximum when

$$(1) \frac{N_y}{N_x - N_y} = \frac{\lambda_y}{\lambda_x - \lambda_y} \quad (2) \frac{N_x}{N_x - N_y} = \frac{\lambda_x}{\lambda_x - \lambda_y}$$

$$(3) \lambda_y N_y = \lambda_x N_x \quad (4) \lambda_y N_x = \lambda_x N_y$$

30. There are two radio nuclei A and B. A is an alpha emitter and B a beta emitter. Their disintegration constants are in the ratio of 1:2. What should be the ratio of number of atoms of A and B at any time t so that probabilities of getting alpha and beta particles are same at that instant?

$$(1) 2:1 \quad (2) 1:2$$

$$(3) e \quad (4) e^{-1}$$

31. Half-life of a radioactive substance A is two times the half-life of another radioactive substance B. Initially, the number of A and B are N_A and N_B , respectively. After three half-lives of A, number of nuclei of both are equal. Then, the ratio N_A/N_B is

$$(1) 1/4 \quad (2) 1/8$$

$$(3) 1/3 \quad (4) 1/6$$

32. There are two radioactive substances A and B. Decay constant of B is two times that of A. Initially, both have equal number of nuclei. After n half-lives of A, rates of disintegration of both are equal. The value of n is

$$(1) 1 \quad (2) 2$$

$$(3) 4 \quad (4) \text{all of these}$$

33. A radioactive nucleus is being produced at a constant rate α per second. Its decay constant is λ . If N_0 are the number of nuclei at time $t = 0$, then maximum number of nuclei possible are

$$(1) \frac{\alpha}{\lambda} \quad (2) N_0 \frac{\alpha}{\lambda}$$

$$(3) N_0 \quad (4) \frac{\lambda}{\alpha} + N_0$$

34. In a sample of a radioactive substance, what fraction of the initial nuclei will remain undecayed after a time $t = T/2$, where T = half-life of radioactive substance?

$$(1) \frac{1}{\sqrt{2}} \quad (2) \frac{1}{2\sqrt{2}}$$

$$(3) \frac{1}{4} \quad (4) \frac{1}{\sqrt{2} - 1}$$

35. The activity of a radioactive substance is R_1 at time t_1 and R_2 at time t_2 ($> t_1$). Its decay constant is λ . Then

$$(1) R_1 t_1 \quad (2) R_2 = R_1 e^{\lambda(t_1 - t_2)}$$

$$(3) \frac{R_1 - R_2}{t_2 - t_1} = \text{constant} \quad (4) R_2 = R_1 e^{\lambda(t_2 - t_1)}$$

36. In problem 35, number of atoms decayed between time interval t_1 and t_2 are

$$(1) \frac{\ln(2)}{\lambda} (R_1 R_2) \quad (2) R_1 e^{-\lambda t_2} - R_2 e^{-\lambda t_2}$$

$$(3) \lambda (R_1 - R_2) \quad (4) \left(\frac{R_1 - R_2}{\lambda} \right)$$

37. The ratio of molecular mass of two radioactive substances is 3/2 and the ratio of their decay constants is 4/3. Then, the ratio of their initial activity per mole will be

$$(1) 2 \quad (2) \frac{8}{9}$$

$$(3) \frac{4}{3} \quad (4) \frac{9}{8}$$

38. N_1 atoms of a radioactive element emit N_2 beta particles per second. The decay constant of the element is (in s^{-1})

$$(1) \frac{N_1}{N_2} \quad (2) \frac{N_2}{N_1}$$

$$(3) N_1 \ln(2) \quad (4) N_2 \ln(2)$$

39. The binding energies of nuclei X and Y are E_1 and E_2 , respectively. Two atoms of X fuse to give one atom of Y and an energy Q is released. Then,

$$(1) Q = 2E_1 - E_2 \quad (2) Q = E_2 - 2E_1$$

$$(3) Q < 2E_1 - E_2 \quad (4) Q > E_2 - 2E_1$$

40. Binding energy per nucleon of ${}_1\text{H}^2$ and ${}_2\text{He}^4$ are 1.1 MeV and 7.0 MeV, respectively. Energy released in the process ${}_1\text{H}^2 + {}_1\text{H}^2 = {}_2\text{He}^4$ is

$$(1) 20.8 \text{ MeV} \quad (2) 16.6 \text{ MeV}$$

$$(3) 25.2 \text{ MeV} \quad (4) 23.6 \text{ MeV}$$

41. ${}_{92}\text{U}^{238}$ absorbs a neutron. The product emits an electron. This product further emits an electron. The result is

$$(1) {}_{94}\text{Pu}^{239} \quad (2) {}_{90}\text{Pu}^{239}$$

$$(3) {}_{93}\text{Pu}^{237} \quad (4) {}_{94}\text{Pu}^{237}$$

42. The activity of a radioactive element decreases to one-third of the original activity A_0 in a period of 9 years. After a further lapse of 9 years, its activity will be

$$(1) A_0 \quad (2) \frac{2}{3} A_0$$

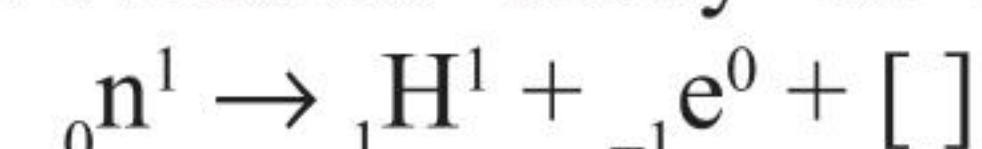
$$(3) \frac{A_0}{9} \quad (4) \frac{A_0}{6}$$

43. The half-life of a radioactive decay is x times its mean life. The value of x is

$$(1) 0.3010 \quad (2) 0.6930$$

$$(3) 0.6020 \quad (4) \frac{1}{0.6930}$$

44. Neutron decay in the free space is given as follows:



Then, the parenthesis represents

$$(1) \text{photon} \quad (2) \text{graviton}$$

$$(3) \text{neutrino} \quad (4) \text{antineutrino}$$

45. A nucleus ${}_Z^AX$ emits an α -particle. The resultant nucleus emits a β^+ particle. The respective atomic and mass numbers of the final nucleus will be

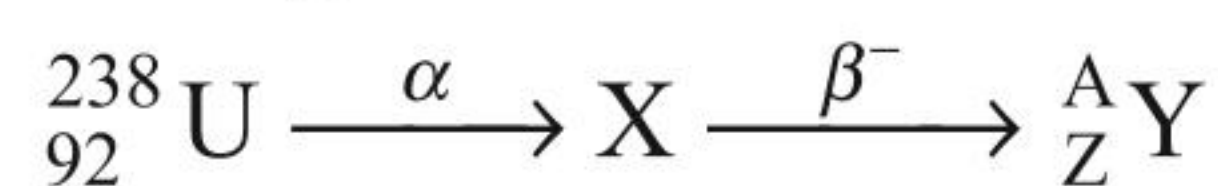
$$(1) Z - 3, A - 4 \quad (2) Z - 1, A - 4$$

$$(3) Z - 2, A - 4 \quad (4) Z, A - 2$$

46. Certain radioactive substance reduces to 25% of its value in 16 days. Its half-life is

- (1) 32 days (2) 8 days
(3) 64 days (4) 28 day

47. In the disintegration series



the values of Z and A , respectively, will be

- (1) 92, 326 (2) 88, 230
(3) 90, 234 (4) 91, 234

48. In the case of thorium ($A = 232$ and $Z = 90$), we obtain an isotope of lead ($A = 208$ and $Z = 82$) after some radioactive disintegrations. The number of α - and β -particles emitted are, respectively,

- (1) 6, 3 (2) 6, 4
(3) 5, 5 (4) 4, 6

49. The initial activity of a certain radioactive isotope was measured as $16000 \text{ counts min}^{-1}$. Given that the only activity measured was due to this isotope and that its activity after 12 h was $2100 \text{ counts min}^{-1}$, its half-life, in hours, is nearest to [Given $\log_e (7.2) = 2$]

- (1) 9.0 (2) 6.0
(3) 4.0 (4) 3.0

50. The minimum frequency of a γ -ray that causes a deuteron to disintegrate into a proton and a neutron is ($m_d = 2.0141 \text{ amu}$, $m_p = 1.0078 \text{ amu}$, $m_n = 1.0087 \text{ amu}$.)

- (1) $2.7 \times 10^{20} \text{ Hz}$ (2) $5.4 \times 10^{20} \text{ Hz}$
(3) $10.8 \times 10^{20} \text{ Hz}$ (4) $21.6 \times 10^{20} \text{ Hz}$

51. The fission of a heavy nucleus gives, in general, two smaller nuclei, two or three neutrons, some β -particles, and some γ -radiation. It is always true that the nuclei produced

- (1) have a total rest-mass that is greater than that of the original nucleus
(2) have large kinetic energies that carry off the greater part of the energy released
(3) travel in exactly opposite directions
(4) have neutron-to-proton ratios that are too low for stability

52. The rest mass of a deuteron is equivalent to an energy of 1876 MeV, that of a proton to 939 MeV, and that of a neutron to 940 MeV. A deuteron may disintegrate to a proton and a neutron if it

- (1) emits an γ -ray photon of energy 2 MeV
(2) captures an γ -ray photon of energy 2 MeV
(3) emits an γ -ray photon of energy 3 MeV
(4) captures an γ -ray photon of energy 3 MeV

53. A newly prepared radioactive nuclide has a decay constant λ of 10^{-6} s^{-1} . What is the approximate half-life of the nuclide?

- (1) 1 hour (2) 1 day
(3) 1 week (4) 1 month

54. Samples of two radioactive nuclides, X and Y, each have equal activity A at time $t = 0$. X has a half-life of 24 years and Y a half-life of 16 years. The samples are mixed together. What will be the total activity of the mixture at $t = 48$ years?

- (1) $\frac{1}{2} A_0$ (2) $\frac{1}{4} A_0$
(3) $\frac{3}{16} A_0$ (4) $\frac{3}{8} A_0$

55. A sample of a radioactive element has a mass of 10 g at an instant $t = 0$. The approximate mass of this element in the sample after two mean lives is

- (1) 1.35 g (2) 2.50 g
(3) 3.70 g (4) 6.30 g

56. After an interval of one day, $1/16$ th initial amount of a radioactive material remains in a sample. Then, its half-life is

- (1) 6 h (2) 12 h
(3) 1.5 h (4) 3 h

57. The half-life of At is 100 μs . The time taken for the radioactivity of a sample of At to decay to $1/16$ th of its initial value is

- (1) 400 μs (2) 6.3 μs
(3) 40 μs (4) 300 μs

58. A stationary thorium nucleus ($A = 220$, $Z = 90$) emits an alpha particle with kinetic energy E_α . What is the kinetic energy of the recoiling nucleus?

- (1) $\frac{E_\alpha}{108}$ (2) $\frac{E_\alpha}{110}$
(3) $\frac{E_\alpha}{55}$ (4) $\frac{E_\alpha}{54}$

59. The fraction of a radioactive material which remains active after time t is $9/16$. The fraction which remains active after time $t/2$ will be

- (1) $\frac{4}{5}$ (2) $\frac{7}{8}$
(3) $\frac{3}{5}$ (4) $\frac{3}{4}$

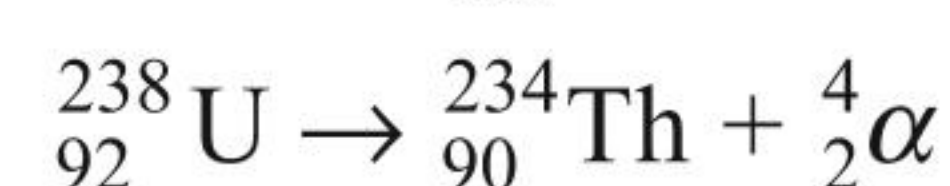
60. The rate of decay of a radioactive element at any instant is $10^3 \text{ disintegrations s}^{-1}$. If the half-life of the elements is 1 s, then the rate of decay after 1 s will be

- (1) 500 s^{-1} (2) 1000 s^{-1}
(3) 250 s^{-1} (4) 2000 s^{-1}

61. If 10% of a radioactive substance decays in every 5 year, then the percentage of the substance that will have decayed in 20 years will be

- (1) 40% (2) 50%
(3) 65.6% (4) 34.4%

62. Stationary nucleus ${}^{238}_{92}\text{U}$ decays by a emission generating a total kinetic energy T :



What is the kinetic energy of the α -particle?

- (1) Slightly less than $T/2$
(2) $T/2$
(3) Slightly less than T
(4) Slightly greater than T

63. A freshly prepared radioactive source of half-life 2 h emits radiation of intensity which is 64 times the permissible safe level. The minimum time after which it would be possible to work safely with the source is
 (1) 6 h (2) 12 h
 (3) 24 h (4) 128 h
64. Uranium ores contain one radium-226 atom for every 2.8×10^6 uranium-238 atoms. Calculate the half-life of ${}_{92}\text{U}^{238}$, given that the half-life of ${}_{88}\text{Ra}^{226}$ is 1600 years (${}_{88}\text{Ra}^{226}$ is a decay product of ${}_{92}\text{U}^{238}$).
 (1) 1.75×10^3 years (2) $1600 \times \frac{238}{92}$ years
 (3) 4.5×10^9 years (4) 1600×238 years
65. Plutonium has atomic mass 210 and a decay constant equal to $5.8 \times 10^{-8} \text{ s}^{-1}$. The number of α -particles emitted per second by 1 mg plutonium is
 (Avogadro's constant = 6.0×10^{23})
 (1) 1.7×10^9 (2) 1.7×10^{11}
 (3) 2.9×10^{11} (4) 3.4×10^9
66. At any instant, the ratio of the amounts of two radioactive substances is 2:1. If their half-lives be, respectively, 12 h and 16 h, then after two days, what will be the ratio of the substances?
 (1) 1:1 (2) 2:1
 (3) 1:2 (4) 1:4
67. The radioactivity of a sample is R_1 at a time T_1 and R_2 at a time T_2 . If the half-life of the specimen is T , the number of atoms that have disintegrated in the time $(T_2 - T_1)$ is proportional to
 (1) $R_1 T_1 = R_2 T_2$ (2) $R_1 - R_2$
 (3) $\frac{(R_1 - R_2)}{T}$ (4) $(R_1 - R_2)T$
68. Half-lives of two radioactive substances A and B are, respectively, 20 min and 40 min. Initially, the samples of A and B have equal number of nuclei. After 80 min, the ratio of the remaining number of A and B nuclei is
 (1) 1:16 (2) 4:1
 (3) 1:4 (4) 1:1
69. A radioactive nucleus can decay by two different processes. The mean value period for the first process is t_1 and that for the second process is t_2 . The effective mean value period for the two processes is
 (1) $\frac{t_1 + t_2}{2}$ (2) $t_1 + t_2$
 (3) $\sqrt{t_1 t_2}$ (4) $\frac{t_1 t_2}{t_1 + t_2}$
70. The half-life of radium is 1620 years and its atomic weight is 226. The number of atoms that will decay from its 1 g sample per second will be
 (1) 3.6×10^{10} (2) 3.6×10^{12}
 (3) 3.1×10^{15} (4) 31.1×10^{15}
71. The nuclear radius of a nucleus with nucleon number 16 is $3 \times 10^{-15} \text{ m}$. Then, the nuclear radius of a nucleus with nucleon number 128 is
 (1) $3 \times 10^{-15} \text{ m}$ (2) $1.5 \times 10^{-15} \text{ m}$
 (3) $6 \times 10^{-15} \text{ m}$ (4) $4.5 \times 10^{-15} \text{ m}$
72. The nuclear radius of ${}_{8}\text{O}^{16}$ is $3 \times 10^{-15} \text{ m}$. If an atomic mass unit is $1.67 \times 10^{-27} \text{ kg}$, then the nuclear density is approximately
 (1) $2.35 \times 10^{17} \text{ g cm}^{-3}$ (2) $2.35 \times 10^{17} \text{ kg m}^{-3}$
 (3) $2.35 \times 10^{17} \text{ g m}^{-3}$ (4) $2.35 \times 10^{17} \text{ kg mm}^{-3}$
73. What would be the energy required to dissociate completely 1 g of Ca-40 into its constituent particles?
 Given: Mass of proton = 1.007277 amu,
 Mass of neutron = 1.00866 amu,
 Mass of Ca-40 = 39.97545 amu
 (take 1 amu = 931 MeV)
 (1) $4.813 \times 10^{24} \text{ MeV}$ (2) $4.813 \times 10^{24} \text{ eV}$
 (3) $4.813 \times 10^{23} \text{ MeV}$ (4) None of the above
74. In fission, the percentage of mass converted into energy is about
 (1) 10% (2) 1%
 (3) 0.1% (4) 0.01%
75. In the nuclear reaction ${}_1\text{H}^2 + {}_1\text{H}^2 \rightarrow {}_2\text{He}^3 + {}_0\text{n}^1$ if the mass of the deuterium atom = 2.014741 amu, mass of ${}_2\text{He}^3$ atom = 3.016977 amu, and mass of neutron = 1.008987 amu, then the Q value of the reaction is nearly
 (1) 0.00352 MeV (2) 3.27 MeV
 (3) 0.82 MeV (4) 2.45 MeV
76. Assuming that about 20 MeV of energy is released per fusion reaction
 ${}_1\text{H}^2 + {}_1\text{H}^2 \rightarrow {}_2\text{He}^4 + \text{E} + \text{other particles}$
 then the mass of ${}_1\text{H}^2$ consumed per day in a fusion reactor of power 1 megawatt will approximately be
 (1) 0.001 g (2) 0.1 g
 (3) 10.0 g (4) 1000 g
77. Assuming that about 200 MeV of energy is released per fission of ${}_{92}\text{U}^{235}$ nuclei, the mass of U^{235} consumed per day in a fission reactor of power 1 megawatt will be approximately
 (1) 10^{-2} g (2) 1 g
 (3) 100 g (4) 10,000 g
78. If mass of $\text{U}^{235} = 235.12142 \text{ amu}$, mass of $\text{U}^{236} = 236.1205 \text{ amu}$, and mass of neutron = 1.008665 amu, then the energy required to remove one neutron from the nucleus of U^{236} is nearly about
 (1) 75 MeV (2) 6.5 MeV
 (3) 1 eV (4) zero
79. The binding energies per nucleon of deuteron (${}_1\text{H}^2$) and helium (${}_2\text{He}^4$) atoms are 1.1 MeV and 7 MeV. If two deuteron atoms react to form a single helium atom, then the energy released is
 (1) 13.9 MeV (2) 26.9 MeV
 (3) 23.9 MeV (4) 19.2 MeV
80. In the fusion reaction ${}_1^2\text{He} + {}_1^2\text{H} \rightarrow {}_2^3\text{He} + {}_0^1\text{n}$, the masses of deuteron, helium, and neutron expressed in amu are

2.015, 3.017 and 1.009, respectively. If 1 kg of deuterium undergoes complete fusion, find the amount of total energy released. ($1 \text{ amu} = 931.5 \text{ MeV}/c^2$)

- (1) $\approx 6.02 \times 10^{13} \text{ J}$ (2) $\approx 5.6 \times 10^{13} \text{ J}$
 (3) $\approx 9.0 \times 10^{13} \text{ J}$ (4) $\approx 0.9 \times 10^{13} \text{ J}$

81. The half-life of radium is 1500 years. In how many years will 1 g of pure radium be reduced to one centigram?

- (1) $3.927 \times 10^2 \text{ years}$
 (2) $9.972 \times 10^2 \text{ years}$
 (3) $99.927 \times 10^2 \text{ years}$
 (4) $0.927 \times 10^2 \text{ years}$

82. A proton and a neutron are both shot at 100 m s^{-1} towards a $^{12}_6\text{C}$ nucleus. Which particle, if either, is more likely to be absorbed by the nucleus?

- (1) The proton.
 (2) The neutron.
 (3) Both particles are about equally likely to be absorbed.
 (4) Neither particle will be absorbed.

83. A container is filled with a radioactive substance for which the half-life is 2 days. A week later, when the container is opened, it contains 5 g of the substance. Approximately how many grams of the substances were initially placed in the container?

- (1) 40 (2) 60
 (3) 80 (4) 100

84. The half-life of ^{131}I is 8 days. Given a sample of ^{131}I at time $t = 0$, we can assert that

- (1) no nucleus will decay before $t = 4$ days
 (2) no nucleus will decay before $t = 8$ days
 (3) all nuclei will decay before $t = 16$ days
 (4) a given nucleus may decay at any time after $t = 0$

85. When an atom undergoes β^+ decay,

- (1) a neutron changes into a proton
 (2) a proton changes into a neutron
 (3) a neutron changes into an antiproton
 (4) a proton changes into an antineutron

86. Calculate the binding energy of a deuteron atom, which consists of a proton and a neutron, given that the atomic mass of the deuteron is 2.014102 u.

- (1) 0.002388 MeV (2) 2.014102 MeV
 (3) 2.16490 MeV (4) 2.224 MeV

87. The compound unstable nucleus $^{236}_{92}\text{U}$ often decays in accordance with the following reaction
 $^{236}_{92}\text{U} \rightarrow ^{140}_{54}\text{Xe} + ^{94}_{38}\text{Sr} + \text{other particles}$

In the nuclear reaction presented above, the “other particle” might be

- (1) an alpha particle, which consists of two protons and two neutrons
 (2) two protons
 (3) one proton and one neutron

(4) two neutrons

88. Why is a ^4_2He nucleus more stable than a ^4_3Li nucleus?

- (1) The strong nuclear force is larger when the neutron to proton ratio is higher.
 (2) The laws of nuclear physics forbid a nucleus from containing more protons than neutrons.
 (3) Forces other than the strong nuclear force make the lithium nucleus less stable.
 (4) None of the above.

89. What is the power output of $^{235}_{92}\text{U}$ reactor if it takes 30 days to use up 2 kg of fuel and if each fission gives 185 MeV of usable energy? Avogadro's number = 6.02×10^{26} per kilomole.

- (1) 45 megawatt (2) 58.46 megawatt
 (3) 72 megawatt (4) 92 megawatt

90. Consider the following reaction



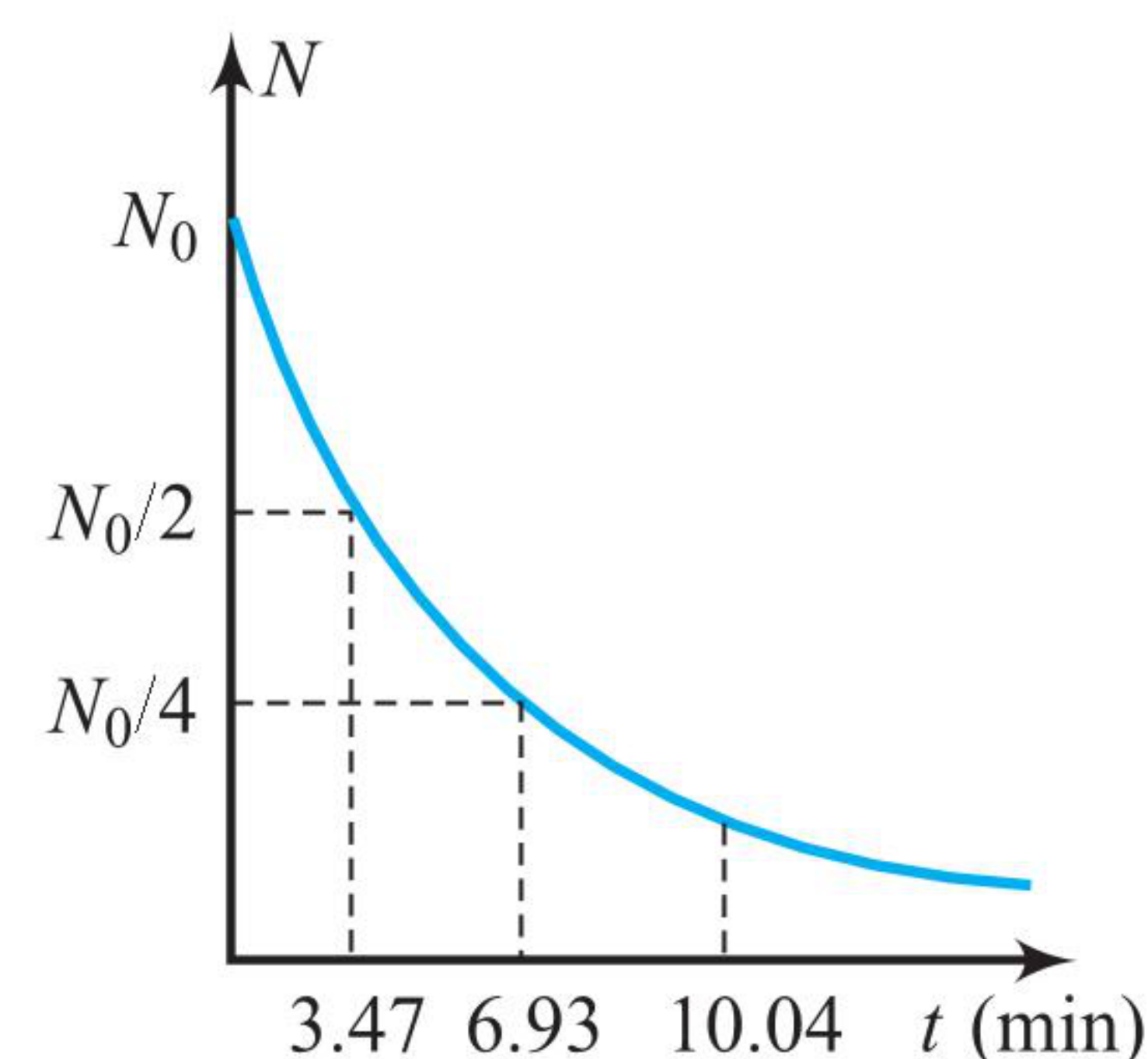
If $m(^1_1\text{H}_2) = 2.0141 \text{ u}$; $m(^4_2\text{He}^4) = 4.0024 \text{ u}$, the energy Q released (in MeV) in this fusion reaction is

- (1) 12 (2) 6
 (3) 24 (4) 48

91. A radioactive nuclide is produced at the constant rate of n per second (say, by bombarding a target with neutrons). The expected number N of nuclei in existence t s after the number is N_0 is given by

- (1) $N = N_0 e^{-\lambda t}$
 (2) $N = \frac{n}{\lambda} + N_0 e^{-\lambda t}$
 (3) $N = \frac{n}{\lambda} + \left(N_0 - \frac{n}{\lambda}\right) e^{-\lambda t}$
 (4) $N = \frac{n}{\lambda} + \left(N_0 + \frac{n}{\lambda}\right) e^{-\lambda t}$

92. A radioactive sample undergoes decay as per the following graph. At time $t = 0$, the number of undecayed nuclei is N_0 . Calculate the number of nuclei left after 1 h.



- (1) N_0/e^8 (2) N_0/e^{10}
 (3) N_0/e^{12} (4) N_0/e^{14}

93. The luminous dials of watches are usually made by mixing a zinc sulphide phosphor with an α -particle emitter. The mass

of radium (mass number 226, half-life 1620 years) that is needed to produce an average of 10 α -particles per second for this purpose is

- (1) 2.77 mg (2) 2.77 g
(3) 2.77×10^{-23} g (4) 2.77×10^{-13} kg

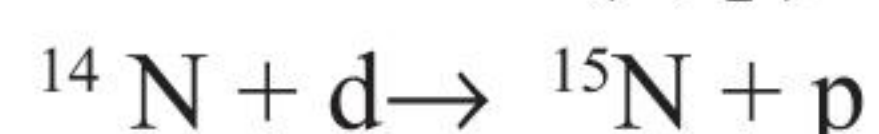
94. The following deuterium reactions and corresponding reaction energies are found to occur.

$$^{14}\text{N}(\text{d}, \text{p})^{15}\text{N}, \quad Q = 8.53 \text{ MeV}$$

$$^{15}\text{N}(\text{d}, \alpha)^{13}\text{C}, \quad Q = 7.58 \text{ MeV}$$

$$^{13}\text{C}(\text{d}, \alpha)^{11}\text{B}, \quad Q = 5.16 \text{ MeV}$$

The notation $^{14}\text{N}(\text{d}, \text{p})^{15}\text{N}$ represents the reaction



$$^4_2\text{He} = 4.0026 \text{ amu}, \quad ^2_1\text{He} = 2.014 \text{ amu}, \quad ^1_1\text{H} = 1.0078 \text{ amu}, \\ n = 1.0087 \text{ amu} \quad (1 \text{ amu} = 931 \text{ MeV})$$

The Q values of the reaction $^{11}\text{B}(\alpha, n)^{14}\text{N}$ is

- (1) 0.5 eV (2) 0.5 MeV
(3) 0.05 MeV (4) 0.05 eV

95. A neutron of energy 1 MeV and mass 1.6×10^{-27} kg passes a proton at such a distance that the angular momentum of the neutron relative to the proton approximately equals 10^{-33} J s. The distance of closest approach neglecting the interaction between particles is

- (1) 0.44 nm (2) 0.44 mm
(3) 0.44 Å (4) 0.44 fm

96. Rank the following nuclei in order from largest to smallest value of the binding energy per nucleon: (i) ^4_2He , (ii) $^{52}_{24}\text{Cr}$, (iii) $^{152}_{62}\text{Sm}$, (iv) $^{200}_{80}\text{Hg}$, (v) $^{252}_{92}\text{Cf}$.

- (1) $E_{(v)} > E_{(iv)} > E_{(iii)} > E_{(ii)} > E_{(i)}$
(2) $E_{(i)} > E_{(ii)} > E_{(iii)} > E_{(iv)} > E_{(v)}$
(3) $E_{(ii)} > E_{(iii)} > E_{(iv)} > E_{(v)} > E_{(i)}$
(4) $E_{(i)} = E_{(ii)} = E_{(iii)} = E_{(iv)} = E_{(v)}$

97. A nucleus with atomic number Z and neutron number N undergoes two decay processes. The result is a nucleus with atomic number $Z - 3$ and neutron number $N - 1$. Which decay processes took place?

- (1) Two β^- decays
(2) Two β^+ decays
(3) An α -decay and a β^- decay
(4) An α -decay and a β^+ decay

98. Gold $^{198}_{79}\text{Au}$ undergoes β^- decay to an excited state of $^{198}_{80}\text{Hg}$. If the excited state decays by emission of a γ -photon with energy 0.412 MeV, the maximum kinetic energy of the electron emitted in the decay is (This maximum occurs when the antineutrino has negligible energy. The recoil energy of the $^{198}_{80}\text{Hg}$ nucleus can be ignored. The masses of the neutral atoms in their ground states are 197.968225 u for $^{198}_{79}\text{Au}$ and 197.966752 u for $^{198}_{80}\text{Hg}$.)

- (1) 0.412 MeV (2) 1.371 MeV
(3) 0.959 MeV (4) 1.473 MeV

Multiple Correct Answers Type

1. For a certain radioactive substance, it is observed that after 4 h, only 6.25% of the original sample is left undecayed. It follows that

- (1) the half-life of the sample is 1 h
(2) the mean life of the sample is $\frac{1}{\ln 2}$ h
(3) the decay constant of the sample is $\ln(2) \text{ h}^{-1}$
(4) after a further 4 h, the amount of the substance left over would be only 0.39% of the original amount

2. Mark out the correct statement(s).

- (1) Higher binding energy per nucleon means the nucleus is more stable.
(2) If the binding energy of nucleus were zero, then it would spontaneously break apart.
(3) Binding energy of a nucleus can be negative.
(4) Binding energy of a nucleus is always positive.

3. Mark out the correct statement(s).

- (1) In alpha decay, the energy released is shared between alpha particle and daughter nucleus in the form of kinetic energy and share of alpha particle is more than that of the daughter nucleus.
(2) In beta decay, the energy released is in the form of kinetic energy of beta particles.
(4) In beta minus decay, the energy released is shared between electron and antineutrino.
(4) In gamma decay, the energy released is in the form of energy carried by photons termed as gamma rays.

4. Mark the correct statement(s).

- (1) For an exothermic reaction, if Q value is +12.56 MeV and the KE of incident particle is 2.44 MeV, then the total KE of products of reaction is 15.00 MeV.
(2) For an exothermic reaction, if Q value is +12.56 MeV and the KE of incident particle is 2.44 MeV, then the total KE of products of reaction is 12.56 MeV.
(3) For an endothermic reaction, if we give the energy equal to $|Q|$ value of reaction, then the reaction will be carried out.
(4) For an exothermic reaction, the BE per nucleon of products should be greater than the BE per nucleon of reactants.

5. Mark out the correct statement(s).

- (1) In both fission and fusion processes, the mass of reactant nuclide is greater than the mass of product nuclide.
(2) In fission process, BE per nucleon of reactant nuclide is less than the binding energy per nucleon of product nuclide.
(3) In fusion process, BE per nucleon of reactant nuclide is less than the binding energy per nucleon of product nuclide.
(4) In fusion process, BE per nucleon of reactant nuclide is greater than the binding energy per nucleon of product nuclide.

6. During β -decay (beta minus), the emission of antineutrino particle is supported by which of the following statement(s)?
- (1) Angular momentum conservation holds good in any nuclear reaction.
 - (2) Linear momentum conservation holds good in any nuclear reaction.
 - (3) The KE of emitted β -particle is varying continuously to a maximum value.
 - (4) None of the above.
7. Two samples A and B of same radioactive nuclide are prepared. Sample A has twice the initial activity of sample B. For this situation, mark out the correct statement(s).
- (1) The half-lives of both the samples would be same.
 - (2) The half-lives of the samples are different.
 - (3) After each has passed through 5 half-lives, the ratio of activity of A to B is 2:1.
 - (4) After each has passed through 5 half-lives, ratio of activities of A to B is 64:1.
8. The decay constant of a radioactive substance is 0.173 year^{-1} . Therefore,
- (1) nearly 63% of the radioactive substance will decay in $(1/0.173) \text{ year}$
 - (2) half-life of the radioactive substance is $(1/0.173) \text{ year}$
 - (3) one-fourth of the radioactive substance will be left after 8 years
 - (4) all of the above
9. A nuclide A undergoes α -decay and another nuclide B undergoes β -decay. Then,
- (1) all the α -particles emitted by A will have almost the same speed
 - (2) the α -particles emitted by B will have widely different speeds
 - (3) all the β -particles emitted by B will have almost the same speed
 - (4) the β -particles emitted by B may have widely different speeds
10. If A , Z , and N denote the mass number, the atomic number, and the neutron number for a given nucleus, we can say that
- (1) $N = Z + A$
 - (2) isobars have the same A but different Z and N
 - (3) isotopes have the same Z but different N and A
 - (4) isotopes have the same N but different A and Z
11. It has been found that nuclides with 2, 8, 20, 50, 82, and 126 protons or neutrons are exceptionally stable. These numbers are referred to as the magic numbers and their existence has led us to
- (1) the idea of periodicity in nuclear properties similar to the periodicity of chemical elements in periodic table
 - (2) the so-called “liquid drop model of the nucleus”
 - (3) the so-called “shell model of the nucleus”
 - (4) have a convenient explanation of “nuclear fission”
12. The phenomenon of nuclear fission can be carried out both in a controlled and in an uncontrolled way. Out of the following, the correct statements vis-à-vis these phenomena are:
- (1) The fission energy released per reaction is much more than conventional nuclear reactions and one of the products of the reaction is that very particle which initiates the reaction.
 - (2) It is the “surface to volume” ratio of the sample of nuclear fuel used which determines whether or not the reaction would sustain itself as a “chain reaction”.
 - (3) The “control rods” in a nuclear reactor must be made of a material that absorbs neutrons effectively.
 - (4) The energy released per fission as well as energy released per unit mass of the fuel in nuclear fission are both greater than the corresponding quantities for nuclear fusion.
13. Choose the correct statements from the following:
- (1) Like other light nuclei, the ${}^4_2\text{He}$ nuclei also have a low value of the binding energy per nucleon.
 - (2) The binding energy per nucleon decreases for nuclei with small as well as large atomic number.
 - (3) The energy required to remove one neutron from ${}^7_3\text{Li}$ to transform it into the isotope ${}^6_3\text{Li}$ is 5.6 MeV, which is the same as the binding energy per nucleon of ${}^6_3\text{Li}$.
 - (4) When two deuterium nuclei fuse together, they give rise to a tritium nucleus accompanied by a release of energy.
14. It is observed that only 0.39% of the original radioactive sample remains undecayed after eight hours. Hence,
- (1) the half-life of that substance is 1 h
 - (2) the mean-life of the substance is $[1/(\log 2)] \text{ h}$
 - (3) decay constant of the substance is $(\log 2) \text{ h}^{-1}$
 - (4) if the number of radioactive nuclei of this substance at a given instant is 10, then the number left after 30 min would be 7.5
15. In a nuclear reactor
- (1) the chain reaction is kept under control by rods of cadmium, which reduces the rate
 - (2) the thick concrete shield is used to slow down the speed of fast neutrons
 - (3) heavy water (or graphite) moderate the activity of the reactor
 - (4) out of U^{238} and U^{235} natural uranium has less than 1% of U^{235}
16. A radioactive sample has initial concentration N_0 of nuclei. Then,
- (1) the number of undecayed nuclei present in the sample decays exponentially with time
 - (2) the activity (R) of the sample at any instant is directly proportional to the number of undecayed nuclei present in the sample at that time
 - (3) the number of decayed nuclei grows exponentially with time
 - (4) the number of decayed nuclei grows linearly with time
17. An O^{16} nucleus is spherical and has a radius R and a volume $V = \frac{4}{3}\pi R^3$. According to the empirical observations, the

volume of the ${}_{54}^{128}\text{X}$ nucleus assumed to be spherical is V' and radius is R' . Then

- (1) $V' = 8V$ (2) $V' = 2V$
(3) $R' = 2R$ (4) $R' = 8R$

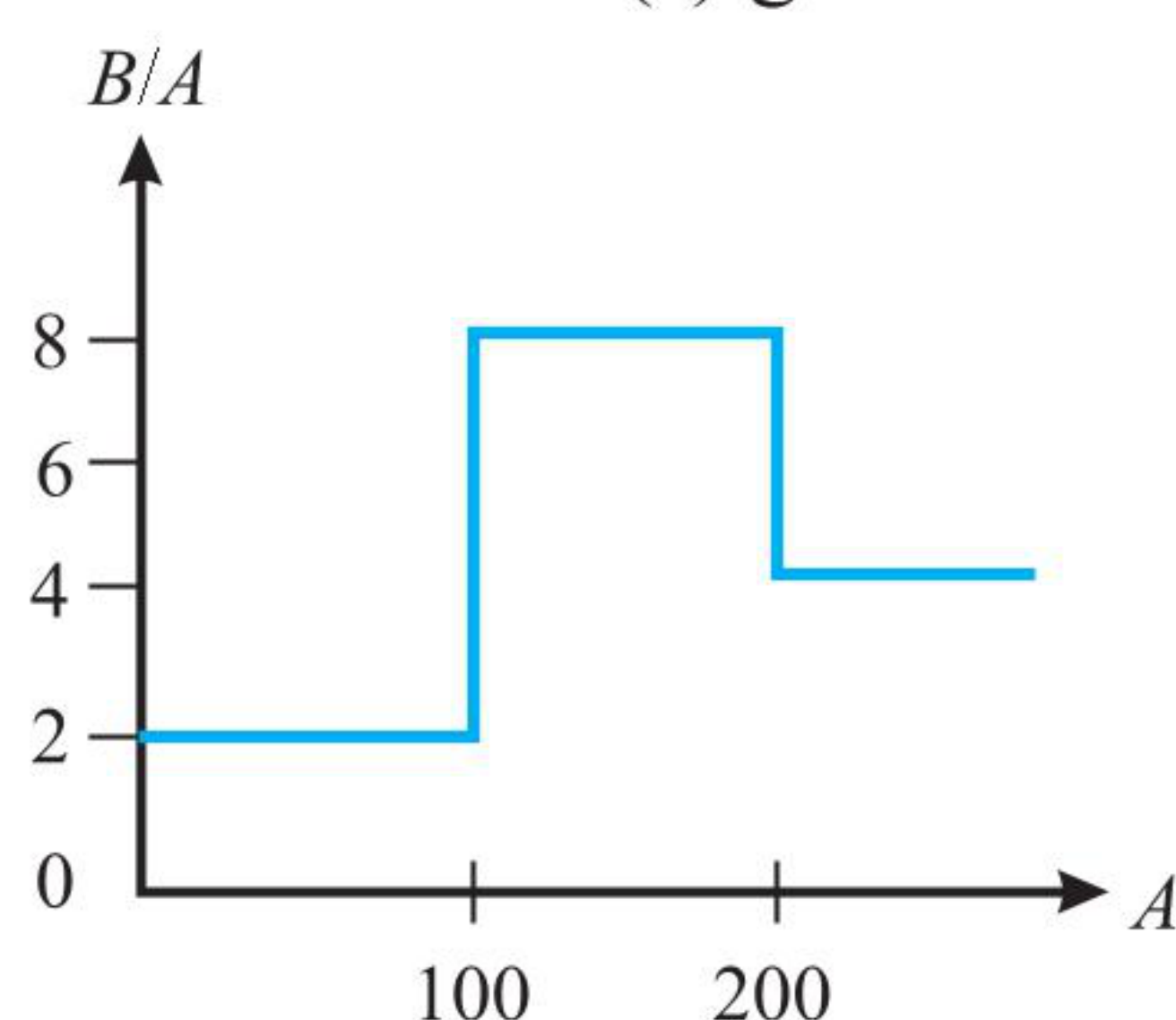
18. Which of the following statements is(are) correct?

- (1) The rest mass of a stable nucleus is greater than the sum of the rest masses of its separated nucleons.
(2) The rest mass of a stable nucleus is less than the sum of the rest masses of its separated nucleons.
(3) In nuclear fission, energy is released by fusing two nuclei of medium mass (approximately 100 amu).
(4) In nuclear fission, energy is released by fragmentation of a very heavy nucleus.

19. Let m_p be the mass of proton, m_n the mass of a neutron, M_1 the mass of a ${}_{10}^{20}\text{Ne}$ nucleus, and M_2 the mass of a ${}_{20}^{40}\text{Ca}$ nucleus. Then,

- (1) $M_2 = 2M_1$ (2) $M_2 > 2M_1$
(3) $M_2 < 2M_1$ (4) $M_1 < 10(m_p + m_n)$

20. Assume that the nuclear binding energy per nucleon (B/A) versus mass number (A) is as shown in figure. Use this plot to choose the correct choice(s) given below:



- (1) Fusion of two nuclei with mass numbers lying in the range of $1 < A < 50$ will release energy.
(2) Fusion of two nuclei with mass numbers lying in the range of $51 < A < 100$ will release energy.
(3) Fission of a nucleus lying in the mass range of $100 < A < 200$ will release energy when broken into equal fragments.
(4) Fission of a nucleus lying in the mass range of $200 < A < 260$ will release energy when broken into equal fragments.

21. Two radioactive materials A and B decay constant 3λ and 2λ respectively. At $t = 0$, the numbers of nuclei of A and B are $4N_0$ and $2N_0$ respectively, then

- (1) their number of radioactive nuclei will be equal at $t = \frac{1}{\lambda} \ln 2$
(2) their decay rate will be equal at $t = \frac{1}{\lambda} \ln 4$
(3) their decay rate will be equal at $t = \frac{1}{\lambda} \ln 3$
(4) At $t = \frac{1}{\lambda} \ln 4$, the decay rate of A will be greater than that of B .

22. A nucleus X of mass M , initially at rest, undergoes alpha decay according to the equation ${}_Z^AX \rightarrow {}_{Z-2}^{A-4}Y + {}_2^4\text{He}$

The alpha particle emitted in the above process is found to move in a circular track of radius r in a uniform magnetic field B . Then (mass and charge of α -particle are m and q respectively)

(1) The ratio of kinetic energies of the alpha particle and the

daughter nucleus Y approximately equals $\frac{M-m}{m}$

(2) The ratio of kinetic energies of the alpha particle and the

daughter nucleus Y approximately equals $\frac{m}{M}$

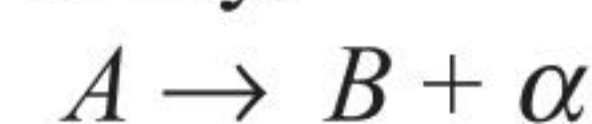
(3) The energy released in the process approximately equals

$$\frac{M}{M-m} \left(\frac{r^2 q^2 B^2}{2m} \right)$$

(4) The energy released in the process approximately equals

$$\frac{M-m}{M} \left(\frac{r^2 q^2 B^2}{2m} \right)$$

23. The half life of a radioactive sample can be experimentally found by the following method. A pure radioactive sample A decays into B by a decay.



N_A and N_B are measured at different times and $\frac{N_B}{N_A}$ is

plotted against time. If time interval is much less than half life,

- (1) graph is straight line with $-ve$ slope.
(2) graph is straight line with $+ve$ slope.
(3) graph has slope whose magnitude is the disintegration constant.
(4) graph has slope whose magnitude is the reciprocal of disintegration constant.

Linked Comprehension Type

For Problems 1–3

Nuclei of a radioactive element X are being produced at a constant rate K and this element decays to a stable nucleus Y with a decay constant λ and half-life $T_{1/2}$. At time $t = 0$, there are N_0 nuclei of the element X .

1. The number N_X of nuclei of X at time $t = T_{1/2}$ is

- (1) $\frac{K + \lambda N_0}{2\lambda}$ (2) $(2\lambda N_0 - K) \frac{1}{\lambda}$
(3) $\left[\lambda N_0 + \frac{K}{2} \right] \frac{1}{\lambda}$ (4) Data insufficient

2. The number N_Y of nuclei of Y at time t is

- (1) $Kt - \left(\frac{K - \lambda N_0}{\lambda} \right) e^{-\lambda t} + \frac{K - \lambda N_0}{\lambda}$
(2) $Kt + \left(\frac{K - \lambda N_0}{\lambda} \right) e^{-\lambda t} - \frac{K - \lambda N_0}{\lambda}$
(3) $Kt + \left(\frac{K - \lambda N_0}{\lambda} \right) e^{-\lambda t}$

$$(4) Kt - \left(\frac{K - \lambda N_0}{\lambda} \right) e^{-\lambda t}$$

3. The number N_Y of nuclei of Y at $t = T_{1/2}$ is

$$(1) K \frac{\ln 2}{\lambda} + \frac{3}{2} \left(\frac{K - \lambda N_0}{\lambda} \right) \quad (2) K \frac{\ln 2}{\lambda} + \frac{1}{2} \left(\frac{K - \lambda N_0}{\lambda} \right)$$

$$(3) K \frac{\ln 2}{\lambda} - \frac{1}{2} \left(\frac{K - \lambda N_0}{\lambda} \right) \quad (4) K \frac{\ln 2}{\lambda} - 2 \left(\frac{K - \lambda N_0}{\lambda} \right)$$

For Problems 4–6

A radionuclide with decay constant λ is being produced in a nuclear reactor at a rate $q_0 t$ per second, where q_0 is a positive constant and t is the time. During each decay, E_0 energy is released. The production of radionuclide starts at time $t = 0$.

4. Which differential equation correctly represents the above process?

$$(1) \frac{dN}{dt} + \lambda N = q_0 t \quad (2) \frac{dN}{dt} - \lambda N = q_0 t$$

$$(3) \frac{dN}{dt} + q_0 t = \lambda N \quad (4) \frac{dN}{dt} + q_0 t = -\lambda N$$

5. Instantaneous power developed at time t due to the decay of the radionuclide is

$$(1) \left(q_0 t - \frac{q_0}{\lambda} + \frac{q_0}{\lambda} e^{-\lambda t} \right) E_0 \quad (2) \left(q_0 t + \frac{q_0}{\lambda} - \frac{q_0}{\lambda} e^{-\lambda t} \right) E_0$$

$$(3) \left(q_0 t + \frac{q_0}{\lambda} + \frac{q_0}{\lambda} e^{-\lambda t} \right) E_0 \quad (4) \left(q_0 t - \frac{q_0}{\lambda} - \frac{q_0}{\lambda} e^{-\lambda t} \right) E_0$$

6. Average power developed in time t due to the decay of the radionuclide is

$$(1) \left(\frac{q_0 t}{2} - \frac{q_0}{\lambda} + \frac{q_0}{\lambda^2 t} - \frac{q_0}{\lambda^2 t} e^{-\lambda t} \right) E_0$$

$$(2) \left(\frac{q_0 t}{2} + \frac{q_0}{\lambda} + \frac{q_0}{\lambda^2 t} - \frac{q_0}{\lambda^2 t} e^{-\lambda t} \right) E_0$$

$$(3) \left(\frac{q_0 t}{2} - \frac{q_0}{\lambda} + \frac{q_0}{\lambda^2 t} + \frac{q_0}{\lambda^2 t} e^{-\lambda t} \right) E_0$$

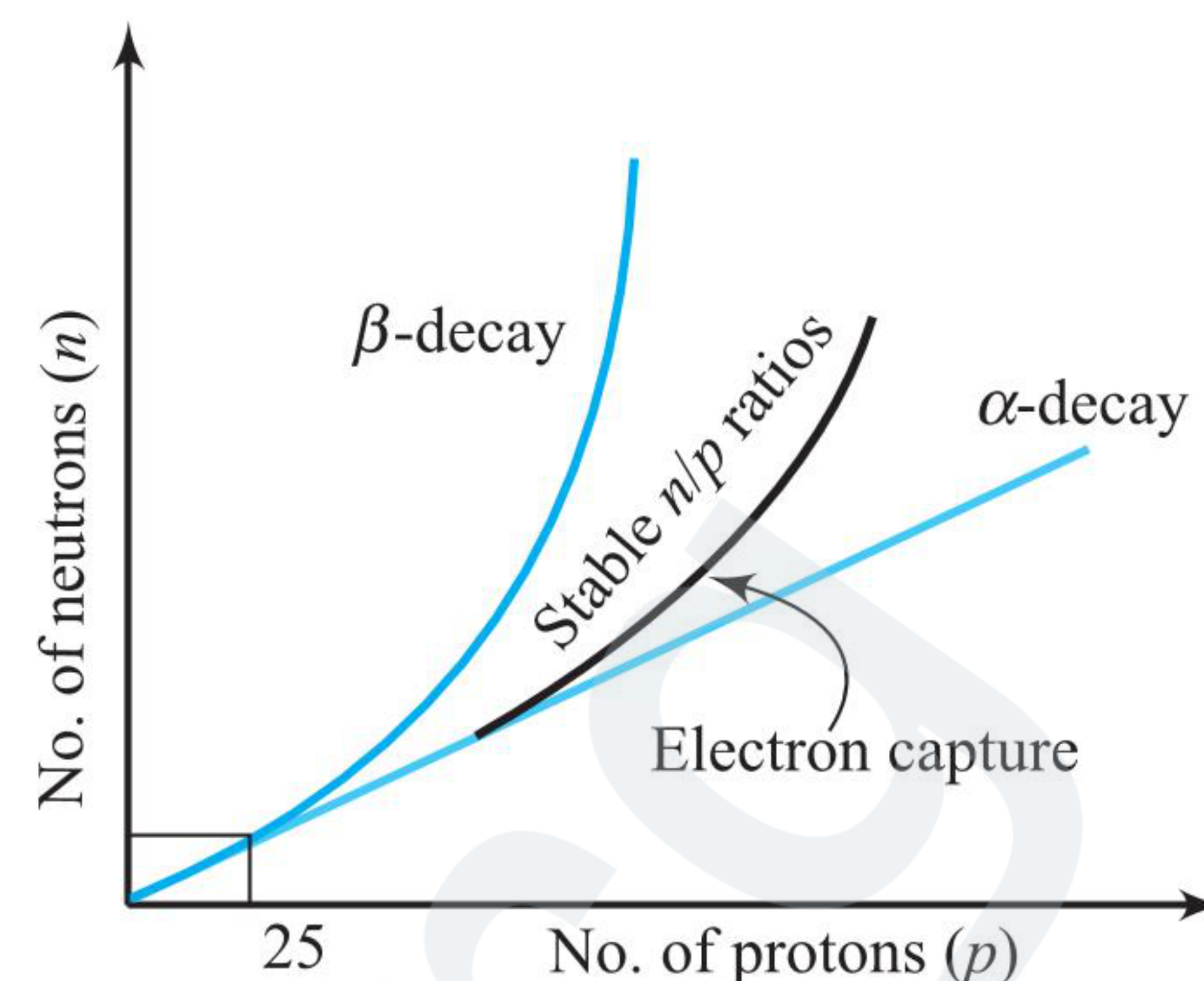
$$(4) \left(\frac{q_0 t}{2} + \frac{q_0}{\lambda} + \frac{q_0}{\lambda^2 t} + \frac{q_0}{\lambda^2 t} e^{-\lambda t} \right) E_0$$

For Problems 7–9

Various rules of thumb have been proposed by the scientific community to explain the mode of radioactive decay by various radioisotopes. One of the major rules is called the n/p ratio. If all the known isotopes of the elements are plotted on a graph of number of neutrons (n) versus number of protons (p), it is observed that all isotopes lying outside of a “stable” n/p ratio region are radioactive as shown in figure.

The graph exhibits straight line behavior with unit slope up to $p = 25$. Above $p = 25$, those isotopes with an n/p ratio lying below the stable region usually undergo electron capture while those with n/p ratios lying above the stable region usually undergo beta decay. Very heavy isotopes ($p > 83$) are unstable because of their relatively large nuclei and they undergo alpha decay. Gamma ray emission does not involve the release of a particle. It represents a

change in an atom from a higher energy level to a lower energy level.



7. How would the radioisotope of magnesium with atomic mass 27 undergo radioactive decay?

- (1) Electron capture (2) Alpha decay
(3) Beta decay (4) Gamma ray emission

8. Th-230 undergoes a series of radioactive decay processes resulting in Bi-214 being the final product. What was the sequence of the processes that occurred?

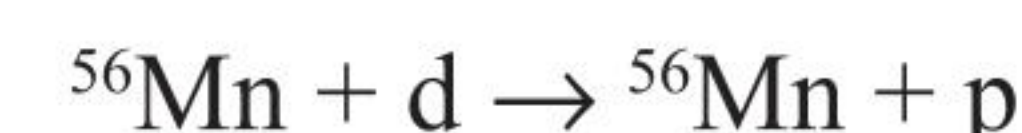
- (1) $\alpha, \alpha, \alpha, \gamma, \beta$ (2) $\alpha, \alpha, \alpha, \alpha, \beta$
(3) $\alpha, \alpha, \beta, \beta$ (4) $\alpha, \beta, \beta, \beta, \gamma$

9. Which of the following represents the relative penetrating power of the three types of radioactive emission in decreasing order?

- (1) $\beta > \alpha > \gamma$ (2) $\beta > \gamma > \alpha$
(3) $\gamma > \alpha > \beta$ (4) $\gamma > \beta > \alpha$

For Problems 10–12

The radionuclide ^{56}Mn is being produced in a cyclotron at a constant rate P by bombarding a manganese target with deuterons. ^{56}Mn has a half-life of 2.5 h and the target contains large number of only the stable manganese isotopes ^{56}Mn . The reaction that produces ^{56}Mn is



After being bombarded for a long time, the activity of ^{56}Mn becomes constant, equal to $13.86 \times 10^{10} \text{ s}^{-1}$. (Use $\ln 2 = 0.693$; Avagadro number = 6×10^{23} ; atomic weight of $^{56}\text{Mn} = 56 \text{ g mol}^{-1}$.)

10. At what constant rate P , ^{56}Mn nuclei are being produced in the cyclotron during the bombardment?

- (1) $2 \times 10^{11} \text{ nuclei s}^{-1}$ (2) $13.86 \times 10^{10} \text{ nuclei s}^{-1}$
(3) $9.6 \times 10^{10} \text{ nuclei s}^{-1}$ (4) $6.93 \times 10^{10} \text{ nuclei s}^{-1}$

11. After the activity of ^{56}Mn becomes constant, number of ^{56}Mn nuclei present in the target is equal to

- (1) 5×10^{11} (2) 20×10^{11}
(3) 1.2×10^{14} (4) 1.8×10^{15}

12. After a long time bombardment, number of ^{56}Mn nuclei present in the target depends upon

- (i) the number of ^{56}Mn nuclei present at the start of the process
(ii) half-life of the ^{56}Mn

- (iii) the constant rate of production P
(1) All (i), (ii), and (iii) are correct.

- (2) Only (i) and (ii) are correct.
(3) Only (ii) and (iii) are correct.

- (4) Only (i) and (iii) are correct.

For Problems 13–15

Many unstable nuclei can decay spontaneously to a nucleus of lower mass but different combination of nucleons. The process of spontaneous emission of radiation is called radioactivity. Three types of radiations are emitted by radioactive substance.

Radioactive decay is a statistical process. Radioactivity is independent of all external conditions. The number of decays per unit time or decay rate is called activity. Activity exponentially decreases with time. Mean lifetime is always greater than half-life time.

13. Choose the correct statement about radioactivity:

- (1) Radioactivity is a statistical process.
- (2) Radioactivity is independent of high temperature and high pressure.
- (3) When a nucleus undergoes α - or β -decay, its atomic number changes.
- (4) All of these.

14. If T_H is the half-life and T_M is the mean life. Which of the following statement is correct.

- (1) $T_M > T_H$
- (2) $T_M < T_H$
- (3) Both are directly proportional to square of the decay constant.
- (4) $T_M \propto \lambda_0$.

15. n number of α -particles per second are being emitted by N atoms of a radioactive element. The half-life of element will be

- (1) $\frac{n}{N}$ s
- (2) $\frac{N}{n}$ s
- (3) $\frac{0.693N}{n}$ s
- (4) $\frac{0.693n}{N}$ s

For Problems 16–18

All nuclei consist of two types of particles—protons and neutrons. Nuclear force is the strongest force. Stability of nucleus is determined by the neutron–proton ratio or mass defect or binding energy per nucleus or packing fraction. Shape of nucleus is calculated by quadrupole moment. Spin of nucleus depends on even or odd mass number. Volume of nucleus depends on the mass number. Whole mass of the atom (nearly 99%) is centered at the nucleus. Magnetic moment of the nucleus is measured in terms of the nuclear magnetons.

16. The correct statements about nuclear force is/are

- (1) charge independent
- (2) short-range force
- (3) non-conservative force
- (4) spin-dependent force

17. Binding energy per nucleon is maximum

- (1) for lighter order element (low mass number)
- (2) for heavier order elements (high mass number)
- (3) for middle order elements
- (4) equal for all order elements

18. Volume (V) of the nucleus is related to mass number (A) as

- (1) $V \propto A^2$
- (2) $V \propto A^{1/3}$
- (3) $V \propto A^{2/3}$
- (4) $V \propto A$

For Problems 19–22

When subatomic particles undergo reactions, energy is conserved, but mass is not necessarily conserved. However, a particle's mass “contributes” to its total energy, in accordance with Einstein's famous equation, $E = mc^2$.

In this equation, E denotes the energy a particle carries because of its mass. The particle can also have additional energy due to its motion and its interactions with other particles.

Consider a neutron at rest, and well separated from other particles. It decays into a proton, an electron, and an undetected third particle:



The table below summarizes some data from a single neutron decay. An MeV (mega electron volt) is a unit of energy. Column 2 shows the rest mass of the particle times the speed of light squared.

Particle	Mass $\times c^2$ (MeV)	Kinetic energy (MeV)
Neutron	940.97	0.00
Proton	939.67	0.01
Electron	0.51	0.39

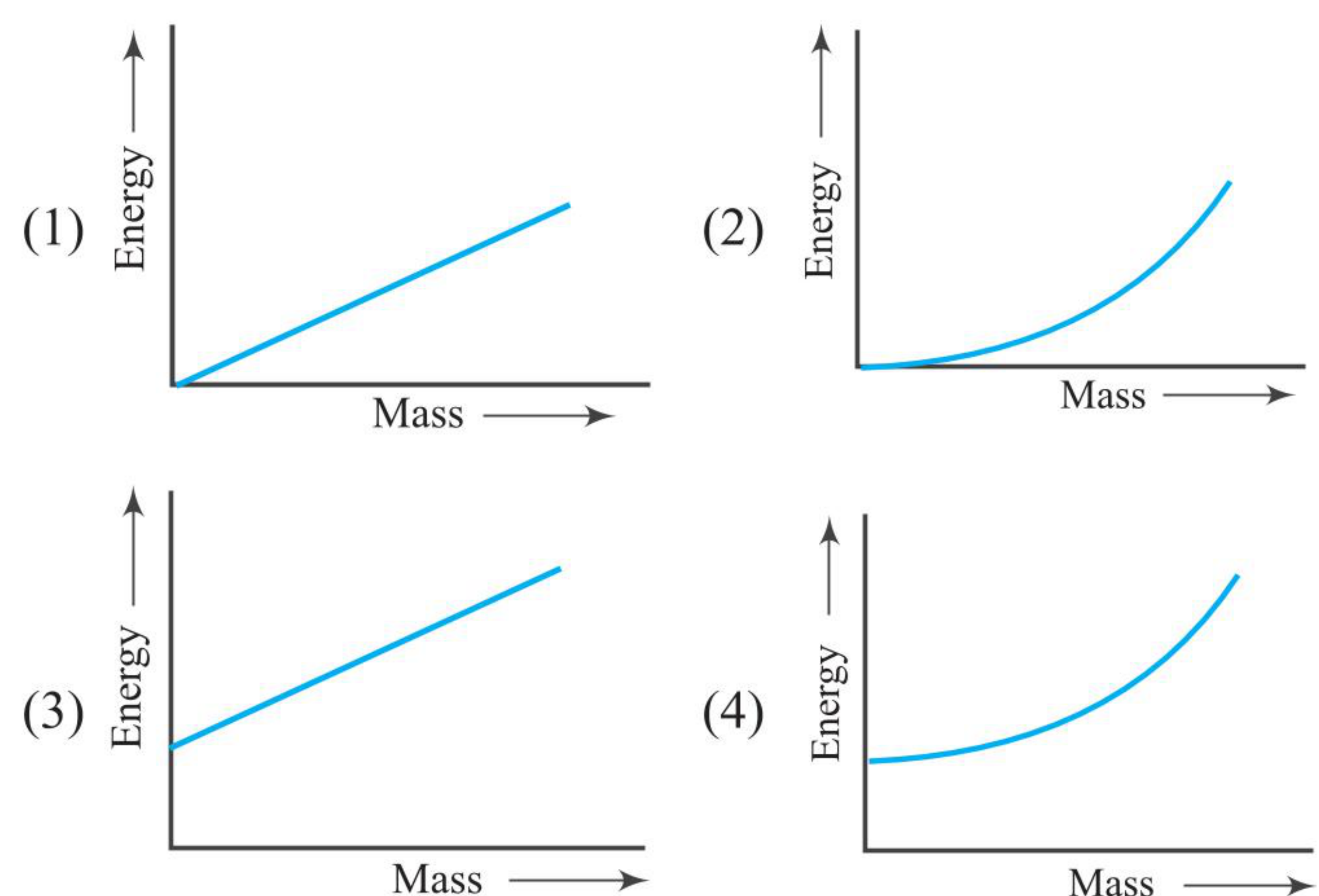
19. Assuming the table contains no major errors, what can we conclude about the (mass $\times c^2$) of the undetected third particle?

- (1) It is 0.79 MeV.
- (2) It is 0.39 MeV.
- (3) It is less than or equal to 0.79 MeV; but we cannot be more precise.
- (4) It is less than or equal to 0.39 MeV; but we cannot be more precise.

20. From the given table, which properties of the undetected third particle can we calculate?

- (1) Total energy, but not kinetic energy.
- (2) Kinetic energy, but not total energy.
- (3) Both total energy and kinetic energy.
- (4) Neither total energy nor kinetic energy.

21. Consider an ensemble of particles, all of which have the same positive kinetic energy but different masses. For this ensemble, which graph best represents the relationship between the particle's mass and its total energy?



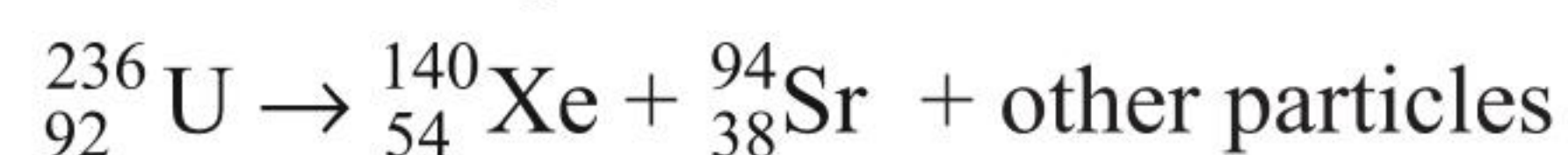
22. Could this reaction occur?

Proton $\rightarrow$ neutron + other particles

- (1) Yes, if the other particles have much more kinetic energy than mass energy.
- (2) Yes, but only if the proton has potential energy (due to interactions with other particles).
- (3) No, because a neutron is more massive than a proton.
- (4) No, because a proton is positively charged while a neutron is electrically neutral.

For Problems 23–26

The compound unstable nucleus ${}_{92}^{236}\text{U}$ often decays in accordance with the following reaction:



During the reaction, the uranium nucleus “fissions” (splits) into the two smaller nuclei. The reaction is energetically favorable because the small nuclei have higher nuclear binding energy per nucleon (although the lighter nuclei have lower total nuclear binding energies, because they contain fewer nucleons).

Inside a nucleus, the nucleons (protons and neutrons) attract each other with a “strong nuclear” force. All nucleons exert approximately the same strong nuclear force on each other. This force holds the nucleus together. Importantly, the strong nuclear force becomes important only when the protons and neutrons are very close together at intranuclear distances.

23. In the nuclear reaction presented above, the “other particles” might be

- (1) an alpha particle, which consists of two protons and two neutrons
- (2) two protons
- (3) one proton and one neutron
- (4) two neutrons

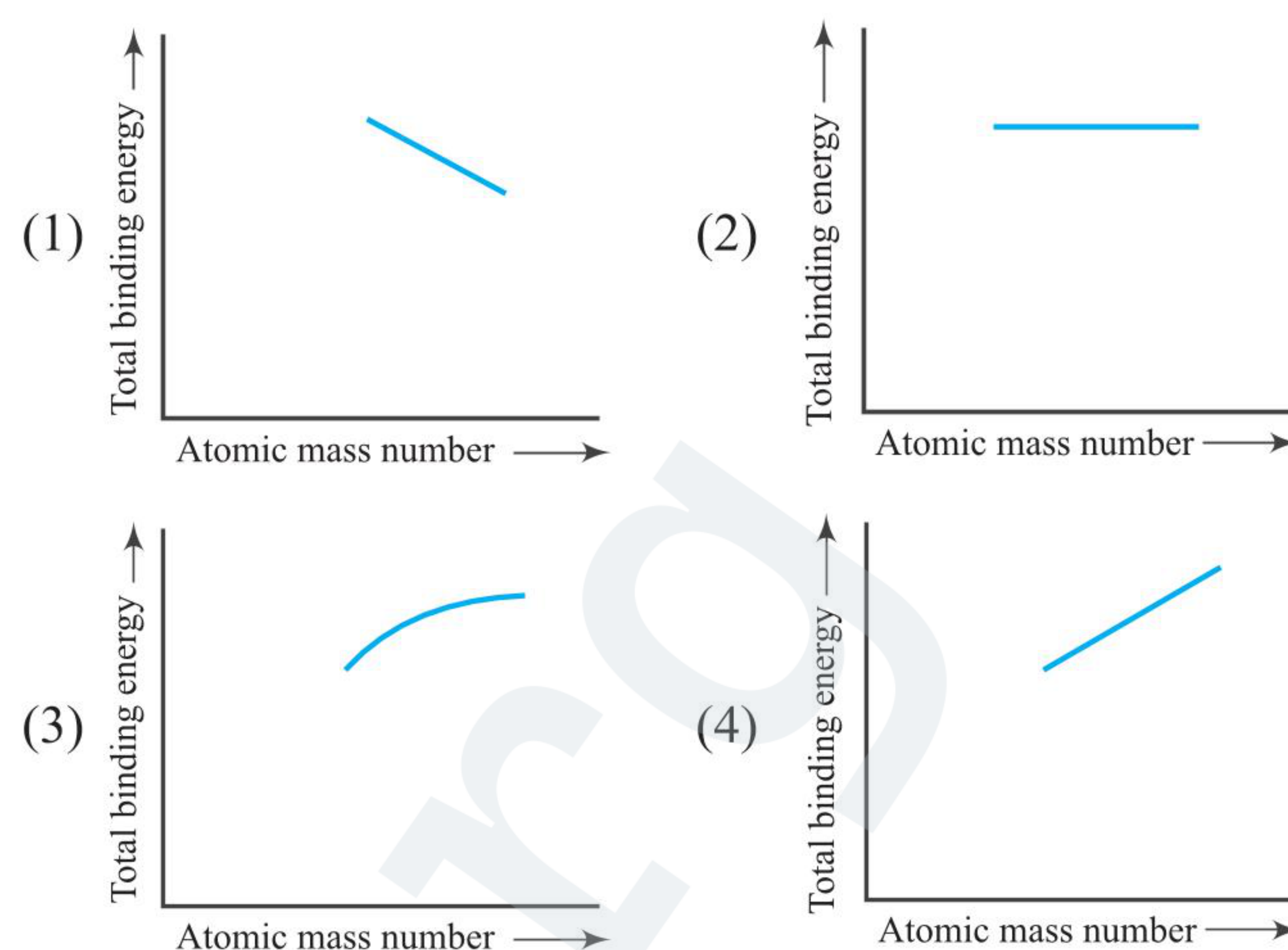
24. Why is a ${}^4_2\text{He}$ nucleus more stable than a ${}^4_3\text{Li}$ nucleus?

- (1) The strong nuclear force is larger when the neutron-to-proton ratio is higher.
- (2) The laws of nuclear physics forbid a nucleus from containing more protons than neutrons.
- (3) Forces other than the strong nuclear force make the lithium nucleus less stable.
- (4) None of the above.

25. A proton and a neutron are both shot at 100 m s^{-1} toward a ${}^{12}_6\text{C}$ nucleus. Which particle, if either, is more likely to be absorbed by the nucleus?

- (1) The proton.
- (2) The neutron.
- (3) Both particles are about equally likely to be absorbed.
- (4) Neither particle will be absorbed.

26. Which of the following graphs might represent the relationship between atomic number (i.e., “atomic weight”) and the total binding energy of the nucleus, for nuclei heavier than ${}^{94}_{38}\text{Sr}$?



For Problems 27–29

A beam of alpha particles is incident on a target of lead. A particular alpha particle comes in ‘head-on’ to a particular lead nucleus and stops $6.50 \times 10^{-14} \text{ m}$ away from the center of the nucleus. (This point is well outside the nucleus.) Assume that the lead nucleus, which has 82 protons, remains at rest. The mass of alpha particle is $6.64 \times 10^{-27} \text{ kg}$.

27. Calculate the electrostatic potential energy at the instant when the alpha particle stops?

- (1) 36.3 MeV
- (2) 45.0 MeV
- (3) 3.63 MeV
- (4) 40.0 MeV

28. What initial kinetic energy (in joule and in MeV) did the alpha particle have?

- (1) 36.3
- (2) 0.36
- (3) 3.63
- (4) 2.63

29. What was the initial speed of the alpha particle?

- (1) $132 \times 10^2 \text{ m s}^{-1}$
- (2) $1.32 \times 10^7 \text{ m s}^{-1}$
- (3) $13.2 \times 10^2 \text{ m s}^{-1}$
- (4) $0.13 \times 10^7 \text{ m s}^{-1}$

For Problems 30–32

A nucleus, kept at rest in free space, break up into two smaller nuclei of masses m and $2m$. Total energy generated in this fission is E . The bigger part is radioactive, emits five gamma ray photons in the direction opposite to its velocity, and finally comes to rest. Now, answer the following questions: (given: $h = 6.6 \times 10^{-34} \text{ J s}$, $m = 1.00 \times 10^{-26} \text{ kg}$, $E = 3.63 \times 10^{-8} \text{ mc}^2$, $c = 3 \times 10^8 \text{ m s}^{-1}$)

30. Fractional loss of mass in the fission is

- (1) 1.21×10^{-8}
- (2) 2.56×10^{-8}
- (3) 1.73×10^{-8}
- (4) 3.52×10^{-8}

31. Velocity of smaller daughter nucleus is

- (1) $5.6 \times 10^4 \text{ m s}^{-1}$
- (2) $6.6 \times 10^4 \text{ m s}^{-1}$
- (3) $7.6 \times 10^4 \text{ m s}^{-1}$
- (4) $8.6 \times 10^4 \text{ m s}^{-1}$

32. The wavelength of the gamma ray is

- (1) $0.02 \text{ \AA}$
- (2) $0.03 \text{ \AA}$
- (3) $0.04 \text{ \AA}$
- (4) $0.05 \text{ \AA}$

For Problems 33–35

The results of activity measurements on a radioactive sample are given in the table below.

Time (h)	Decays (s ⁻¹)
0	20000
0.5	14800
1.0	11000
1.5	8130
2.0	6020
2.5	4460
3.0	3300
4.0	1810
5.0	1000
6.0	550
7.0	300

33. The half-life of the radioactive nuclei is nearly (ln 2 = 0.693, ln 3 = 1.0986)

(1) 2.5 h

(2) 7 h

(3) 5 h

(4) 1.2 h
34. The number of radioactive nuclei that were present in the sample at $t = 0$ is

(1) 2×10^4

(2) 2×10^8

(3) 1.25×10^8

(4) 1.25×10^4
35. The number of radioactive nuclei that were present after 7 h is

(1) 2×10^4

(2) 1.25×10^8

(3) 1.875×10^6

(4) 1.25×10^4

Matrix Match Type

1. Four physical quantities are given in Column I and their order of values in Column II. Match appropriately.

Column I	Column II
i. Thermal energy of air molecules at room temperature	a. 0.02 eV
ii. Binding energy of heavy nuclei per nucleon	b. 2 eV
iii. X-ray photon energy	c. 10 keV
iv. Photon energy of visible light	d. 7 MeV

2. Match the Column I of properties with Column II of reactions

Column I	Column II
i. Mass of products formed is less than the original mass of the system in	a. α -decay
ii. Binding energy per nucleon increase in	b. β -decay
iii. Mass number is conserved in	c. Nuclear fission
iv. Charge number is conserved in	d. Nuclear fusion

3. In Column I consider each process just before and just after it occurs. Initial system is isolated from all other bodies. Consider all product particles (even those having rest mass zero) in the system. Match the system in Column I with the result they produce in Column II:

Column I	Column II
i. Spontaneous radioactive decay of a uranium nucleus initially at rest as given by reaction ${}^{238}_{92}\text{U} \rightarrow {}^{234}_{90}\text{Th} + {}^4_2\text{He} + \dots$	a. Number of protons is increased
ii. Fusion reaction of two hydrogen nuclei as given by reaction ${}_1^1\text{H} + {}_1^1\text{H} \rightarrow {}_1^2\text{H} + \dots$	b. Momentum is conserved
iii. Fission of U^{235} nucleus initiated by a thermal neutron as given by reaction ${}_0^1\text{n} + {}^{235}_{92}\text{U} \rightarrow {}^{144}_{56}\text{Ba} + {}^{89}_{36}\text{K} + 3{}_0^1\text{n} + \dots$	c. Mass is converted to energy or vice versa
iv. β -decay (negative beta decay)	d. Charge is conserved

4.

Column I	Column II
i. Photoelectric effect	a. Photon
ii. Wave	b. Frequency
iii. X-rays	c. K capture
iv. Nucleus	d. γ -rays

5.

Column I	Column II
i. Binding energy per nucleon for middle order of element is	a. Shell model
ii. Nuclear force depends on	b. 8.8 MeV
iii. For nuclear fission, $\frac{Z^2}{A}$ is	c. 2.5 eV
iv. Magic numbers 2, 8, 20, 28, 50, 82, 126 are explained by	d. Spin of nucleons
	e. Greater than 15

6.

Column I	Column II
i. Stability of nucleus decided by	a. -ve
ii. Four radioactive substance spontaneously decays because its	b. Binding energy per nucleon is minimum
iii. For the stable orbit or bound orbit, total energy is	c. Neutron–proton ratio
iv. Stopping potential	d. Packing fraction

e. Mass defect

7.

Column I	Column II
i. Nuclear fusion	a. Satisfies $E = mc^2$
ii. Nuclear fission	b. Generally possible for nuclei with low atomic number
iii. β -decay	c. Generally possible for nuclei with higher atomic number and unstable
iv. Exothermic nuclear reaction	d. Essentially proceeds by weak nuclear forces
	e. Significant momentum conservation

8. In Column I some of the nuclear reactions are given. Match this with the energy involved in these reactions in Column II.

Column I	Column II
i. ${}^2_1\text{H} + {}^2_1\text{H} \rightarrow {}^3_1\text{H} + {}^1_1\text{H} + E_1$	a. 3.3 MeV
ii. ${}^3_1\text{H} + {}^2_1\text{H} \rightarrow {}^4_2\text{He} + {}^1_0\text{n} + E_2$	b. 18.3 MeV
iii. ${}^2_1\text{H} + {}^2_1\text{H} \rightarrow {}^3_2\text{He} + {}^1_0\text{n} + E_3$	c. 4 MeV
iv. ${}^3_2\text{H} + {}^2_1\text{H} \rightarrow {}^4_2\text{He} + {}^1_1\text{H} + E_4$	d. 17.6 MeV
	e. 200 MeV

9. Before the neutrino hypothesis, the beta decay process was thought to be the transition: $n \rightarrow p + e^-$

If the neutron was at rest, the proton and electron would emerge with fixed energies. Given p is the magnitude of the momentum of the electron. ($m_p c^2 = 936$ MeV, $m_n c^2 = 938$ MeV, $m_e c^2 = 0.51$ MeV). Now match the List-I and List-II.

Column I	Column II
i. $m_p c^2 + \frac{p^2 c^2}{2m_p^2 c^4} + pc$	a. 936 MeV
ii. E_p (energy of the proton)	b. 2.06 MeV
iii. E_e (energy of the electron)	c. 2.00 MeV
iv. pc	d. 938 MeV

Codes:

	i.	ii.	iii.	iv.
(1)	d	b	c	a
(2)	c	a	b	d
(3)	d	a	b	c
(4)	c	d	b	a

Numerical Value Type

- Nuclei A and B convert into a stable nucleus C . Nucleus A is converted into C by emitting 2 α -particles and 3 β -particles. Nucleus B is converted into C by emitting one α -particle and 5 β -particles. At time $t = 0$, nuclei of A are $4N_0$ and nuclei of B are N_0 . Initially, number of nuclei of C are zero. Half-life of A (into conversion of C) is 1 min and that of B is 2 min. Find the time (in minutes) at which rate of disintegration of A and B are equal.
- The half-life of a radioactive nuclide is 20 h. It is found that the fraction $(1/x)$ of original activity remains after 40 hours? What is the value of x ?
- A radioactive sample has 8.0×10^{18} active nuclei at a certain instant. How many of these nuclei will still be in the active state after two half-lives (in $\times 10^{18}$)?
- A radioactive sample decays with an average-life of 20 ms. A capacitor of capacitance $100 \mu\text{F}$ is charged to some potential and then the plates are connected through a resistance R . What should be the value of R (in $\times 10^2 \Omega$) so that the ratio of the charge on the capacitor to the activity of the radioactive sample remains constant in time?
- A radioactive sample decays through two different decay processes α -decay and β -decay. Half-life time for α -decay is 3 h and half-life time for β -decay is 6 h. What will be the ratio of number of initial radioactive nuclei to the number of radioactive nuclei present after 6 h?
- ${}_{92}\text{U}^{238}$ changes to ${}_{85}\text{At}^{210}$ by a series of α - and β -decays. Find the number of α -decays undergone (an integer).
- A certain radioactive material can undergo three different types of decay, each with a different decay constant λ , 2λ , and 3λ . Then, the effective decay constant λ_{eff} is equal to $n\lambda$. What is the value of n ?
- A freshly prepared radioactive source of half-life 2 h emits radiation of intensity which is 64 times the permissible safe level. Find the minimum time (in hrs) after which it would be possible to work safely.
- A radioactive sample S_1 having an activity of $5 \mu\text{Ci}$ has twice the number of nuclei as another sample S_2 which has an activity of $10 \mu\text{Ci}$. Find the ratio of the half lives of S_1 and S_2 .
- Nuclei X and Y convert into a stable nucleus Z . At $t = 0$ the number of nuclei of X is 8 times that of Y . Half life of X is 1 hour and half life of Y is 2 hour. Nucleus X is converted into Z by emitting 4α and 8β particles while nucleus Y is converted into Z by emitting 2α and 4β particles. Find the time (in hour) at which rate of disintegration of X and Y are equal.
- A sample of ${}^{18}\text{F}$ is used internally as a medical diagnostic tool to look for the effects of the positron decay ($T_{1/2} = 110$ min). How long (in hr) does it take for 99% of the ${}^{18}\text{F}$ to decay? (Give answer in nearest integer)
- A neutron is scattered through ($\equiv$ deviation from its old direction) θ degree in an elastic collision with an initially stationary deuteron. If the neutron loses $2/3$ of its initial KE to the deuteron then find the value of θ (in degree) (In atomic mass unit, the mass of a neutron is 1u and mass of a deuteron is 2u).

Archives

JEE ADVANCED

Single Correct Answer Type

1. An accident in a nuclear laboratory resulted in deposition of a certain amount of radioactive material of half-life 18 days inside the laboratory. Tests revealed that the radiation was 64 times more than the permissible level required for safe operation of the laboratory. What is the minimum number of days after which the laboratory can be considered safe for use ?

- (1) 64 (2) 90
(3) 108 (4) 120

(JEE Advanced 2016)

2. The electrostatic energy of Z protons uniformly distributed throughout a spherical nucleus of radius R is given by

$$E = \frac{3}{5} \frac{Z(Z-1)e^2}{4\pi\epsilon_0 R}$$

The measured masses of the neutron, ${}^1_1\text{H}$, ${}^{15}_7\text{N}$, and ${}^{15}_8\text{O}$ are 1.008665 u, 1.007825 u, 15.000109 u and 15.003065 u, respectively. Given that the radii of both the ${}^{15}_7\text{N}$ and ${}^{15}_8\text{O}$ nuclei are same, $1 \text{ u} = 931.5 \text{ MeV}/c^2$ (c is the speed of light) and $e^2/(4\pi\epsilon_0) = 1.44 \text{ MeV fm}$. Assuming that the difference between the binding energies of ${}^{15}_7\text{N}$ and ${}^{15}_8\text{O}$ is purely due to the electrostatic energy, the radius of either of the nuclei is ($1 \text{ fm} = 10^{-15} \text{ m}$)

- (1) 2.85 fm (2) 3.03 fm
(3) 3.42 fm (4) 3.80 fm

(JEE Advanced 2016)

3. In a radioactive sample, ${}^{40}_{19}\text{K}$ nuclei either decay into stable ${}^{40}_{20}\text{Ca}$ nuclei with decay constant 4.5×10^{-10} per year or into stable ${}^{40}_{18}\text{Ar}$ nuclei with decay constant 0.5×10^{-10} per year. Given that in this sample all the stable ${}^{40}_{20}\text{Ca}$ and ${}^{40}_{18}\text{Ar}$ nuclei are produced by the ${}^{40}_{19}\text{K}$ nuclei only. In time $t \times 10^9$ years, if the ratio of the sum of stable ${}^{40}_{20}\text{Ca}$ and ${}^{40}_{18}\text{Ar}$ nuclei to the radioactive ${}^{40}_{19}\text{K}$ nuclei is 99, the value of t will be [Given $\ln 10 = 2.3$]

- (1) 9.2 (2) 1.15
(3) 4.6 (4) 2.3

(JEE Advanced 2019)

4. A heavy nucleus Q of half-life 20 minutes undergoes alpha-decay with probability of 60% and beta-decay with probability of 40%. Initially, the number of Q nuclei is 1000. The number of alpha-decays of Q in the first one hour is

- (1) 50 (2) 75
(3) 350 (4) 525

(JEE Advanced 2021)

Multiple Correct Answers Type

1. In a radioactive decay chain, ${}^{232}_{90}\text{Th}$ nucleus decays to ${}^{212}_{82}\text{Pb}$ nucleus. Let N_α and N_β be the number of α and β particles, respectively, emitted in this decay process. Which of the following statements is (are) true ?

- (1) $N_\alpha = 5$ (2) $N_\alpha = 6$
(3) $N_\beta = 2$ (4) $N_\beta = 4$

(JEE Advanced 2018)

2. A heavy nucleus N , at rest, undergoes fission $N \rightarrow P + Q$, where P and Q are two lighter nuclei. Let $\delta = M_N - M_P - M_Q$, where M_P , M_Q and M_N are the masses of P , Q and N , respectively. E_P and E_Q are the kinetic energies of P and Q , respectively. The speed of P and Q are v_P and v_Q , respectively. If c is the speed of light, which of the following statement(s) is(are) correct ?

- (1) $E_P + E_Q = c^2\delta$
(2) $E_P = \left(\frac{M_P}{M_P + M_Q} \right) c^2\delta$
(3) $\frac{v_P}{v_Q} = \frac{M_Q}{M_P}$

- (4) The magnitude of momentum for P as well as Q is $c\sqrt{2\mu\delta}$,

$$\text{where } \mu = \frac{M_P M_Q}{(M_P + M_Q)}$$

(JEE Advanced 2021)

Linked Comprehension Type

For Problems 1–3

Scientists are working hard to develop nuclear fusion reactor. Nuclei of heavy hydrogen, ${}^2_1\text{H}$, known as deuteron and denoted by D can be thought of as a candidate for fusion reactor. The D - D reaction is ${}^2_1\text{H} + {}^2_1\text{H} \rightarrow {}^3_2\text{He} + n + \text{energy}$. In the core of fusion reactor, a gas of heavy hydrogen is fully ionized into deuteron nuclei and electrons. This collection of ${}^2_1\text{H}$ nuclei and electrons is known as plasma. The nuclei move randomly in the reactor core and occasionally come close enough for nuclear fusion to take place. Usually, the temperatures in the reactor core are too high and no material wall can be used to confine the plasma. Special techniques are used which confine the plasma for a time t_0 before the particles fly away from the core. If n is the density (number/volume) of deuterons, the product nt_0 is called Lawson number. In one of the criteria, a reactor is termed successful if Lawson number is greater than $5 \times 10^{14} \text{ s cm}^{-3}$.

It may be helpful to use the following: Boltzmann constant,

$$k = 8.6 \times 10^{-5} \text{ eV K}^{-1}; \frac{e^2}{4\pi\epsilon_0} = 1.44 \times 10^{-9} \text{ eV m.}$$

(IIT-JEE 2009)

- In the core of nuclear fusion reactor, the gas becomes plasma because of
 - strong nuclear force acting between the deuterons
 - Coulomb force acting between the deuterons
 - Coulomb force acting between deuteron–electron pairs
 - the high temperature maintained inside the reactor core
- Assume that two deuteron nuclei in the core of fusion reactor at temperature T are moving toward each other, each with kinetic energy $1.5kT$, when the separation between them is large enough to neglect Coulomb potential energy. Also, neglect any interaction from other particles in the core. The minimum temperature T required for them to reach a separation of 4×10^{-15} m is in the range
 - $1.0 \times 10^9 \text{ K} < T < 2.0 \times 10^9 \text{ K}$
 - $2.0 \times 10^9 \text{ K} < T < 3.0 \times 10^9 \text{ K}$
 - $3.0 \times 10^9 \text{ K} < T < 4.0 \times 10^9 \text{ K}$
 - $4.0 \times 10^9 \text{ K} < T < 5.0 \times 10^9 \text{ K}$
- Results of calculations for four different designs of a fusion reactor using D–D reaction are given below. Which of these is most promising based on Lawson criterion?
 - Deuteron density = $2.0 \times 10^{12} \text{ cm}^{-3}$,
confinement time = $5.0 \times 10^{-3} \text{ s}$
 - Deuteron density = $8.0 \times 10^{14} \text{ cm}^{-3}$,
confinement time = $9.0 \times 10^{-1} \text{ s}$
 - Deuteron density = $4.0 \times 10^{23} \text{ cm}^{-3}$,
confinement time = $1.0 \times 10^{-11} \text{ s}$
 - Deuteron density = $1.0 \times 10^{24} \text{ cm}^{-3}$,
confinement time = $4.0 \times 10^{-12} \text{ s}$

For Problems 4 and 5

The β -decay process, discovered around 1900, is basically the decay of a neutron (n). In the laboratory, a proton (p) and an electron (e^-) are observed as the decay products of the neutron. Therefore, considering the decay of a neutron as a two body decay process, it was predicated theoretically that the kinetic energy of the electron should be a constant. But experimentally, it was observed that the electron kinetic energy has continuous spectrum. Considering a three-body decay process, i.e., $n \rightarrow p + e^- + \bar{\nu}_e$, around 1930, Pauli explained the observed electron energy spectrum. Assuming the anti-neutrino ($\bar{\nu}_e$) to be massless and possessing negligible energy, and the neutron to be at rest, momentum and energy conservation principles are applied. From this calculation, the maximum kinetic energy of the electron is $0.8 \times 10^6 \text{ eV}$. The kinetic energy carried by the proton is only the recoil energy. (IIT-JEE 2012)

- If the anti-neutrino had a mass of $3 \text{ eV}/c^2$ (where c is the speed of light) instead of zero mass, what should be the range of the kinetic energy, K of the electron?
 - $0 \leq K \leq 0.8 \times 10^6 \text{ eV}$
 - $3.0 \text{ eV} \leq K \leq 0.8 \times 10^6 \text{ eV}$
 - $3.0 \text{ eV} \leq K < 0.8 \times 10^6 \text{ eV}$
 - $0 \leq K < 0.8 \times 10^6 \text{ eV}$
- What is the maximum energy of the anti-neutrino?
 - Zero
 - Much less than $0.8 \times 10^6 \text{ eV}$
 - Nearly $0.8 \times 10^6 \text{ eV}$
 - Much larger than $0.8 \times 10^6 \text{ eV}$

For Problems 6 and 7

The mass of nucleus ${}_Z^AX$ is less than the sum of the masses of $(A-Z)$ number of neutrons and Z number of protons in the nucleus. The energy equivalent to the corresponding mass difference is known as the binding energy of the nucleus. A heavy nucleus of mass M can break into two light nuclei of mass m_1 and m_2 only if $(m) < M$. Also two light nuclei of masses m_3 and m_4 can undergo complete fusion and form a heavy nucleus of mass M “only if $(m_3 + m_4) > M$ ”. The masses of some neutral atoms are given in the table below:

(JEE Advanced 2013)

${}_1^1\text{H}$	1.007825 u	${}_1^2\text{H}$	2.014102 u
${}_1^3\text{H}$	3.016050 u	${}_2^4\text{He}$	4.002603 u
${}_3^6\text{Li}$	6.015123 u	${}_3^7\text{Li}$	7.016004 u
${}_{30}^{70}\text{Zn}$	69.925325 u	${}_{34}^{82}\text{Se}$	81.916709 u
${}_{64}^{152}\text{Gd}$	151.919803 u	${}_{82}^{206}\text{Pb}$	205.974455 u
${}_{83}^{209}\text{Bi}$	208.980388 u	${}_{84}^{210}\text{Po}$	209.982876 u

- The correct statement is
 - The nucleus ${}_3^6\text{Li}$ can emit an alpha particle
 - The nucleus ${}_{84}^{210}\text{Po}$ can emit a proton
 - Deuteron and alpha particle can undergo complete fusion
 - The nuclei ${}_{30}^{70}\text{Zn}$ and ${}_{34}^{82}\text{Se}$ can undergo complete fusion
- The kinetic energy (in keV) of the alpha particle, when the nucleus at rest undergoes alpha decay, is
 - 5319
 - 5422
 - 5707
 - 5818

Matrix Match Type

- Column II gives certain systems undergoing a process. Column I suggests changes in some of the parameters related to the system. Match the statements in Column I to the appropriate process(es) from Column II.

Column I	Column II
a. The energy of the system is increased.	p. System: A capacitor, initially uncharged Process: It is connected to a battery
b. Mechanical energy is provided to the system, which is converted into energy of random motion of its parts.	q. System: A gas in an adiabatic container fitted with an adiabatic piston Process: The gas is compressed by pushing the piston

Column I	Column II
c. Internal energy of the system is converted into its mechanical energy.	r. System: A gas in a rigid container Process: The gas gets cooled due to colder atmosphere surrounding it
d. Mass of the system is decreased.	s. System: A heavy nucleus, initially at rest Process: The nucleus fissions into two fragments of nearly equal masses and some neutrons are emitted
	t. System: A resistive wire loop Process: The loop is placed in a time-varying magnetic field perpendicular to its plane

(IIT-JEE 2009)

2. Match List I of the nuclear processes with List II containing parent nucleus and one of the end products of each process and then select the correct answer using the codes given below the lists:

List I	List II
a. Alpha decay	p. $^{15}_8\text{O} \rightarrow ^{15}_7\text{N} + \dots$
b. β^+ decay	q. $^{238}_{92}\text{U} \rightarrow ^{234}_{90}\text{Th} + \dots$
c. Fission	r. $^{185}_{83}\text{Bi} \rightarrow ^{184}_{82}\text{Pb} + \dots$
d. Proton-emission	s. $^{239}_{94}\text{Pu} \rightarrow ^{140}_{57}\text{La} + \dots$

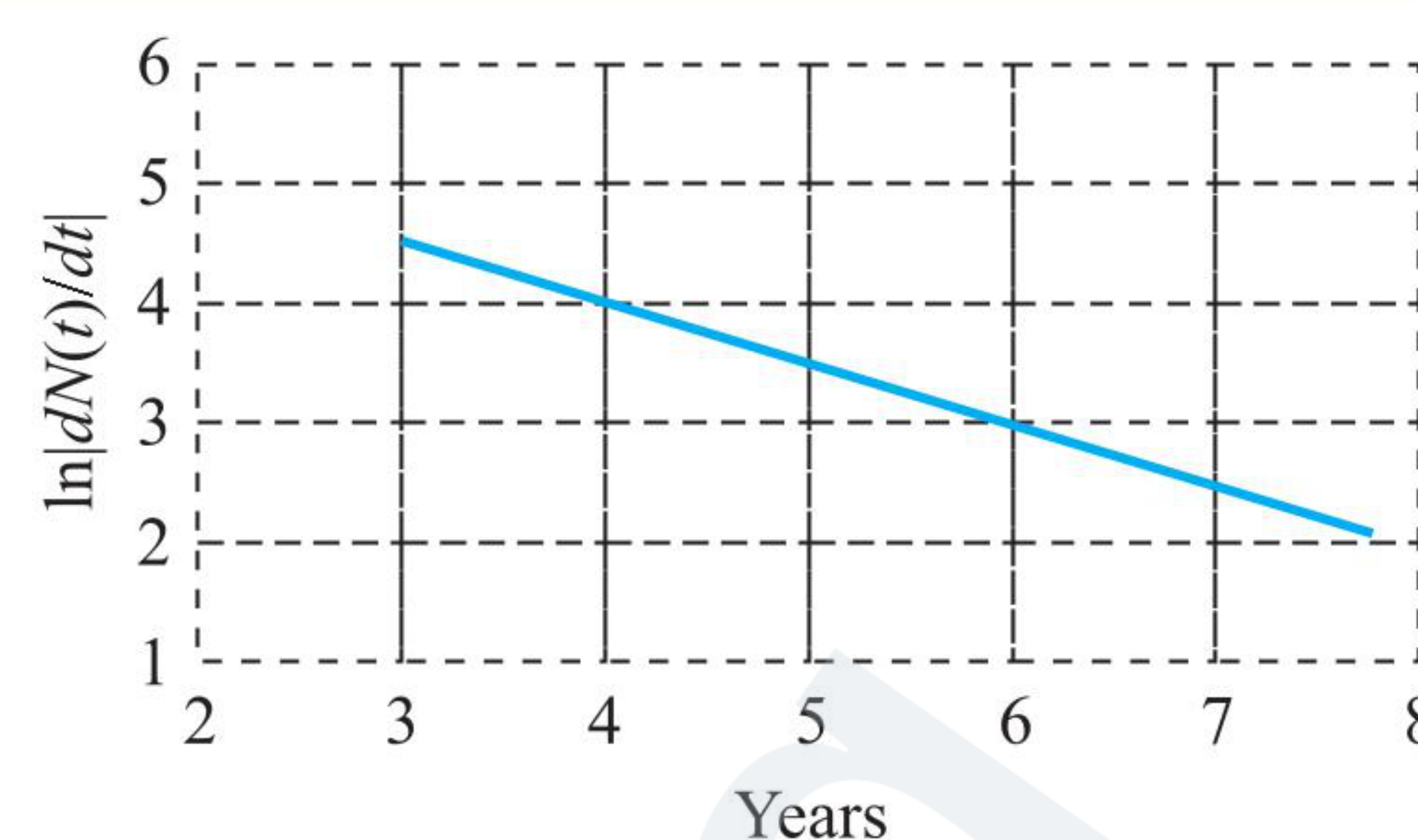
Codes:

	a	b	c	d
(1)	s	q	p	r
(2)	p	r	q	s
(3)	q	p	s	r
(4)	s	r	q	p

(JEE Advanced 2013)

Numerical Value Type

1. To determine the half life of a radioactive element, student plots a graph of $\ln|dN(t)/dt|$ versus t . Here $dN(t)/dt$ is the rate of radioactive decay at time t . If the number of radioactive nuclei of this element decreases by a factor of p after 4.16 years, the value of p is _____. (IIT-JEE 2010)



2. The activity of a freshly prepared radioactive sample is 10^{10} disintegrations per second, whose mean life is 10^9 s. The mass of an atom of this radioisotope is 10^{-25} kg. The mass (in mg) of the radioactive sample is _____. (IIT-JEE 2011)
3. A freshly prepared sample of a radioisotope of half-life 1386 s has activity 10^3 disintegrations per second. Given that $\ln 2 = 0.693$, the fraction of the initial number of nuclei (expressed in nearest integer percentage) that will decay in the first 80 s after preparation of the sample is _____. (JEE Advanced 2013)
4. The isotope $^{12}_5\text{B}$ having a mass 12.014 u undergoes β -decay to $^{12}_6\text{C}$. $^{12}_6\text{C}$ has an excited state of the nucleus ($^{12}_6\text{C}^*$) at 4.041 MeV above its ground state. If $^{12}_5\text{B}$ decays to $^{12}_6\text{C}^*$, the maximum kinetic energy of the β -particle in units of MeV is $(1\text{u} = 931.5\text{ MeV}/c^2, \text{ where } c \text{ is the speed of light in vacuum})$. (JEE Advanced 2016)
5. $^{131}_{53}\text{I}$ is an isotope of Iodine that β decays to an isotope of Xenon with a half-life of 8 days. A small amount of a serum labelled with $^{131}_{53}\text{I}$ is injected into the blood of a person. The activity of the amount of $^{131}_{53}\text{I}$ injected was 2.4×10^5 becquerel (Bq). It is known that the injected serum will get distributed uniformly in the blood stream in less than half an hour. After 11.5 hours, 2.5 ml of blood is drawn from the person's body, and gives an activity of 115 Bq. The total volume of blood in the person's body, in liters is approximately (you may use $e^x \approx 1 + x$ for $|x| \ll 1$ and $\ln 2 \approx 0.7$). (JEE Advanced 2017)
6. Suppose a $^{226}_{88}\text{Ra}$ nucleus at rest and in ground state undergoes α -decay to a $^{222}_{86}\text{Rn}$ nucleus in its excited state. The kinetic energy of the emitted particle is found to be 4.44 MeV. $^{222}_{86}\text{Rn}$ nucleus then goes to its ground state by γ -decay. The energy of the emitted γ -photon is _____ keV. [Given: atomic mass of $^{226}_{88}\text{Ra} = 226.005$ u, atomic mass of $^{222}_{86}\text{Ra} = 222.000$ u, atomic mass of α particle = 4.000 u, $1\text{u} = 931\text{ MeV}/c^2$, is speed of the light] (JEE Advanced 2019)

Answers Key

EXERCISES

Single Correct Answer Type

- | | | | | |
|---------|---------|---------|---------|---------|
| 1. (4) | 2. (3) | 3. (1) | 4. (4) | 5. (1) |
| 6. (2) | 7. (1) | 8. (2) | 9. (3) | 10. (2) |
| 11. (3) | 12. (2) | 13. (2) | 14. (1) | 15. (3) |
| 16. (3) | 17. (1) | 18. (3) | 19. (3) | 20. (1) |
| 21. (1) | 22. (1) | 23. (1) | 24. (2) | 25. (3) |
| 26. (4) | 27. (3) | 28. (1) | 29. (3) | 30. (1) |
| 31. (2) | 32. (1) | 33. (1) | 34. (1) | 35. (2) |
| 36. (4) | 37. (3) | 38. (2) | 39. (2) | 40. (4) |
| 41. (1) | 42. (3) | 43. (2) | 44. (4) | 45. (1) |
| 46. (2) | 47. (4) | 48. (2) | 49. (3) | 50. (2) |
| 51. (2) | 52. (4) | 53. (3) | 54. (4) | 55. (1) |
| 56. (1) | 57. (1) | 58. (4) | 59. (4) | 60. (1) |
| 61. (4) | 62. (3) | 63. (2) | 64. (3) | 65. (2) |
| 66. (1) | 67. (4) | 68. (3) | 69. (4) | 70. (1) |
| 71. (3) | 72. (2) | 73. (1) | 74. (3) | 75. (2) |
| 76. (2) | 77. (2) | 78. (2) | 79. (3) | 80. (3) |
| 81. (2) | 82. (2) | 83. (3) | 84. (4) | 85. (2) |
| 86. (4) | 87. (4) | 88. (3) | 89. (2) | 90. (3) |
| 91. (3) | 92. (3) | 93. (4) | 94. (3) | 95. (4) |
| 96. (3) | 97. (4) | 98. (3) | | |

Multiple Correct Answers Type

- | | | |
|--------------------|-----------------|-----------------|
| 1. (1),(2),(3),(4) | 2. (1),(2),(4) | 3. (1),(3),(4) |
| 4. (1),(4) | 5. (1),(2),(3) | 6. (1),(2),(3) |
| 7. (1),(3) | 8. (1),(3) | 9. (1),(4) |
| 10. (2),(3),(4) | 11. (1),(3) | 12. (1),(2),(3) |
| 13. (2),(3) | 14. (1),(2),(3) | 15. (1),(4) |
| 16. (1),(2),(3) | 17. (1),(3) | 18. (1),(4) |
| 19. (3),(4) | 20. (2),(4) | 21. (1),(3) |
| 22. (1),(3) | 23. (2),(3) | |

Linked Comprehension Type

- | | | | | |
|---------------------|---------|---------|---------|---------|
| 1. (1) | 2. (2) | 3. (3) | 4. (1) | 5. (1) |
| 6. (1) | 7. (3) | 8. (2) | 9. (4) | 10. (2) |
| 11. (4) | 12. (3) | 13. (4) | 14. (1) | 15. (3) |
| 16. (1),(2),(3),(4) | 17. (3) | 18. (4) | 19. (4) | |
| 20. (1) | 21. (3) | 22. (2) | 23. (4) | 24. (1) |
| 25. (2) | 26. (3) | 27. (3) | 28. (3) | 29. (2) |

30. (1) 31. (2) 32. (4) 33. (4) 34. (3)
35. (3)

Matrix Match Type

- i. $\rightarrow$ a.; ii. $\rightarrow$ d.; iii. $\rightarrow$ c.; iv. $\rightarrow$ b.
- i. $\rightarrow$ a., b., c., d.; ii. $\rightarrow$ a., b., c., d.; iii. $\rightarrow$ a., b., c., d.;
iv. $\rightarrow$ a., b., c., d.
- i. $\rightarrow$ b., c., d.; ii. $\rightarrow$ b., c., d.; iii. $\rightarrow$ b., c., d.; iv. $\rightarrow$ a., c., d.
- i. $\rightarrow$ a.; ii. $\rightarrow$ b.; iii. $\rightarrow$ c.; iv. $\rightarrow$ d.
- i. $\rightarrow$ a.; ii. $\rightarrow$ d.; iii. $\rightarrow$ e.; iv. $\rightarrow$ b.
- i. $\rightarrow$ c., d., e.; ii. $\rightarrow$ b.; iii. $\rightarrow$ a.; iv. $\rightarrow$ a.
- i. $\rightarrow$ a., b., e.; ii. $\rightarrow$ a., c.; iii. $\rightarrow$ d.; iv. $\rightarrow$ b., d.
- i. $\rightarrow$ c.; ii. $\rightarrow$ d.; iii. $\rightarrow$ a.; iv. $\rightarrow$ b.
- (3)

Numerical Value Type

- | | | | | |
|----------|----------|---------|--------|---------|
| 1. (6) | 2. (4) | 3. (2) | 4. (2) | 5. (8) |
| 6. (7) | 7. (6) | 8. (12) | 9. (4) | 10. (8) |
| 11. (12) | 12. (90) | | | |

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JEE Advanced

Single Correct Answer Type

1. (3) 2. (3) 3. (1) 4. (4)

Multiple Correct Answers Type

1. (1),(3) 2. (1),(3),(4)

Linked Comprehension Type

- | | | | | |
|--------|--------|--------|--------|--------|
| 1. (4) | 2. (1) | 3. (2) | 4. (4) | 5. (3) |
| 6. (3) | 7. (1) | | | |

Matrix Match Type

1. a. $\rightarrow$ p., q., t.; b. $\rightarrow$ q.; c. $\rightarrow$ s.; d. $\rightarrow$ s. 2. (3)

Numerical Value Type

- | | | | | |
|-------------|--------|--------|--------|--------|
| 1. (8) | 2. (1) | 3. (4) | 4. (9) | 5. (5) |
| 6. (135.00) | | | | |

Chapter 7

Concept Application Exercises

Exercise 7.1

- Number of protons in nucleus = atomic number = 11
Number of electrons in an atom = Number of protons = 11
Number of neutrons = mass number (A) – atomic number (Z)
 $= 24 - 11 = 13$
- In order to compute binding energy, let us first find the total mass of all protons and neutrons in Nb and subtract mass of the Nb:
Given: $m_p = 1.007276$ u and $m_n = 1.008665$ u
Number of protons: $N_p = 41$
Number of neutrons: $N_n = 93 - 41 = 52$
Mass difference: $\Delta m = 41m_p + 52m_n - m_{Nb}$
 $= 41(1.007276 \text{ u}) + 52(1.008665 \text{ u}) - (92.9063768 \text{ u})$
 $= 0.865028 \text{ u}$

Thus, binding energy per nuclear is

$$\frac{E_b}{A} = \frac{(\Delta m)c^2}{A} = \frac{(0.865028 \text{ u})(931.5 \text{ MeV/u})}{93}$$

$$= 8.66 \text{ MeV nucleon}^{-1}$$

- Nuclei are stable because of the presence of another short-range (about 2 fm) force, the nuclear force. This is an attractive force that acts between all nuclear particles. The protons attract each other via the nuclear force, and at the same time they repel each other through the Coulomb force. The attractive nuclear force also acts between pairs of neutrons and between neutrons and protons.

The nuclear attractive force is stronger than the Coulomb repulsive force within the nucleus (at short ranges). If this were not the case, stable nuclei would not exist.

- $\left(\frac{BE}{A}\right)_P = \frac{100}{10} = 10$, $\left(\frac{BE}{A}\right)_Q = \frac{60}{5} = 12$, and $\left(\frac{BE}{A}\right)_R = \frac{66}{6} = 11$

Therefore, stability order is $Q > R > P$.

- After releasing 10 MeV, it will become more stable and its binding energy will increase.

$$\text{New binding energy} = 100 + 10 = 110 \text{ MeV}$$

- $(B_3 + B_4) - (B_1 + B_2)$
- The alpha particle contains 2 protons and 2 neutrons. The binding energy is

$$B = (2 \times 1.00726 \text{ u} + 2 \times 1.008665 \text{ u} - 4.00260 \text{ u})c^2$$

$$= (0.03038 \text{ u})c^2 = 0.03038 \times 931 \text{ MeV} = 28.3 \text{ MeV}$$

- The number of protons in $^{56}_{26}\text{Fe}$ = 26 and the number of neutrons = $56 - 26 = 30$. The binding energy of $^{56}_{26}\text{Fe}$
 $= [26 \times 1.00783 \text{ u} + 30 \times 1.00867 \text{ u} - 55.9349 \text{ u}]c^2$
 $= (0.52878 \text{ u})c^2 = (0.52878 \text{ u})(931 \text{ MeV/u}) = 492 \text{ MeV}$

- Mass of Avogadro number of atoms of $^{12}_6\text{C}$ is exactly 12 g. Avogadro's number, N_A , has the value $6.02 \times 10^{23} \text{ atoms mol}^{-1}$. Thus, the mass of one carbon atom is

$$\frac{0.012 \text{ kg}}{6.02 \times 10^{23} \text{ atom}} = 1.99 \times 10^{-26} \text{ kg}$$

Since one atom of $^{12}_6\text{C}$ is defined to have a mass of 12 u, we find that

$$1 \text{ u} = \frac{1.99 \times 10^{-26} \text{ kg}}{12} = 1.66 \times 10^{-27} \text{ kg}$$

- The nuclear density is nearly same in all atoms. The difference in densities between solid lead and gaseous oxygen arises mainly because of the difference in how closely the atoms themselves are packed together in solid and gaseous phase.

- The binding energy of ^8Be is determined by the equation

$$B(^8\text{Be}) = [4m_n + 4M(^1\text{H}) - M(^8\text{Be})]c^2$$

The binding energy

$$B(^8\text{Be}) = [4 \times 1.008665 + 4 \times 1.007825 - 8.005305] \times 931.5$$

$$= 56.5 \text{ MeV}$$

Now, we calculate the binding energy (B) of the decay of ^8Be into two α -particles.

$$^8\text{Be} \rightarrow 2\alpha,$$

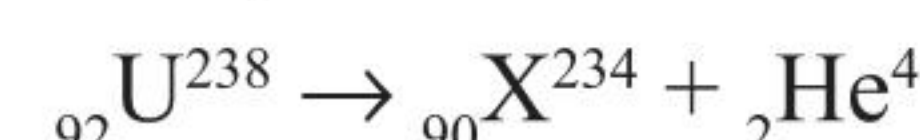
$$B = [2M(^4\text{He}) - M(^8\text{Be})]c^2$$

$$= [2 \times 4.002603 - 8.005305] \times 931.5 = -0.092 \text{ MeV}$$

Because B is negative for this reaction, hence ^8Be is unstable against decay to two alpha particles.

Exercise 7.2

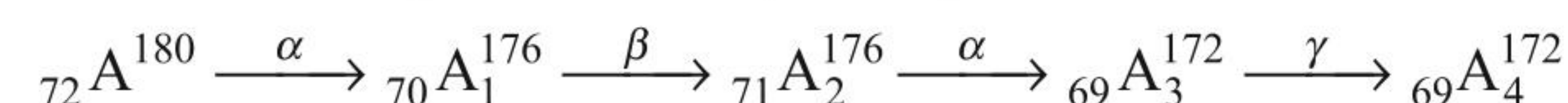
- When an α -particle is emitted, we have the product nucleus X:



When the product nucleus emits a β -particle, the final nucleus is Y (say) given by $^{234}_{90}\text{X} \rightarrow ^{234}_{91}\text{Y} + ^0_{-1}\beta$.

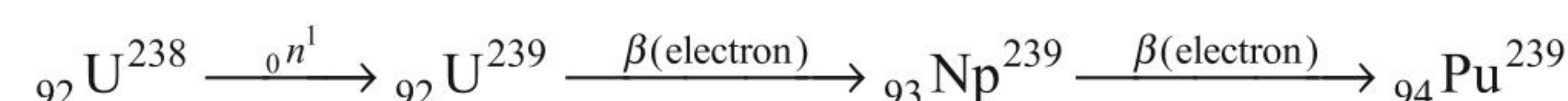
Thus, atomic number of final nucleus = 91, mass number of final nucleus = 234.

- The successive processes may be expressed as



Thus, the mass number of A_4 = 172 and the atomic number of A_4 = 69.

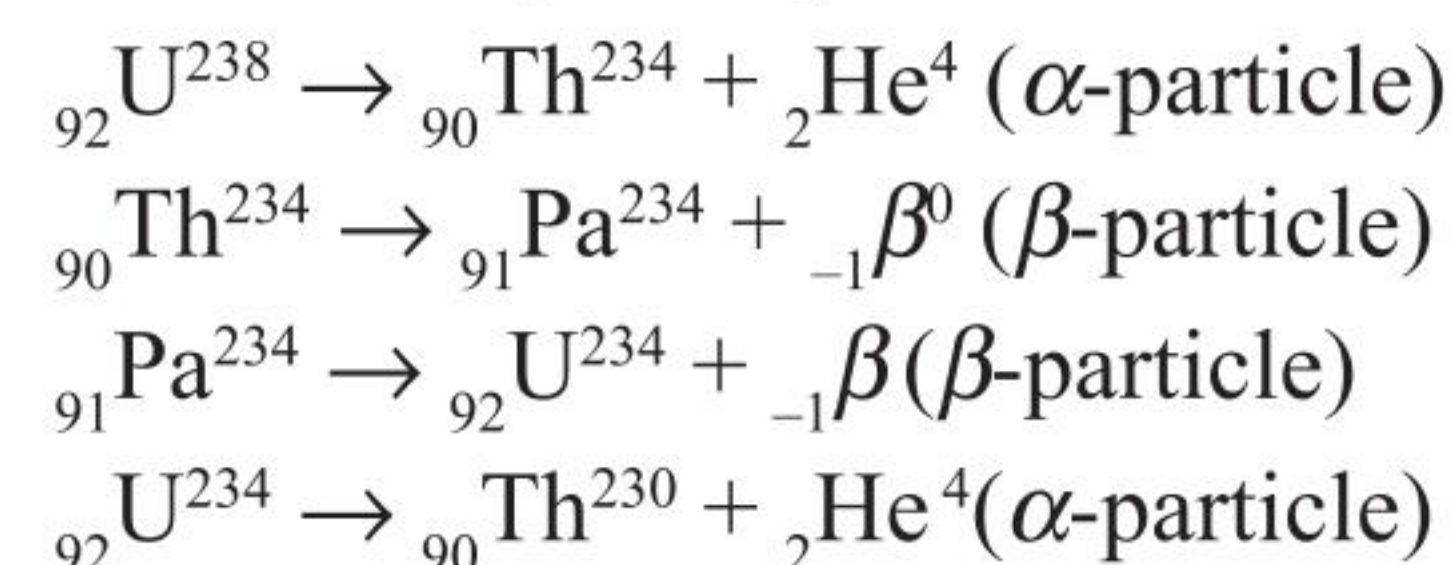
- The successive processes may be expressed as



Thus, the resulting plutonium has atomic number 94 and mass number 239 and is expressed as $^{239}_{94}\text{Pu}$.

- The process is expressed as $^{238}_{92}\text{U} \rightarrow ^{230}_{90}\text{Th} + 2(^4_2\text{He}) + 2(^0_{-1}\beta)$.

The process may be expressed as



The mass number of thorium nucleus is 230 and atomic number is 90.

- Mass number (or atomic weight) of $^{12}_6\text{C}$ is 12.

$$\text{Therefore, the number of atoms in 12 g of } ^{12}_6\text{C}$$

$$= \text{Avogadro number} = 6 \times 10^{23}$$

$$\text{The number of electrons in 12 g of } ^{12}_6\text{C} = 6 \times 6 \times 10^{23} = 36 \times 10^{23}$$

$$\text{The number of protons in 12 g of } ^{12}_6\text{C} = 36 \times 10^{23}$$

$$\text{The number of neutrons in 12 g of } ^{12}_6\text{C}$$

$$= (A - Z) \times 6 \times 10^{23} = (12 - 6) \times 6 \times 10^{23} = 36 \times 10^{23}$$

$$\text{Similarly, number of electrons in 14 g of } ^{14}_6\text{C} = 3.6 \times 10^{23}$$

$$\text{Number of protons in 14 g of } ^{14}_6\text{C}$$

$$= (A - Z) \times 6 \times 10^{23} = (14 - 6) \times 6 \times 10^{23} = 48 \times 10^{23}$$

The emitted particles are α , β , β , α and α , respectively.

6. Given: Reaction

In order to balance the reaction, the total amount of nucleons (sum of A -numbers) must be the same on both sides. Same for the Z -number.

$$\text{Number of nucleons (A): } 7 + 4 = X + 1 \Rightarrow X = 10$$

$$\text{Number of protons (Z): } 3 + 2 = Y + 0 \Rightarrow Y = 5$$

$$\text{Thus, it is B, i.e., } {}^7_3\text{Li} + {}^4_2\text{He} \rightarrow {}^{10}_5\text{B} + {}^1_0\text{n}$$

7. Let a and b be the number of α - and β -particles emitted when ${}^{238}_{92}\text{U}$ decays to ${}^{206}_{82}\text{Pb}$.

We know that

- (a) The emission of an α -particle (${}_2\text{He}^4$) decreases the charge number by two and mass number by four.

Therefore, emission of a α -particles will reduce the charge number by $2a$ and mass number by $4a$.

- (b) The emission of a β -particle increases the charge number by one and leaves the mass number unchanged.

Therefore, emission of b β -particles will increase the charge number by $b \times 1 = b$.

$$\text{Thus, } {}^{238}_{92}\text{U} \rightarrow {}^{206}_{82}\text{Pb} + a({}_2\text{He}^4) + b({}_{-1}\beta^0)$$

Applying the law of conservation of charge number and mass number before and after the decay, we have

$$92 = 82 + 2a - b \quad \dots(i)$$

$$238 = 206 + 4a \quad \dots(ii)$$

From Eq. (ii), $a = 8$, i.e., number of emitted α -particles = 8

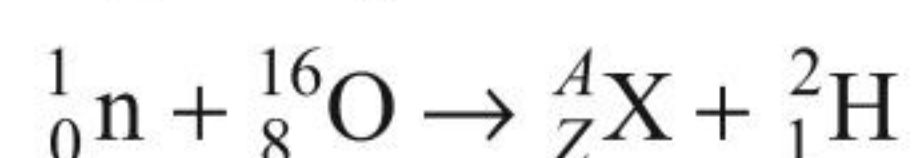
From Eq. (i), $b = 6$, i.e., number of emitted β -particles = 6

8. As the net electric charge is conserved, hence number of protons before the reaction is equal to the total number after the reaction. The total number of nucleons is conserved as well, so we can equate the total number before and after the reaction.

The results are listed in the table as follows:

Conserved quantity	Before reaction	After reaction
Total electric charge (number of protons)	$2 + 13$	$= Z + 0$
Total number of nucleons	$4 + 27$	$= A + 1$

Thus, we get $Z = 15$ and $A = 30$. Since $Z = 15$ identifies the element as phosphorus, the nucleus produced is ${}^{30}_{15}\text{P}$.

9. A deuteron is the nucleus of deuterium, the isotope of hydrogen containing one proton and one neutron, ${}^2_1\text{H}$, we have the reaction

Proceeding similar to previous problem,

Conserved quantity	Before reaction	After reaction
Total electric charge (number of protons)	$1 + 16$	$= Z$
Total number of nucleons	$0 + 8$	$= A$

Hence, the product nucleus must have $Z = 7$ and $A = 15$. From the periodic table we find that it is nitrogen that has $Z = 7$, so the nucleus is ${}^{15}_7\text{N}$. The reaction can be written ${}^{16}_8\text{O} (\text{n}, \text{d}) {}^{15}_7\text{N}$, where d represents deuterium ${}^2_1\text{H}$.

10. The equation for decay is ${}^{238}_{92}\text{U} \rightarrow {}^A_Z\text{X} + {}^4_2\text{He}$

Proceed similar to previous problem, to get $Z = 90$ and $A = 234$.

Thus, the nucleus is ${}^{234}_{90}\text{Th}$.

11. Each α -particle carries off two protons; the total number of protons carried by the α -particles is $2N_\alpha$.

Each β -particle is an electron ${}^0_{-1}\text{e}$ and is emitted when a neutron in the nucleus decays into a proton and an electron. Thus, the total number of protons left behind by the β -particles is N_β .

Each α -particle carries four nucleons; therefore the total number of nucleons carried off by the α -particles is $4N_\alpha$.

With the emission of each β -particle, a neutron is replaced by a proton. But since each is a nucleon, the number of nucleons is not changed.

$$\text{The overall decay process is } {}^{228}_{90}\text{Th} \rightarrow {}^{212}_{83}\text{Bi} + N_\alpha({}_2\text{He}^4) + N_\beta({}_{-1}\text{e})$$

Now, we can apply conservation of charge and nucleon number.

Conserved quantity	Before reaction	After reaction
Total electric charge (number of protons)	90	$83 + 2N_\alpha + (-1)N_\beta$
Total number of nucleons	228	$212 + 4N_\alpha + (0)N_\beta$

On solving the two equations, we get $N_\alpha = 4$ and $N_\beta = 1$.

Exercise 7.3

$$1. \quad R_1 = \left| \frac{dN_1}{dt} \right| = \lambda N_1 \quad \text{and} \quad R_2 = \left| \frac{dN_2}{dt} \right| = \lambda N_2$$

where N_1 and N_2 are the numbers of undecayed nuclei present at times t_1 and t_2 respectively

$$\text{Thus, } R_1 - R_2 = \lambda(N_1 - N_2)$$

$$\text{Or } (N_1 - N_2) = \frac{(R_1 - R_2)}{\lambda} = (R_1 - R_2)\tau \quad \left(\text{as } \frac{1}{\lambda} = \tau \right)$$

Thus, number of nuclei that have disintegrated in time interval $(t_1 - t_2)$, i.e., $(N_1 - N_2) = (R_1 - R_2)\tau$.

2. The activity is proportional to the number of undecayed atoms. In each half-life, the remaining sample decays to half of its initial

value. Since $\left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) = \frac{1}{8}$, therefore, three half-lives or 15.75 years are required for the sample to decay to 1/8th its original strength.

3. Here, $T_{1/2} = 24.1$ days, $N_0 = 100$, $N = (100 - 90) = 10$

$$\text{Thus, } \frac{N_0}{N} = \frac{100}{10} = 10$$

$$\text{As } \frac{N_0}{N} = 2^{t/T_{1/2}} \Rightarrow 10 = 2^{t/24.1}$$

$$\text{or } \log 10 = \log 2^{t/24.1} \Rightarrow t = \frac{1}{0.3010} \times 24.1 \text{ d} = 80 \text{ d}$$

4. Let t be the time taken by the activity (R) to drop to (1/50) of its present unsafe value (R_0), i.e., $\frac{R}{R_0} = \frac{1}{50}$

$$\text{As } R = \lambda N \text{ and } R_0 = \lambda N_0$$

$$\frac{N}{N_0} = \frac{R}{R_0} = \frac{1}{50}$$

$$\text{or } \frac{N_0}{N} = 50$$

$$\text{As } \frac{N_0}{N} = 2^{t/T_{1/2}}; 50 = 2^{t/10}$$

$$\text{Thus, } \log 50 = \log 2^{t/10}$$

$$\text{or } \log 50 = \frac{t}{10} \log 2$$

$$\text{or } t = \frac{10 \times \log 50}{\log 2} = \frac{10 \times 1.6990}{0.3010} = 56.45 \text{ d}$$

5. Here, $R = 1.24 \times 10^4$ dps

Number of nuclei in 1 g of ^{238}U , i.e., $N = \frac{6.023 \times 10^{23}}{238}$

As $R = \lambda N$,

$$\lambda = \frac{R}{N} = \frac{(1.24 \times 10^4) \times 238}{6.023 \times 10^{23}} = 4.9 \times 10^{-18} \text{ s}^{-1}$$

$$\text{Thus, } T_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{4.9 \times 10^{-18} \text{ s}^{-1}} = 1.41 \times 10^{17} \text{ s}$$

$$\Rightarrow T = 4.5 \times 10^9 \text{ y} \quad (\text{as } 1 \text{ y} = 3.15 \times 10^7 \text{ s})$$

6. We know that $\lambda = 0.693/T_{1/2}$

Here, $T_{1/2} = 3.8$ day

$$\therefore \lambda = \frac{0.693}{3.8} = 0.182 \text{ per day}$$

If initially (at $t = 0$) the number of atoms present be N_0 , then the number of atoms N left after a time given by

$$N = N_0 e^{-\lambda t}$$

$$\frac{N}{N_0} = e^{-\lambda t} \text{ or } \frac{1}{20} = e^{-\lambda t}$$

Taking log, $\lambda t = \log_e 20 = 2.3026 \log_{10} 20$

$$\therefore t = \frac{2.3026 \log_{10} 20}{\lambda} = \frac{2.3026 \log_{10} 20}{0.182} = 16.45 \text{ days}$$

7. (a) The initial number of nuclei is $N_0 = \frac{mN_A}{M}$

(b) The number of undecayed nuclei after a time t is

$$N = N_0 e^{-\lambda t} = \left(\frac{mN_A}{M} \right) e^{-\lambda t}$$

Therefore, the number of decayed nuclei is

$$N' = N_0 - N = \frac{mN_A}{M} (1 - e^{-\lambda t})$$

(c) The activity of the sample is $A = A_0 e^{-\lambda t} = \lambda N_0 e^{-\lambda t}$

$$A = \left(\frac{mN_A \lambda}{M} \right) e^{-\lambda t}$$

8. We know that $N = N_0 e^{-\lambda t}$, where $\lambda = \frac{\ln 2}{T} = \frac{1}{T_{\text{av}}}$

Here, $N = (1 - \eta)N_0$

$$\Rightarrow (1 - \eta)N_0 = N_0 e^{-\lambda t} \text{ or } t = \frac{1}{\lambda} \ln \left| \frac{1}{(1 - \eta)} \right|$$

$$(a) \text{ Thus, } t = \frac{T}{\ln 2} \left[\ln \left| \frac{1}{1 - \eta} \right| \right] = \frac{T}{0.693} \ln \left| \frac{1}{(1 - \eta)} \right|$$

$$(b) t = T_{\text{av}} \ln \left| \frac{1}{(1 - \eta)} \right|$$

9. $A_1 = A_0 e^{-\lambda t_1}$ and $A_2 = A_0 e^{-\lambda t_2}$

$$\therefore \frac{A_1}{A_2} = e^{\lambda(t_2 - t_1)}$$

$$\Rightarrow \lambda(t_2 - t_1) = \ln \left| \frac{A_1}{A_2} \right| \Rightarrow T = \frac{1}{\lambda} = \frac{t_2 - t_1}{\ln \left| \frac{A_1}{A_2} \right|}$$

10. The decay constant is $\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{5730 \times 5.26 \times 10^5} = 2.30 \times 10^{-10} \text{ min}^{-1}$

As 1 year = 3.01×10^9 min and for 1.0 g of carbon, the number of moles is $n = 1/12$, so $N = nN_A = \left(\frac{1}{12} \right) (6.02 \times 10^{23}) = 5.0 \times 10^{22}$ nuclei g^{-1} .

The given concentration factor is $\frac{^{14}\text{C}}{^{12}\text{C}} = 1.4 \times 10^{-12}$

Thus, the number of ^{14}C nuclei per gram is

$$N \left(\frac{^{14}\text{C}}{^{12}\text{C}} \right) = (5.0 \times 10^{22})(1.4 \times 10^{-12}) = 7.0 \times 10^{10} \text{ nuclei g}^{-1}$$

The activity, number of decays per gram per minute, is

$$\frac{\Delta N}{\Delta t} = \lambda N = (2.30 \times 10^{-10})(7.0 \times 10^{10}) = 16 \text{ decays g}^{-1} \text{ min}^{-1}$$

11. (a) This part has already been discussed in section on successive disintegrations with the result

$$N_2(t) = \frac{\lambda_1 N_0}{\lambda_2 - \lambda_1} (e^{-\lambda_1 t} - e^{-\lambda_2 t})$$

(b) The activity of nuclide A_2 is $\lambda_2 N_2$. This is maximum when N_2 is maximum, i.e., when $dN_2/dt = 0$, activity is maximum.

$$\frac{d[N_2(t)]}{dt} = \frac{d}{dt} \left[\frac{\lambda_1 N_0}{\lambda_2 - \lambda_1} (e^{-\lambda_1 t} - e^{-\lambda_2 t}) \right]$$

$$\frac{\lambda_1 N_0}{\lambda_2 - \lambda_1} (\lambda_1 e^{-\lambda_1 t} + \lambda_2 e^{-\lambda_2 t}) = 0$$

$$e^{-(\lambda_1 - \lambda_2)t} = \frac{\lambda_2}{\lambda_1}$$

$$t = \frac{\ln(\lambda_2/\lambda_1)}{\lambda_2 - \lambda_1}$$

12. In 5730 years, half the sample will have decayed, leaving 500 radioactive $^{14}_6\text{C}$ nuclei. In another 5730 years (for a total elapsed time of 11460 years), the number will be reduced to 250 nuclei. After another 5730 years (total time 17190 years), 125 nuclei remain.

These numbers represent ideal circumstances. Radioactive decay is an averaging process over a very large number of atoms, and the actual outcome depends on statistics. Our original sample in this example contained only 1000 nuclei, certainly not a very large number. Thus, if we were actually to count the number remaining after one half-life for this small sample, it probably would not be exactly 500.

13. First, let us convert the half-life to seconds.

$$T_{1/2} = (1.6 \times 10^3 \text{ years})(3.16 \times 10^7 \text{ s/years}) = 5.0 \times 10^{10} \text{ s}$$

$$\therefore \lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{5.0 \times 10^{10} \text{ s}} = 1.4 \times 10^{-11} \text{ s}^{-1}$$

We can calculate the activity of the sample at $t = 0$ using $R_0 = \lambda N_0$, where R_0 is the decay rate at $t = 0$ and N_0 is the number of radioactive nuclei present at $t = 0$.

$$R_0 = \lambda N_0 = (1.4 \times 10^{-11} \text{ s}^{-1})(3.0 \times 10^{16}) = 4.1 \times 10^5 \text{ decays s}^{-1}$$

Because $\text{Ci} = 3.7 \times 10^{10} \text{ decays s}^{-1}$, the activity or decay rate at $t = 0$ is $R_0 = 11.1 \text{ } \mu\text{Ci}$.

Exercise 7.4

1. Energy released in the fission of a single nucleus of $^{235}_{92}\text{U}$
 $= 200 \text{ MeV} = 200(1.6 \times 10^{-13}) \text{ J} = 3.2 \times 10^{-11} \text{ J}$

To produce a power of 1 kW, 1000 J of energy should be released per second.

If n is the number of fissions required per second to produce an energy of 1000 J,

$$n = \frac{1000 \text{ J}}{3.2 \times 10^{-11} \text{ J}} = 3.125 \times 10^{13}$$

2. Energy output of the reactor per day

$$= 200 \times 10^6 \text{ (J/s)} \times (24 \times 60 \times 60 \text{ s}) = 1728 \times 10^{10} \text{ J}$$

$$\text{Energy input} = \frac{1728 \times 10^{10}}{(25/100)} = 6912 \times 10^{10} \text{ J}$$

$$\begin{aligned} \text{Energy released by the fusion of two } {}^2_1\text{H nuclei} \\ &= [(2 \times 2.0141) - 4.0026] \times 931.5 \text{ MeV} \\ &= 23.85 \text{ MeV} = 38.15 \times 10^{-13} \text{ J} \end{aligned}$$

$$\text{Number of } {}^2_1\text{H nuclei required} = \frac{6912 \times 10^{10} \text{ J}}{(38.15 \times 10^{-13} \text{ J})/2} = 362.3 \times 10^{23}$$

$$\text{Mass of fuel required} = (362.3 \times 10^{23}) \times \frac{2}{6.023 \times 10^{23}} = 120.3 \text{ g}$$

3. Energy released in one year = $(10^6 \times 10^3)(3.154 \times 10^7)$

$$\text{Number of } {}^{235}_{92}\text{U nuclei fissioned} = \frac{10^9 (3.154 \times 10^7)}{200(1.6 \times 10^{-13})} = 9.856 \times 10^{26}$$

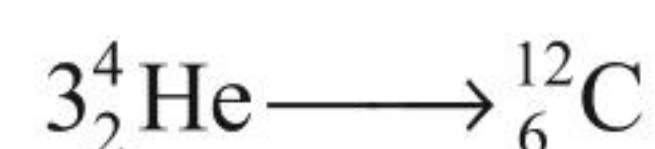
$$\text{Mass of } {}^{235}_{92}\text{U released in the reactor} = \frac{(9.856 \times 10^{26})235}{6.023 \times 10^{23}} = 384.5 \text{ kg}$$

4. Number of atoms in 2 kg of ${}^{235}_{92}\text{U}$

$$= \frac{2000}{235} \times (6.023 \times 10^{23}) = 5.126 \times 10^{24}$$

$$\begin{aligned} \text{Energy released due to fission of 2 kg of } {}^{235}_{92}\text{U} \\ &= (5.126 \times 10^{24}) \times 185 \text{ MeV} = 15.173 \times 10^{13} \text{ J} \end{aligned}$$

$$\text{Power output of the reactor} = \frac{15.173 \times 10^{13}}{30(24 \times 60 \times 60)} = 58.54 \text{ MW}$$

5. According to the assumption, as three alpha particles form a ${}^{12}_6\text{C}$ nucleus,

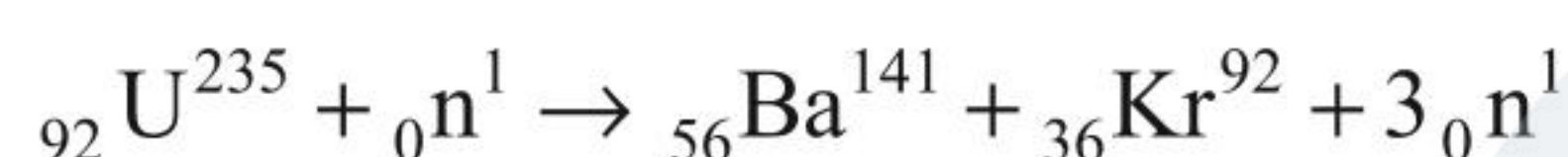
$$\text{Mass of three alpha particles} = 3 \times 4.002604 = 12.007812 \text{ u}$$

$$\text{Mass of carbon nucleus} = 12 \text{ u}$$

$$\text{Mass defect} = 12.007812 \text{ u} - 12 \text{ u} = 0.007812 \text{ u}$$

$$\begin{aligned} \text{Energy released in the reaction} \\ &= 0.007812 \times 931.5 \text{ MeV} = 7.277 \text{ MeV} \end{aligned}$$

6. The nuclear fission reaction is



$$\begin{aligned} \text{Mass defect } \Delta m &= [(m_{\text{U}} + m_{\text{n}}) - (m_{\text{Ba}} + m_{\text{Kr}} + 3m_{\text{n}})] \\ &= 256.0526 - 235.8373 = 0.2153 \text{ u} \end{aligned}$$

$$\text{Energy released } Q = 0.2153 \times 931 = 200 \text{ MeV}$$

$$\text{Number of atoms in 1 g} = \frac{6.02 \times 10^{23}}{235} = 2.56 \times 10^{21}$$

$$\begin{aligned} \text{Energy released in fission of 1 g of } {}^{235}_{92}\text{U} \\ &= 200 \times 2.56 \times 10^{21} = 5.12 \times 10^{23} \text{ MeV} \\ &= 5.12 \times 10^{23} \times 1.6 \times 10^{-13} = 8.2 \times 10^{10} \text{ J} \\ &= \frac{8.2 \times 10^{10}}{3.6 \times 10^6} \text{ kWh} = 2.28 \times 10^4 \text{ kWh} \end{aligned}$$

7. Energy released in the nuclear fusion is $Q = \Delta mc^2 = \Delta m(931) \text{ MeV}$ (where Δm is in amu)

$$Q = (2 \times 2.0141 - 4.0026) \times 931 \text{ MeV} = 23.834 \times 10^6 \text{ eV}$$

Since efficiency of reactor is 25%

So effective energy used

$$= \frac{25}{100} \times 23.834 \times 10^6 \times 1.6 \times 10^{-19} \text{ J} = 9.534 \times 10^{-13} \text{ J}$$

Since the two deuterium nuclei are involved in a fusion reaction, therefore, energy released per deuterium is $\frac{9.534 \times 10^{-13}}{2}$.

For 200 MW power per day, number of deuterium nuclei required

$$= \frac{200 \times 10^6 \times 86400}{\frac{9.534 \times 10^{-23}}{2}} = 3.624 \times 10^{25}$$

Since 2 g of deuterium constitute 6×10^{23} nuclei, therefore amount of deuterium required is

$$\frac{2 \times 3.624 \times 10^{25}}{6 \times 10^{23}} = 120.83 \text{ g/day}$$

8. Number of deuterium atoms in 2.0 kg of deuterium

$$= (6.023 \times 10^{23}) \left(\frac{2.0 \times 1000 \text{ g}}{2 \text{ g}} \right) = 6.023 \times 10^{26}$$

Energy released when 2 deuterium nuclei fuse = 3.27 MeV

$$\begin{aligned} \text{Total energy released} &= \left(\frac{3.27}{2} \right) (6.023 \times 10^{26}) \text{ MeV} \\ &= \left(\frac{3.27}{2} \right) (6.023 \times 10^{26}) (1.6 \times 10^{-13} \text{ J}) \\ &= 15.76 \times 10^{13} \text{ J} \end{aligned}$$

Energy consumed by bulb per second = 100 J/s

$$\text{Time for which the bulb can be kept glowing} = \frac{15.76 \times 10^{13} \text{ J}}{100 \text{ J/s}}$$

$$= 15.76 \times 10^{11} \text{ s} = \frac{15.76 \times 10^{11} \text{ s}}{3.154 \times 10^7 \text{ s/y}} = 5 \times 10^4 \text{ y}$$

9. For 1000 MW output, the total power generation is 3000 MW, of which 2000 MW is rejected as “waste” heat. Thus, the total energy released in 1 year ($3 \times 10^7 \text{ s}$) from fission is about $(3 \times 10^9) (3 \times 10^7) \approx 10^{17}$.

If each fission releases 200 MeV of energy, the number of fissions required is

$$\frac{(10^{17})}{(2 \times 10^8)(1.6 \times 10^{-19})} \approx 3 \approx 10^{27} \text{ fissions}$$

The mass of a single uranium atom is about (235 u) ($1.66 \times 10^{-27} \text{ kg/u}$) $\approx 4.5 \times 10^{-25} \text{ kg}$, so that mass needed is $(4.5 \times 10^{-25} \text{ kg}) (3 \times 10^{27} \text{ fissions}) \approx 1000 \text{ kg}$ or about a ton. Since ${}^{235}_{92}\text{U}$ is only a fraction of normal uranium, and even when enriched it is never more than 10 per cent of the total, the yearly requirement for uranium is of the order of tens of tons.

10. After decay, the mass of the daughter m_d plus the mass of the alpha particle m_α is

$$m_d + m_\alpha = 222.017571 \text{ u} + 4.002602 \text{ u} = 226.020173 \text{ u}$$

Thus, calling the mass of the parent nucleus M_p , we find that the mass lost during decay is

$$\begin{aligned} \Delta m &= M_p - (m_d + m_\alpha) \\ &= 226.025402 \text{ u} - 226.020173 \text{ u} = 0.005229 \text{ u} \end{aligned}$$

Using the relationship that 1 u corresponds to 931.494 MeV, we find that the energy liberated is

$$E = (0.005229 \text{ u}) (931.494 \text{ MeV/u}) = 4.87 \text{ MeV}$$

11. The mass of a ${}^{12}_6\text{C}$ atom is exactly 12 u. The energy released in the reaction is $3({}^4_2\text{He}) \rightarrow {}^{12}_6\text{C}$

$$\begin{aligned} [3m({}^4_2\text{He}) - m({}^{12}_6\text{C})] c^2 &= [3 \times 4.002603 \text{ u} - 12 \text{ u}] \\ (931 \text{ MeV/u}) &= 7.27 \text{ MeV} \end{aligned}$$

Exercises

Single Correct Answer Type

1. (4) For decay (i):

$$Q = [230.033927 - 229.033496 - 1.008665] \times 931.5 \\ = -7.7 \text{ MeV}$$

For decay (ii):

$$Q = [230.033927 - 229.032089 - 1.007825] \times 931.5 \\ = -5.6 \text{ MeV}$$

As Q is negative for both the decays, so none of the decays is allowed.

2. (3) Suppose an initial radionuclide I decays to a final product F with a half-life
- $T_{1/2}$
- .

At any time, $N_I = N_0 e^{-\lambda t}$ Number of product nuclei = $N_F = N_0 - N_I$

$$\frac{N_F}{N_I} = \frac{N_0 - N_I}{N_I} = \left(\frac{N_0}{N_I} - 1 \right)$$

$$\frac{N_0}{N_I} = \left(1 + \frac{N_F}{N_I} \right) = 1 + 0.5 = 1.5$$

$$e^{\lambda t} = 1.5 \Rightarrow \lambda t = \ln 1.5$$

$$\therefore \frac{T_{1/2} \ln(1.5)}{\ln 2} = 4.5 \times 10^9 \frac{\ln\left(\frac{3}{2}\right)}{\ln 2} \text{ year}$$

3. (1) Given,
- $N_2 = \frac{N_0}{e} = N_0 e^{-\lambda t} \Rightarrow t = \frac{1}{\lambda} = 10 \text{ s}$

$$\therefore T_{1/2} = \frac{\ln 2}{\lambda} = 0.693 \times 10 \approx 7 \text{ s}$$

4. (4) Since no external force is present, so momentum conservation principle is completely applicable.

$$\therefore m\vec{v} = m_1\vec{v}_1 + m_2\vec{v}_2$$

$$\text{or } (m_1 + m_2) \vec{v} = m_1\vec{v}_1 + m_2\vec{v}_2$$

5. (1)
- $\frac{dN_A}{dt} = (-\lambda N_A) + (-2\lambda N_A) + (-2\lambda N_A) = -6\lambda N_A$

6. (2) Let,
- ${}_Z^AX \rightarrow {}_{A-2}^{A-4}Y + {}_2^4\text{He}$

$$K_\alpha = \frac{m_y}{m_y + m_\alpha} Q = \frac{A-4}{A} Q$$

$$\text{or } 48 = \frac{A-4}{A} \times 50 \Rightarrow A = 100$$

7. (1) After first half hours,
- $N = N_0 \frac{1}{2}$

For $t = \frac{1}{2} \text{ h}$ to $t = 1 \frac{1}{2} \text{ h}$, 1 h = four half-lives

$$\text{Hence, } N = \left(N_0 \frac{1}{2} \right) \left[\frac{1}{2} \right]^4 = N_0 \left(\frac{1}{2} \right)^5$$

For $t = \frac{1}{2}$ to $t = 2 \text{ h}$

$$\left[\text{for both } A \text{ and } B, \frac{1}{t_{1/2}} = \frac{1}{t_{1/2}} + \frac{1}{t_{1/4}} = 2 + 4 = 6 \Rightarrow t_{1/2} = \frac{1}{6} \right]$$

 $\frac{1}{2} \text{ h}$ = three half-lives

$$\therefore N = \left[N_0 \left(\frac{1}{2} \right)^5 \right] \left(\frac{1}{2} \right)^3 = N_0 \left(\frac{1}{2} \right)^8$$

8. (2) For
- α
- decay:
- ${}_x^A Y \rightarrow {}_{x-2}^{A-4} B + \alpha$

For β^- decay: ${}_x^A Y \rightarrow {}_{x+1}^A B + {}_{-1}^0 \beta$ For β^+ decay: ${}_x^A Y \rightarrow {}_{x-1}^A B + {}_{+1}^0 \beta$

For k-capture, there will be no change in the number of protons. Hence, only case in which number of protons increases is β^- decay.

9. (3)
- $N = N_0 e^{-\lambda t}$
- ,
- $N_Y = N_0 (1 - e^{-\lambda t})$

Rate of formation of Y is $\frac{dN}{dt} = +\lambda N_0 e^{-\lambda t}$

which decreases exponentially with time.

10. (2) 90% of the sample is left undecayed after time
- t
- .

$$\therefore \frac{9}{10} N_0 = N_0 e^{-\lambda t}$$

$$\lambda = \frac{1}{t} \ln \left(\frac{10}{9} \right) \quad \dots(i)$$

After time $2t$,

$$N_c = N_0 e^{-\lambda(2t)} = N_0 e^{-\frac{1}{t} \left[\ln \left(\frac{10}{9} \right) \right] 2t} \quad \dots(ii)$$

$$N = N_0 e^{-\ln \left(\frac{10}{9} \right)^2} = N_0 \left(\frac{9}{10} \right)^2 \approx 81\% \text{ of } N_0 \quad \dots(iii)$$

Therefore, 19% of initial value will decay in time $2t$.

11. (3) Let
- N_2
- be the number of atoms of X at time
- $t = 0$
- . Then, at
- $t = 4 \text{ h}$
- (two half-lives),

$$N_x = \frac{N_0}{4} \quad \text{and} \quad N_y = \frac{3N_0}{4}$$

$$\therefore \frac{N_x}{N_y} = \frac{1}{3} \approx 0.33$$

At $t = 6 \text{ h}$ (three half-lives),

$$N_x = \frac{N_0}{8} \quad \text{and} \quad N_y = \frac{7N_0}{8} \quad \text{or} \quad \frac{N_x}{N_y} = \frac{1}{7} \approx 0.142$$

The given ratio $\frac{1}{4}$ lies between $\frac{1}{3}$ and $\frac{1}{7}$.Therefore, t lies between 4 h and 6 h.

12. (2)
- $\frac{dN_2}{dt} = \lambda N_1 - 2\lambda N_2$

For N_2 to be maximum, $\frac{dN_2}{dt} = 0$

$$\Rightarrow \lambda N_1 = 2\lambda N_2 \quad \text{or} \quad \frac{N_1}{N_2} = 2$$

13. (2) The total energy required to make the electron free from nucleus is the sum of the energy required to separate the electrons from the influence of each other and the energy required to separate the electrons from the influence of nucleus, i.e.,

Total required energy = BE of electron in He atom + ionization energy of He atom = $(24.6 + 13.6 \times 2^2) \text{ eV} = (24.6 + 54.4) \text{ eV} = 79 \text{ eV}$

14. (1) From given information,
- $\frac{dN}{dt} = \frac{-0.04N}{3600}$

Comparing above equation with standard decay equation,

$$\frac{dN}{dt} = -\lambda N$$

$$\lambda = 1.1 \times 10^{-5} \text{ s}^{-1}$$

$$\therefore \tau = \frac{1}{\lambda} = \frac{3600}{0.04} \text{ s} = 25 \text{ h}$$

15. (3) Let n collisions are required for the given condition. Then,

$$\left(\frac{1}{2}\right)^n \times 2 \text{ MeV} = 0.04 \times 10^{-6} \text{ MeV}$$

$$2^n = \frac{2}{0.04} \times 10^6 = 50 \times 10^6$$

After solving above equation, $n = 26$.

16. (3) Expected atomic mass of Cu must be less than that of zinc, but it is not so. So, it means Cu is radioactive and unstable and decays to Zn through β -decay.
17. (1) As the alpha particle decays, the daughter nucleus recoils. In such a process, the momentum conservation holds good.

So, $p_\alpha = p_D = p$

$$K_\alpha = \frac{p^2}{2M_\alpha} \quad \text{and} \quad K_D = \frac{p^2}{2M_D}$$

As $M_D > M_\alpha$, so, $K_\alpha > K_D$.

18. (3) The minimum energy needed to carry out an endothermic reaction is greater than the Q value of the reaction. This is because to conserve the momentum some extra energy has to be provided.

$KE_{\min} = \left(1 + \frac{m}{M}\right) \times |Q|$, where m is the mass of the incident particle and M is the mass of target.

19. (3) The number of nuclei in 1 kg ^{235}U is

$$N = \frac{N_A}{235} \times (1 \times 10^3) = \frac{6.023 \times 10^{23}}{235} \times 10^3 = 2.56 \times 10^{24} \text{ nuclei}$$

Total energy released is $E = N \times 200 \text{ MeV} = 5.12 \times 10^{26} \text{ MeV}$

20. (1) When a free neutron decays to a proton along with an electron and an antineutrino, the Q value of the reaction is positive which means the reaction is possible all by itself, while a free proton cannot convert itself into a neutron due to negative Q value.

In beta minus decay, the electron originates from nucleus only, by the transformation of neutron into a proton, with simultaneous emission of an antineutrino.

21. (1) Since scheme A releases more energy than scheme B, scheme A is more likely to occur. This is because the more the energy released, the more stable the daughter nucleus is. A heavy nucleus undergoes fission such that its products will be more stable than the parent nucleus.

22. (1) At present, $\frac{\text{Number of K atoms}}{\text{Number of Ar atoms}} = \frac{1}{7}$

Let age of rock be n half-lives of K-nuclide. Then,

$$\left(\frac{1}{2}\right)^n = \frac{\text{Number of K-atoms present now}}{\text{Number of K-atom present initially}} = \frac{1}{1+7}$$

where number of K atoms present initially = number of K atoms + number of Ar atoms present now.

$$\therefore n = 3$$

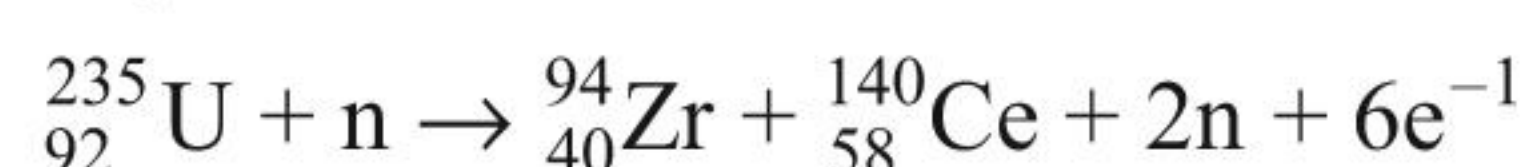
So, age of rock is 3 half-lives of K nuclides, i.e., 4.2×10^9 years.

23. (1) Probability of survival for any nucleus at time t is

$$P = \frac{N}{N_0} = \frac{N_0 e^{-\lambda t}}{N_0} = e^{-\lambda t}$$

So, in one mean life, required probability is $e^{-\lambda \times \frac{1}{\lambda}} = \frac{1}{e}$

24. (2) The complete fission reaction is



$$Q = [m(^{235}\text{U}) - m(^{94}\text{Zr}) - m(^{140}\text{Ce}) - m(\text{n})]c^2 = 208 \text{ MeV}$$

25. (3) The net reaction is $3(^2_1\text{H}) \rightarrow ^4_2\text{He} + \text{n} + \text{p}$

$$Q = [3 \times m(^2\text{H}) - m(^4\text{He}) - m(\text{n}) - m(\text{p})] \times 931 \text{ MeV}$$

$$= 3.87 \times 10^{-12} \text{ J}$$

This is the energy produced by the consumption of 3 deuteron atoms. So, the total energy released by 10^{40} deuterons is

$$\frac{3.87 \times 10^{-12}}{3} \times 10^{40} = 1.29 \times 10^{28} \text{ J}$$

Let total supply of deuterons in star be exhausted in t seconds.

$$\text{Then, } 10^{16} \times t = 1.29 \times 10^{28}$$

$$\Rightarrow t = 1.29 \times 10^{12} \text{ s}$$

26. (4) $N_{x_1} = N_0 e^{-10\lambda t}$

$$N_{x_2} = N_0 e^{-\lambda t}$$

$$\frac{N_{x_1}}{N_{x_2}} = \frac{1}{e} = \frac{e^{-10\lambda t}}{e^{-\lambda t}} = e^{-9\lambda t}$$

$$9\lambda t = 1 \Rightarrow t = \frac{1}{9\lambda}$$

27. (3) Let N be the number of nuclei at any time t . Then,

$$\frac{dN}{dt} = 200 - \lambda N$$

$$\therefore \int_0^N \frac{dN}{200 - \lambda N} = \int_0^t dt$$

$$\text{or } N = \frac{200}{\lambda} (1 - e^{-\lambda t})$$

Given: $N = 100$ and $\lambda = 1 \text{ s}^{-1}$

$$\therefore 100 = 200 (1 - e^{-t})$$

$$\text{or } e^{-t} = \left(\frac{1}{2}\right) \therefore t = \ln(2) \text{ s}$$

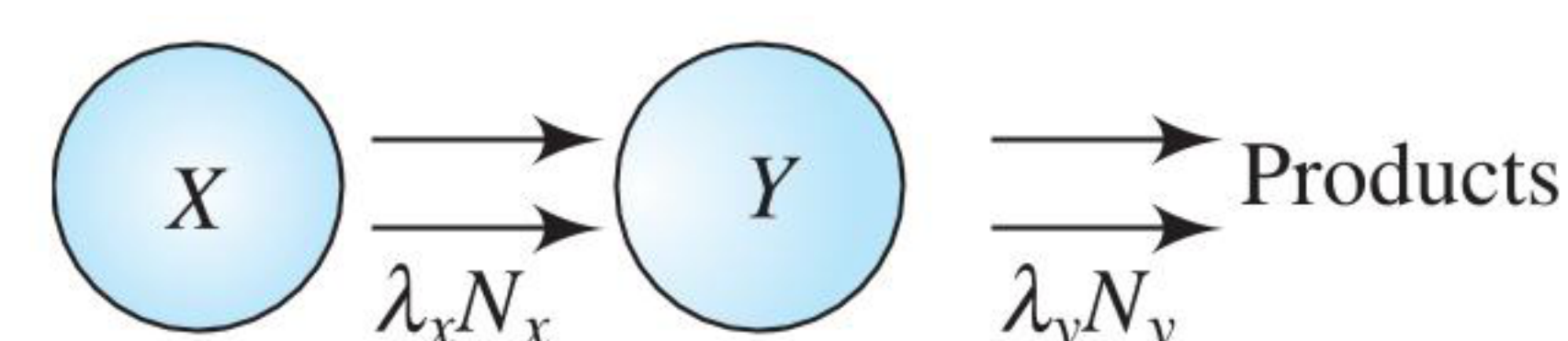
28. (1) Number of radio nuclei become constant, when rate of production becomes equal to rate of decay.

$$X = \lambda N$$

$$\text{or } N = \frac{X}{\lambda}. \text{ Given, } Y = \frac{\ln 2}{\lambda}$$

$$\Rightarrow N = \frac{XY}{\ln(2)}$$

29. (3)



Net rate of formation of Y at any time t is $\frac{dN_y}{dt} = \lambda_x N_x - \lambda_y N_y$

N_y is maximum when $\frac{dN_y}{dt} = 0$

$$\text{or } \lambda_x N_x = \lambda_y N_y$$

30. (1) $\frac{\lambda_A}{\lambda_B} = \frac{1}{2}$

Probabilities of getting α - and β -particles are same. Thus, rates of disintegration are equal.

$$\therefore \lambda_A N_A = \lambda_B N_B$$

$$\text{or } \frac{N_A}{N_B} = \frac{\lambda_B}{\lambda_A} = 2$$

31. (2) Three half-lives of A is equivalent to six half-lives of B. Hence,

$$N_A \left(\frac{1}{2}\right)^3 = N_B \left(\frac{1}{2}\right)^6$$

$$\text{or } \frac{N_A}{N_B} = \frac{1}{8}$$

32. (1) Let $\lambda_A = \lambda$ and $\lambda_B = 2\lambda$. Initially, rate of disintegration of A is λN_0 and that of B is $2\lambda N_0$. After one half-life of A , rate of disintegration of A will become $\lambda N_0/2$ (half-life of B = one-half the half-life of A). So, after one half-life of A or two half-lives of B ,

$$\left(-\frac{dN}{dt}\right) = \left(-\frac{dN}{dt}\right)_B$$

$$\therefore n = 1$$

33. (1) Maximum number of nuclei will be present when rate of decay = rate of formation

$$\text{or } \lambda N = \alpha$$

$$\therefore N = \frac{\alpha}{\lambda}$$

34. (1) Fraction of nuclei which remain undecayed is

$$f = \frac{N}{N_0} = \frac{N_0 e^{-\lambda t}}{N_0} = e^{-\lambda t}$$

$$= e^{-\left(\frac{\ln 2}{T}\right)\left(\frac{T}{2}\right)} = \frac{1}{e^{\ln \sqrt{2}}} = \frac{1}{\sqrt{2}}$$

35. (2) Let R_0 be the initial activity. Then, $R_1 = R_0 e^{-\lambda t_1}$

$$\text{and } R_2 = R_0 e^{-\lambda t_2}$$

$$\therefore \frac{R_2}{R_1} = e^{\lambda(t_1 - t_2)} \quad \text{or } R_2 = R_1 e^{\lambda(t_1 - t_2)}$$

36. (4) $R_1 = \lambda N_1 \Rightarrow N_1 = \frac{R_1}{\lambda}$

$$\text{and } R_2 = \lambda N_2 \Rightarrow N_2 = \frac{R_2}{\lambda}$$

$$\text{Therefore, number of atoms decayed} = N_1 - N_2 = \left(\frac{R_1 - R_2}{\lambda}\right)$$

37. (3) Activity, $R = \lambda N$. Number of nuclei (N) per mole are equal for both the substances.

$$\therefore R \propto \lambda$$

$$\text{or } \frac{R_1}{R_2} = \frac{\lambda_1}{\lambda_2} = \frac{4}{3}$$

38. (2) Activity of a radioactive substance, $R = \lambda N$

$$\therefore \lambda = \frac{R}{N}$$

Here, $R = N_2$ particles per second and $N = N_1$.

$$\therefore \lambda = \frac{N_2}{N_1}$$

39. (2) During fusion, binding energy of daughter nucleus is always greater than the total binding energy of the parent nuclei. The difference of binding energies is released. Hence,

$$Q = E_2 - 2E_1$$

40. (4) Energy released would be

$$\Delta E = \text{total binding energy of } {}_2\text{He}^4 - 2 \times (\text{total binding energy of } {}_1\text{He}^4)$$

$$= 4 \times 7.0 - 2(1.1)(2) = 23.6 \text{ MeV}$$

41. (1) ${}_{92}\text{U}^{238} + {}_0\text{n}^1 \rightarrow {}_{92}\text{U}^{239} \rightarrow {}_{-1}\text{e}^0 + {}_{93}\text{Np}^{239} \rightarrow {}_{-1}\text{e}^0 + {}_{94}\text{Pu}^{239}$

42. (3) $\frac{A_0}{3} = A_0 \left(\frac{1}{2}\right)^{\frac{9}{T_{1/2}}}$ and $A' = \frac{A_0}{3} \left(\frac{1}{2}\right)^{\frac{9}{T_{1/2}}}$

$$\text{Dividing, we get } \frac{A' \times 3}{A_0} = \frac{1}{3} \quad \text{or } A' = \frac{A_0}{9}$$

43. (2) $T_{1/2} = \frac{0.693}{\lambda}$ or $T_{1/2} = 0.693 \left[\frac{1}{\lambda}\right]$

$$\text{or } T_{1/2} = 0.693 \tau$$

Clearly, $x = 0.693$.

44. (4) The emission of antineutrino is a must for the validity of different laws.

45. (1) ${}_Z^AX \rightarrow {}_2^4\text{He} + {}_{Z-2}^{A-4}Y$
 ${}_{Z-2}^{A-4}Y \rightarrow e^+ + {}_{Z-3}^{A-4}Y'$

During β^+ emission, ${}_1^1\text{p}^1 \rightarrow {}_0^1\text{n}^1 + \beta^+$

The proton changes into neutron. So, charge number decreases by 1 but mass number remains unchanged.

46. (2) $\frac{N_0}{4} = \frac{N_0}{2^n} \Rightarrow n = 2$

Thus, 10 days = 2 half-lives

$\therefore$ Half-life = 8 days

47. (4) α -decay decreases mass number by 4 and reduces charge number by 2. β -decay keeps mass number unchanged and increases charge by 1. Clearly, option (4) is the right choice.

48. (2) Decrease in mass number = $232 - 208 = 24$

$$\text{Number of } \alpha\text{-particles emitted} = \frac{24}{4} = 6$$

Due to emission of 6 particles, decrease in charge number is 12. But actual decrease in charge number is 8. Clearly, 4 β -particles are emitted.

49. (3) $A = A_0 e^{-\lambda t}$; $2100 = 16000 e^{-12\lambda} \Rightarrow e^{12\lambda} = 7.6$

$$\Rightarrow 12\lambda = \log_e 7.6 = 2 \Rightarrow \lambda = \frac{2}{12} = \frac{1}{6}$$

$$\therefore T = \frac{0.6931 \times 6}{1} = 4$$

50. (2) Total mass of the products = 2.0165 amu, which is greater than the mass of the deuteron by 0.0024 amu. The extra mass must be provided by the energy of the photon so that minimum possible frequency must be given by

$$\nu = \frac{0.0024 \text{ u } c^2}{h} \quad (1 \text{ amu} = 1.66 \times 10^{-27} \text{ kg})$$

$$= 5.4 \times 10^{20} \text{ Hz}$$

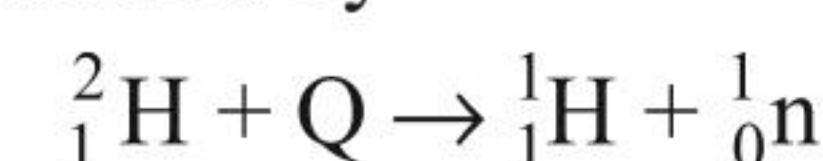
51. (2) The nuclear fission differs from other nuclear reactions in three respects.

(a) The nucleus is deeply divided into two large fission fragments or nuclei of roughly equal mass. The nuclei or fission fragments fly apart at great speed and thus possess large kinetic energies that carry off the greater part of the energy released.

(b) The mass decrease is appreciable and hence large energy is released.

(c) Other neutrons, called fission neutrons, are emitted in the process. Small amount of energy is released in the form of radiation.

52. (4) Disintegration of deuteron to a proton and a neutron can be represented by



The energy captured is the γ -ray photon E_γ is given by

$$E_\gamma + 1876 = 939 + 940$$

$$\Rightarrow E_\gamma = (939 + 940) - 1876 = 3 \text{ MeV.}$$

53. (3) Decay constant, $\lambda = 10^{-6} \text{ s}^{-1}$. The half-life $T_{1/2}$ is thus given by

$$T_{1/2} = \frac{0.639}{\lambda} = \frac{0.639}{10^{-6}} = 0.639 \times 10^6 \text{ s}$$

$$= 192.5 \text{ h} \approx 8 \text{ days} = 1.14 \text{ week} \approx 1 \text{ week}$$

54. (4) Given:

X has activity A_0 at $t = 0$ and its half-life is 24 years.

Y has activity A_0 at $t = 0$ and its half-life is 16 years.

At $t = 48$ years, activity of X = $\frac{1}{4} A_0$ (2 half-lives have elapsed)

At $t = 48$ years, activity of Y = $\frac{1}{8} A_0$ (3 half-lives have elapsed)

Thus, total activity of the mixture of X and Y at $t = 48$ years is

$$\frac{1}{4} A_0 + \frac{1}{8} A_0 = \frac{3}{8} A_0$$

55. (1) Let $t = 0, M_0 = 10 \text{ g}$

$$t = 2\tau = 2 \left(\frac{1}{\lambda} \right)$$

$$\text{Then, } M = M_0 e^{-\lambda t} = 10 e^{-\lambda \left(\frac{2}{\lambda} \right)} = 10 \left(\frac{1}{e} \right)^2 = 1.35 \text{ g}$$

$$56. (1) N = \frac{N_0}{2^{t/T}}$$

$$\frac{N_0}{16} = \frac{N_0}{2^{t/T}}$$

$$2^{t/T} = 16 = 2^4$$

$$\text{or } \frac{t}{T} = 4$$

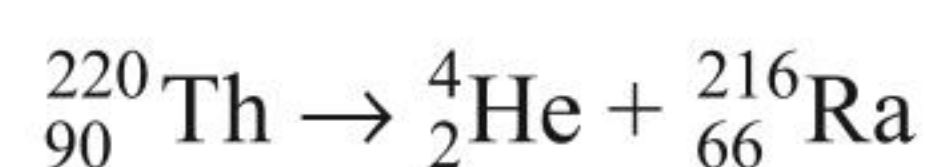
$$\text{or } T = \frac{t}{4} = \frac{24}{4} \text{ h} = 6 \text{ h}$$

$$57. (1) \frac{1}{16} = \frac{1}{2^{t/100}}$$

$$\text{or } \frac{1}{2^4} = \frac{1}{2^{t/100}} \quad \text{or } 4 = \frac{t}{100}$$

$$\text{or } t = 400 \mu\text{s}$$

58. (4) The α -particle emitting radioactive gas, thoron-220, decays to radium-216 and emits an α -particle. The reaction can be represented by



By conservation of momentum, we have momentum of α -particle = momentum of recoiling nucleus Ra

$$\Rightarrow m_\alpha v_\alpha = m_R v_R$$

$$\Rightarrow \frac{v_R}{v_\alpha} = \frac{m_\alpha}{m_R} = \frac{4}{216} = \frac{1}{54}$$

The kinetic energy of Ra, E_R , is related to the kinetic energy of alpha particle E_α by

$$\frac{E_R}{E_\alpha} = \frac{\frac{1}{2} m_R v_R^2}{\frac{1}{2} m_\alpha v_\alpha^2} = \left(\frac{m_R}{m_\alpha} \right) \left(\frac{v_R}{v_\alpha} \right)^2 = \left(\frac{m_R}{m_\alpha} \right) \left(\frac{m_\alpha}{m_R} \right)^2 = \frac{m_\alpha}{m_R} = \frac{1}{54}$$

$$\therefore E_R = \frac{E_\alpha}{54}$$

$$59. (4) \frac{9}{16} = \left(\frac{1}{2} \right)^{t/T}$$

$$\frac{N}{N_0} = \left(\frac{1}{2} \right)^{t/T}$$

$$\left(\frac{N}{N_0} \right)^2 = \left(\frac{1}{2} \right)^{t/T}$$

$$\text{or } \left(\frac{N}{N_0} \right)^2 = \frac{9}{16} \quad \text{or } \frac{N}{N_0} = \frac{3}{4}$$

Note the special technique used in the problem.

$$60. (1) \frac{A_2}{A_1} = \frac{N_2}{N_1}$$

$$\frac{A_2}{10^3} = \frac{1}{2} \quad \text{or } A_2 = \frac{1000}{2} = 500 \text{ s}^{-1}$$

$$61. (4) N = N_0 e^{-\lambda t}$$

$$\Rightarrow \frac{N_0}{N} = e^{\lambda t}$$

$$\text{or } \lambda t = \log_e \frac{N_0}{N} \quad \text{or } t = \frac{1}{\lambda} \log_e \frac{N_0}{N}$$

$$\therefore t \propto \log_e \frac{N_0}{N}$$

$$\Rightarrow 5 \propto \log_e \frac{100}{90} \quad \text{and } 20 \propto \log_e \frac{N_0}{N}$$

$$\text{Dividing, } \frac{5}{20} = \frac{\log_{10} \frac{100}{90}}{\log_{10} \frac{N_0}{N}}$$

$$\text{or } \log_{10} \frac{N_0}{N} = 4 \log_{10} \frac{10}{9}$$

$$\text{or } \frac{N_0}{N} = \left(\frac{10}{9} \right)^4 \Rightarrow \frac{N}{N_0} = 0.6561$$

Percentage of substance decayed is $(1 - 0.6561) \times 100 = 34.39\%$

62. (3) Let the kinetic energy of the α -particle be E_α and that of the thorium Th be E_{th} . The ratio of kinetic energies is

$$\frac{E_\alpha}{E_{th}} = \frac{\frac{1}{2} m_\alpha v_\alpha^2}{\frac{1}{2} m_{th} v_{th}^2} = \left(\frac{m_\alpha}{m_{th}} \right) \left(\frac{v_\alpha}{v_{th}} \right)^2 \quad \dots(i)$$

By conservation of momentum, the momentum of α -particle and that of the recoiling thorium must be equal. Thus,

$$m_\alpha v_\alpha = m_{th} v_{th}$$

$$\text{or } \frac{v_\alpha}{v_{th}} = \frac{m_{th}}{m_\alpha} \quad \dots(ii)$$

Substituting Eq. (ii) in Eq. (i), we have

$$\frac{E_\alpha}{E_{th}} = \left(\frac{m_\alpha}{m_{th}} \right) \left(\frac{m_{th}}{m_\alpha} \right)^2 = \frac{m_{th}}{m_\alpha} = \frac{234}{4} = 58.5$$

Thus, the kinetic energy of the α -particle expressed as the fraction of the total kinetic energy T is given by

$$E_\alpha = \frac{58.5}{1 + 58.5} T = \frac{58.5}{59.5} T = 0.98 T$$

which is slightly less than T .

$$63. (2) \frac{A}{A_0} = \frac{N}{N_0}$$

Let safe level activity be A . Initial activity = $64A$.

$$\text{Hence, } \frac{N}{N_0} = \frac{A}{A_0} = \frac{A}{64A} = \frac{1}{64}$$

$$\text{or } \left(\frac{1}{2}\right)^n = \frac{1}{64} \quad \text{or } n = 6$$

$$\text{Hence, } \frac{t}{T} = n = 6$$

$$\therefore T = 2 \text{ h}$$

$$\therefore t = 12 \text{ h}$$

$$64. (3) N_1 \lambda_1 = N_2 \lambda_2$$

$$T = \frac{0.693}{\lambda}$$

$$\text{Hence, } 2.8 \times 10^6 \times \frac{0.693}{T_1(\text{U})} = 1 \times \frac{0.693}{T_2(\text{Ra})}$$

$$\therefore T_1(\text{U}) = 1600 \times 2.8 \times 10^6 = 4.48 \times 10^9 \text{ years} \\ \approx 4.5 \times 10^9 \text{ years}$$

$$65. (2) \text{ Number of } \alpha\text{-particles per second} = \text{Activity} = (-dN/dt) = N\lambda$$

$$\text{where } N = \frac{6.0 \times 10^{23}}{210} \times 1 \times 10^{-3}$$

$$\lambda = 5.8 \times 10^{-8} \text{ s}^{-1}$$

$$\text{So, } A = N\lambda = \frac{6.0 \times 10^{23}}{210} \times 1 \times 10^{-3} \times 5.8 \times 10^{-8} = 1.7 \times 10^{11}$$

$$66. (1) \text{ Let } \frac{M_1}{M_2} \text{ (mass ratio)} = \frac{1}{2}$$

$$2 \text{ days} = 2 \times 24 \text{ h} = 48 \text{ h}$$

For first substance, 4 half-life periods and for second substance 3 half-life periods are passed; the masses are reduced to

$$M'_1 = M_1 \times \left(\frac{1}{2}\right)^4$$

$$M'_2 = M_2 \times \left(\frac{1}{2}\right)^3$$

$$\therefore \frac{M'_1}{M'_2} = \frac{M_1}{M_2} \times \frac{1}{2} = \frac{2}{1} \times \frac{1}{2} = \frac{1}{1}$$

$$67. (4) R_1 = N_1 \lambda, \quad R_2 = N_2 \lambda$$

$$\text{Also, } T = \frac{\log_e 2}{\lambda} \quad \text{or } \lambda = \frac{\log_e 2}{T}$$

$$\therefore R_1 - R_2 = (N_1 - N_2) \lambda = (N_1 - N_2) \frac{\log_e 2}{T}$$

$$\therefore (N_1 - N_2) = \frac{(R_1 - R_2)T}{\log_e 2} \text{ i.e., } (N_1 - N_2) \propto (R_1 - R_2)T$$

$$68. (3) \text{ We know that } N = N_0 \left(\frac{1}{2}\right)^n$$

$$\text{For A, } N = N_0 \left(\frac{1}{2}\right)^{n_A} = N_0 \left(\frac{1}{2}\right)^4 = \frac{N_0}{16}$$

$$\left[\because n_A = \frac{t}{T_A} = \frac{80}{20} = 4 \right]$$

$$\text{For B, } N_B = N_0 \left(\frac{1}{2}\right)^{n_B} = N_0 \left(\frac{1}{2}\right)^2 = \frac{N_0}{4}$$

$$\therefore \frac{N_A}{N_B} = \frac{1}{4} \quad \text{or } N_A : N_B = 1:4$$

$$69. (4) \text{ Let the decay constants for the first and second processes be } \lambda_1 \text{ and } \lambda_2 \text{ and the effective decay constant for the combined process be } \lambda. \text{ Then,}$$

$$\lambda_1 = \frac{\log_e 2}{t_1}, \quad \lambda_2 = \frac{\log_e 2}{t_2} \quad \text{and} \quad \lambda = \frac{\log_e 2}{t}$$

Now, the probability for decay through first process in a small time interval dt is $\lambda_1 dt$ and the probability for decay through second process in the same time interval dt is $\lambda_2 dt$.

The probability for decay by the combined process in the same time interval dt is $\lambda_1 dt + \lambda_2 dt$.

But this is also equal to λdt .

$$\therefore \lambda dt = \lambda_1 dt + \lambda_2 dt$$

$$\therefore \lambda = \lambda_1 + \lambda_2$$

$$\text{or } \frac{\log_e 2}{t} = \frac{\log_e 2}{t_1} + \frac{\log_e 2}{t_2}$$

$$\text{or } \frac{1}{t} = \frac{1}{t_1} + \frac{1}{t_2} \quad \text{or } t = \frac{t_1 t_2}{t_1 + t_2}$$

$$70. (1) \text{ According to Avogadro's hypothesis,}$$

$$N_0 = \frac{6.02 \times 10^{23}}{226} = 2.66 \times 10^{21}$$

$$\text{Half-life, } T = \frac{0.693}{\lambda} = 1620 \text{ years}$$

$$\therefore \lambda = \frac{0.6931}{1620 \times 3.16 \times 10^7} = 1.35 \times 10^{-11} \text{ s}^{-1}$$

Because half-life is very much large as compared to its time interval, hence $N \approx N_0$. Now,

$$\frac{dN}{dt} = \lambda N \approx \lambda N_0$$

$$\text{or } dN = \lambda N_0 dt = (1.35 \times 10^{-11}) (2.66 \times 10^{21}) \times 1 = 3.61 \times 10^{10}$$

$$71. (3) \text{ Radius } R \text{ of a nucleus changes with the nucleon number } A \text{ of the nucleus as } R = 1.3 \times 10^{-15} \times A^{1/3} \text{ m}$$

$$\text{Hence, } \frac{R_2}{R_1} = \left(\frac{A_2}{A_1}\right)^{1/3} = \left(\frac{128}{16}\right)^{1/3} = (8)^{1/3} = 2$$

$$\therefore R_2 = 2R_1 = 2(3 \times 10^{-15}) \text{ m} = 6 \times 10^{-15} \text{ m}$$

$$72. (2) \text{ For nucleus of } {}_8\text{O}^{16}: \text{Mass} = (16)(1.67 \times 10^{-27}) \text{ kg}$$

$$\text{Volume} = \frac{4}{3} \pi R^3 = \frac{4}{3} \pi (3 \times 10^{-15})^3 \text{ m}^3 = 36\pi \times 10^{-45} \text{ m}^3$$

$$\text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{16 \times 67 \times 10^{-27} \text{ kg}}{36\pi \times 10^{-46} \text{ m}^3} = 2.35 \times 10^{17} \text{ kg m}^{-3}$$

$$73. (1) \text{ Mass defect, } \Delta m = 20(1.007277 + 1.00866) - 39.97545 \\ = 40.31874 - 39.97545 = 0.34329 \text{ amu}$$

$$\therefore \text{Binding energy} = 0.34329 \times 931 = 319.6 \text{ MeV}$$

When one atom of Ca-40 completely dissociates, the energy to be supplied = 319.6 MeV.

$$1 \text{ g of Ca-40 contains } \frac{6.023 \times 10^{23}}{40} = 1.506 \times 10^{22} \text{ atoms}$$

$$\text{The energy required for the dissociation of 1 g of Ca-40} \\ = 319.6 \times 1.506 \times 10^{22} = 4.813 \times 10^{24} \text{ MeV}$$

$$74. (3) \text{ Binding energy per nucleon of fission products is } 8.5 \text{ MeV}$$

$$\text{Binding energy per nucleon of reactants} = 7.6 \text{ MeV}$$

$$\text{Increase in binding energy per nucleon is } 8.5 - 7.6 = 0.9 \text{ MeV}$$

$$\text{Energy released per nucleon in fission is } 0.9 \text{ MeV}$$

$$\text{Therefore, fractional energy released} = \frac{0.9}{931} = \frac{1}{1000}$$

Percentage of mass converted into energy during fission

$$= \frac{1}{1000} \times 100 = 0.1\%$$

75. (2) $Q = (\Sigma B_r - \Sigma B_p) c^2$

where ΣB_r = sum of the masses of reactants

and ΣB_p = sum of the masses of the products

$$\Sigma B_r = 2 \times 2.014741 \text{ amu} = 4.029482 \text{ amu}$$

$$\Sigma B_p = (3.016977 + 1.008987) \text{ amu} = 4.025964 \text{ amu}$$

$$\Sigma B_r - \Sigma B_p = (4.029482 - 4.025964) \text{ amu}$$

$$= 0.003518 \text{ amu}$$

Decrease in mass appears as equivalent energy.

$$\therefore Q = 0.003518 \times 931 \text{ MeV} = 3.27 \text{ MeV}$$

76. (2) $P = 10^6 \text{ W}$

$$\text{Time} = 1 \text{ day} = 24 \times 36 \times 10^2 \text{ s}$$

$$\text{Energy produced, } U = Pt = 10^6 \times 24 \times 36 \times 10^2 = 24 \times 36 \times 10^8 \text{ J}$$

$$\text{Energy released per fusion reaction is } 20 \text{ MeV} = 32 \times 10^{-13} \text{ J}$$

$$\text{Energy released per atom of } {}_1\text{H}^2 \text{ is } 32 \times 10^{-13} \text{ J}$$

$$\text{Number of } {}_1\text{H}^2 \text{ atoms used is } \frac{24 \times 36 \times 10^8}{32 \times 10^{-12}} = 22 \times 10^{21}$$

$$\text{Mass of } 6 \times 10^{23} \text{ atoms} = 2 \text{ g}$$

$$\text{Mass of } 27 \times 10^{21} \text{ atoms} = \frac{2}{6 \times 10^{23}} \times 27 \times 10^{21} = 0.1 \text{ g}$$

77. (2) Power P of fission reactor, $P = 10^6 \text{ W} = 10^6 \text{ J s}^{-1}$

$$\text{Time} = t = 1 \text{ day} = 24 \times 36 \times 10^2 \text{ s}$$

$$\text{Energy produced, } U = Pt$$

$$\text{or } U = 10^6 \times 24 \times 36 \times 10^2 = 24 \times 36 \times 10^8 \text{ J}$$

$$\text{Energy released per fission of } \text{U}^{235} \text{ is } 200 \text{ MeV} = 32 \times 10^{-12} \text{ J}$$

$$\text{Number of } \text{U}^{235} \text{ atoms used is } \frac{24 \times 36 \times 10^8}{32 \times 10^{-12}} = 27 \times 10^{20}$$

$$\text{Mass of } 6 \times 10^{23} \text{ atoms of } \text{U}^{235} = 235 \text{ g}$$

$$\text{Mass of } 27 \times 10^{20} \text{ atoms of } \text{U}^{235} \text{ is}$$

$$\left(\frac{235}{6 \times 10^{23}} \right) (27 \times 10^{20}) = 1.058 \text{ g} = 1 \text{ g}$$

78. (2) Mass of one atom of U^{235} is 235.121420 amu

$$\text{Mass of one neutron} = 1.008665 \text{ amu}$$

$$\text{Sum of the masses of } \text{U}^{235} \text{ and neutron}$$

$$= 236.130085 = 236.130 \text{ amu}$$

$$\text{Mass of one atom of } \text{U}^{236} \text{ is } 236.123050 \text{ amu} = 236.123 \text{ amu}$$

$$\text{Mass defect} = 236.136 - 236.123 = 0.007 \text{ amu}$$

Therefore, energy required to remove one neutron is

$$0.007 \times 931 \text{ MeV} = 6.517 \text{ MeV} = 6.5 \text{ MeV}$$

79. (3) Total binding energy of helium atom (${}_2\text{He}^4$) is $4 \times 7 = 28 \text{ MeV}$

$$\text{Total binding energy of deuteron } {}_1\text{H}^2 (1p + 1n) \text{ is}$$

$$2 \times 1.1 = 2.2 \text{ MeV}$$

$$\text{Hence, binding energy of 2 deuterons is } 2 \times 2.2 = 4.4 \text{ MeV}$$

$$\text{So, the energy released in forming helium nucleus from two deuterons is } 28 - 4.4 \text{ MeV} = 23.6 \text{ MeV}$$

80. (3) Mass defect, $\Delta m = 2(2.015) - (3.017 + 1.009) = 0.004 \text{ amu}$

$$\text{As } 1 \text{ amu} = 931.5 \text{ MeV}/c^2, \text{ energy released will be}$$

$$0.004 \times 931.5 \text{ MeV} = 3.726 \text{ MeV}$$

$$\text{Energy released per deuteron is } \frac{3.726}{2} = 1.863 \text{ MeV}$$

Number of molecules in 1 kg deuterons is

$$\frac{6.02 \times 10^{26}}{2} = 3.01 \times 10^{26}$$

Therefore, energy released per kg of deuterium fusion

$$= (3.01 \times 10^{26} \times 1.863) = 5.6 \times 10^{26} \text{ MeV} \approx 9.0 \times 10^{13} \text{ J}$$

81. (2) Here, half-life of radium, $t = 1500 \text{ years}$

$$\text{Disintegration constant } \lambda = \frac{0.693}{T} = \frac{0.693}{1500} \text{ year}^{-1}$$

$$N_0 = 1 \text{ g}, N = 1 \text{ centigram} = 10^{-2} \text{ g}$$

$$\text{Now apply } N = N_0 e^{-\lambda t}$$

$$\Rightarrow 10^{-2} = e^{-\lambda t} \text{ or } t = 9.972 \times 10^2 \text{ yrs}$$

82. (2) Once the neutron gets sufficiently close to the nucleus, the strong nuclear force sucks it in. Same happens with the proton except it is electrostatically repelled by the six protons already inside the carbon nucleus. The repulsion prevents a 100 ms^{-1} proton from getting close enough to the nucleus. Therefore, the answer is (2).

83. (3) For this substance 7 days correspond to 3.5 half-lives. Over 3

$$\text{half-lives the sample reduces to } \frac{1}{2^3} = \frac{1}{8} \text{ of its initial mass. After}$$

$$4 \text{ half-lives, the sample has only } \frac{1}{2^4} = \frac{1}{16} \text{ of its initial mass.}$$

Hence, after 3.5 half-lives the sample must contain somewhere between $1/8$ and $1/16$ of its initial mass.

Hence, 5 g is somewhere between $1/8$ and $1/16$ of the initial mass.

So, the initial mass is somewhere between $8 \times 5 = 40 \text{ g}$ and $16 \times 5 = 80 \text{ g}$.

84. (4) As we regard the decay process as a spontaneous and statistical process, therefore the decay can start any time after $t = 0$. Therefore, the answer is (4).

85. (2) A nucleus contains protons and neutrons with no antiprotons and antineutrons. Hence, answer can be either (2) or (4). Due to conservation of spin, the answer is (2).

86. (4) Atomic mass $M(\text{H})$ of hydrogen and nuclear mass (M_n) are

$$M(\text{H}) = 1.007825 \text{ u and } M_n = 1.008665 \text{ u}$$

$$\text{Mass defect, } \Delta m = [M(\text{H}) + M_n - M(\text{D})]$$

$$M(\text{D}) = \text{mass of deuteron} = 2.016490 \text{ u} - 2.014102 \text{ u} = 0.002388 \text{ u}$$

As 1 u corresponds to 931.494 MeV energy, therefore, mass defect corresponds to energy,

$$E_b = 0.002388 \times 931.5 = 2.224 \text{ MeV}$$

87. (4) Nuclear reactions conserve total charge, and also conserve the total approximate mass. The other particles in the reaction will have mass $= 236 - 140 - 94 = 2$

The other particles are two neutrons. Hence, (1) is not correct.

For nuclei, number of protons tells the charge. So, the other particles must have charge Z such that

$$92 = 54 + 38 + Z$$

$$\therefore Z = 0$$

Therefore, the other particles have a total atomic mass 2 and total charge 0. Hence, only (4) is correct.

88. (3) All neutrons attract each other with the same strong nuclear force. So, the strong nuclear force holds together three protons and one neutron (${}_3\text{Li}$) just as vigorously as it holds together two protons and two neutrons (${}_2\text{He}$). Specifically, protons electrostatically repel other protons. This repulsion tries to make a nucleus fly

apart. Since ${}^4_2\text{He}$ contains only two protons, the attractive strong nuclear forces overcome the repulsion of the protons. Hence, the nucleus holds together. But in ${}^4_3\text{Li}$, the mutual repulsion of the three protons overcomes the strong nuclear attractions and the nucleus falls apart (or undergoes radioactive decay into a more stable nucleus). Therefore, the answer will be (3).

$$89. (2) \text{ Number of atoms in 2 kg fuel} = \frac{2}{235} \times 6.02 \times 10^{26} = 5.12 \times 10^{24}$$

Fission rate = Number of atoms fissioned in one second

$$= \frac{5.12 \times 10^{24}}{30 \times 24 \times 60 \times 60} = 1.975 \times 10^{18} \text{ s}^{-1}$$

Each fission gives 185 MeV. Hence, energy obtained in one second,

$$P = 185 \times 1.975 \times 10^{18} \text{ MeV s}^{-1} \\ = 185 \times 1.975 \times 10^{18} \times 1.6 \times 10^{-19} \text{ MJ s}^{-1} = 58.46 \text{ MW}$$

$$90. (3) {}_1\text{H}^2 + {}_1\text{H}^2 \rightarrow {}_2\text{He}^4 + Q$$

$$\Delta m = m({}_2\text{He}^4) - 2m({}_1\text{H}^2)$$

$$\Delta m = 4.0024 - 2(2.0141)$$

$$\Delta m = -0.0258 \text{ u}$$

$$\text{Now, } Q = c^2 \Delta m = (0.0258)(931.5) \text{ MeV} \approx 24 \text{ MeV}$$

$$91. (3) \frac{dN}{dt} = n - \lambda N$$

Because the population N is simultaneously increasing at rate n and decreasing due to decay at rate λN .

$$\int_{N_0}^N \frac{dN}{n - \lambda N} = \int_0^t dt$$

$$\frac{1}{\lambda} \ln \left(\frac{n - \lambda N_0}{n - \lambda N} \right) = t$$

$$N = \frac{n}{\lambda} + \left(N_0 - \frac{n}{\lambda} \right) e^{-\lambda t}$$

$$92. (3) t = 0, N = N_0$$

$$t = 6.93, N = N_0/4$$

$N_0/4$ is the sample left after two half-lives

$$\therefore 2t_{1/2} = 6.93$$

$$\Rightarrow 2 \times \frac{0.693}{\lambda} = 6.93 \Rightarrow \lambda = 0.2 \text{ min}^{-1}$$

$$\Rightarrow t = 60 \text{ min}$$

$$\therefore N = N_0 e^{-\lambda t} = N_0 e^{-0.2 \times 60} = \frac{N_0}{e^{12}}$$

$$93. (4) \left| \frac{dN}{dt} \right| = \lambda N$$

$$\text{Number of radium nuclei in } m \text{ g} = \frac{N_A m}{226}$$

$$\text{Decay constant, } \lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{1620 \times 3.16 \times 10^7}$$

$$\left| \frac{dN}{dt} \right| = 10 = \frac{6.02 \times 10^{23} m}{226} \times \frac{0.693}{1620 \times 3.16 \times 10^7}$$

$$\therefore m = \frac{10 \times 226 \times 1620 \times 3.16 \times 10^7}{6.02 \times 10^{23} \times 0.693} \\ = 2.77 \times 10^{-10} \text{ g} = 2.77 \times 10^{-13} \text{ kg}$$

94. (3) The Q value of the first reaction implies that

$${}^{14}\text{N} + d = {}^{15}\text{N} + p + 8.53 \text{ MeV}$$

where ${}^{14}\text{N}$ represents the mass of ${}^{14}\text{N}$ nucleus in energy units. This can be rearranged to give

$${}^{14}\text{N} - {}^{15}\text{N} = p - d + 8.53 \text{ MeV}$$

The second reaction similarly implies as

$${}^{15}\text{N} - {}^{13}\text{C} = \alpha - d + 7.58 \text{ MeV}$$

and the third reaction gives

$${}^{13}\text{C} - {}^{11}\text{B} = \alpha - d + 5.16 \text{ MeV}$$

Adding these three equations, we have

$${}^{14}\text{N} - {}^{11}\text{B} = p + 2\alpha - 3d + 21.27 \text{ MeV}$$

$${}^{11}\text{B} (\alpha, n) {}^{14}\text{N} = {}^{11}\text{B} - {}^{14}\text{N} + \alpha - n$$

$$= 3d - \alpha - p - n - 21.27 \text{ MeV}$$

$$\text{Now, } 3d - \alpha - p - n = (3 \times 2.014 - 4.0020 - 1.0078 - 1.0087) \text{ amu} \\ = 0.0229 \text{ amu}$$

$$\therefore Q = (0.0229 \times 931 - 21.27) \text{ MeV} = 0.05 \text{ MeV}$$

95. (4) If d is the distance of closest approach given, then the angular momentum = $mvd = 10^{-33} \text{ J s}$

$$E = \frac{1}{2}mv^2 = 1 \text{ MeV} = 1.6 \times 10^{-13} \text{ J}$$

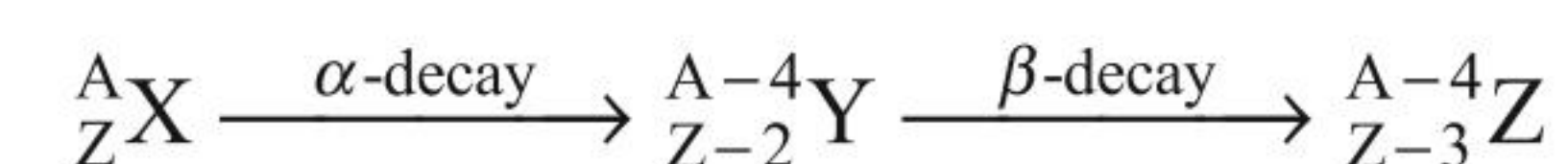
$$\text{Momentum, } p = \sqrt{2m_n E} = \sqrt{2 \times 1.6 \times 10^{-27} \times 1.6 \times 10^{-13}} \\ = 1.6\sqrt{2} \times 10^{-20} \text{ kg m s}^{-1}$$

$$\text{Distance of closest approach, } d = \frac{10^{-33}}{1.6\sqrt{2} \times 10^{-20}}$$

$$= \frac{1}{1.6\sqrt{2}} \times 10^{-13} = \frac{100}{1.6\sqrt{2}} \text{ fm} = 0.44 \text{ fm}$$

96. (3) The binding energy per nucleon is lowest for very light nuclei such as ${}^4_2\text{He}$, is greatest around $A = 60$, and then decreases with increasing A .

97. (4) Two protons and two neutrons are lost in an α -decays, so Z and N each decrease by 2. A β^+ decay changes a proton to a neutron, so Z decreases by 1 and N increases by 1. The net result is Z decreases by 3 and N decreases by 1.



Initially, number of neutrons $N_i = (A - Z)$

$$\text{Now, number of neutrons } N_f = A - 4 - Z + 3 = N_i - 1$$

$$98. (3) m({}^{198}\text{Au}_{79}) = 197.968225 \text{ u}, m({}^{198}\text{Hg}_{80}) = 197.966752 \text{ u}$$

$$\text{Mass defect, } \Delta m = 1.473 \times 10^{-3} \text{ u} = 1.371 \text{ MeV}$$

$$\text{Energy of } \gamma\text{-photon} = 0.412 \text{ MeV}$$

Maximum kinetic energy of the electron emitted in the decay is

$$E_e = 1.371 \text{ MeV} - 0.412 \text{ MeV} = 0.959 \text{ MeV}$$

Multiple Correct Answers Type

1. (1), (2), (3), (4)

$$\text{We have, } 6.25\% = \frac{6.25}{100} = \frac{1}{16}$$

The given time of 4 h thus equals 4 half-lives so that the half-life is 1 h.

$$\text{Since half-life} = \frac{\ln 2}{\text{decay constant}} \text{ and mean life} = \frac{1}{\text{decay constant}},$$

after further 4 h, the amount left over would be $\frac{1}{2^4} \times \frac{1}{2^4}$, i.e., $\frac{1}{256}$ or $\frac{100}{256}$ or 0.39% of original amount.

2. (1), (2), (4)

It has been observed that total mass of nucleus is always less than the sum of the masses of its nucleons. The energy difference between the nucleus and its constituent particles due to their mass difference is termed as the binding energy of the nucleus.

In other words, we can say that to break the nucleus into its constituent particles, some energy is needed to be supplied. This energy is termed as binding energy of the nucleus.

For (1), more is the binding energy per nucleon, more is the energy required to break the nucleus and hence we can say the more stable the nucleus is.

For (2), (3) and (4), in actual the binding energy is always positive but if it were zero, then nucleus will break spontaneously.

3. (1), (3), (4)

All the statements are very conceptual statements related to different decays.

4. (1), (4)

For an exothermic reaction $X + x \rightarrow Y + y$

If K_i is the kinetic energy of incident particle x , then from energy conservation,

$$K_i + (m_x + M_X)c^2 = K_Y + K_y + (M_Y + m_y)c^2$$

$$K_Y + K_y = K_i + (m_x + M_X - M_Y - m_y)c^2$$

$$K_Y + K_y = K_i + Q$$

In any exothermic reaction, mass of the products is less than the mass of reactants, i.e., in products, the nucleons are more tightly bound and hence have greater BE per nucleon as compared to BE per nucleon of reactants. For endothermic reaction to be carried out, minimum energy given to the reactant must be greater than $|Q|$ value.

5. (1), (2), (3)

In general, fission and fusion processes are exothermic reactions, i.e., energy is released. Hence, mass of products must be less than mass of the reactant nuclide, and BE/A of reactants $< BE/A$ of product nuclides.

6. (1), (2), (3)

If the nuclear reaction involving β -decay is $n \rightarrow p + e^-$, the spins on two sides are not equal as all the three (neutron, proton and electron) have spins of $+1/2$. So, to conserve angular momentum (spin), some other particle must be emitted.

Through experiments it has been observed that direction of emitted electron and recoiling nuclei are almost never exactly opposite as required for linear momentum to be conserved.

During β -decay, the energy of electron is found to vary continuously from 0 to a maximum value (this maximum value is a characteristic of nuclide). To explain this experimental observation, we also need some other particle.

7. (1), (3)

Half-lives of both the samples would be same as half-life is the property of radioactive material and is independent of number of nuclei present or its activity. Let $R_{0B} = R_0$, then $R_{0A} = 2R_0$, where R_0 denotes initial activity.

$$\text{Activity of A after 5 half-lives is } R_A = \frac{R_{0A}}{2^5} = \frac{2R_0}{2^5}$$

$$\text{Activity of B after 5 half-lives is } R_B = \frac{R_{0B}}{2^5} = \frac{R_0}{2^5}$$

$$\therefore \frac{R_A}{R_B} = \frac{2}{1}$$

8. (1), (3)

$$\lambda = (0.173 \text{ year})^{-1}$$

$$N = N_0 e^{-\lambda t}$$

$$\text{As } t = \frac{1}{\lambda}, \text{ hence}$$

$$N = \frac{N_0}{e} = \frac{N_0}{2.178} = 0.37 N_0$$

$$\Rightarrow T = \frac{0.693}{\lambda} = \frac{0.693}{0.173} = 4 \text{ years}$$

9. (1), (4)

In α -decay, the entire energy is carried away by the α -particles as its kinetic energy. In β^- -decay, the energy is shared between the β -particle and the anti-neutrino. Hence, the speed of the β -particle will vary, depending on the energy of the anti-neutrino.

10. (2), (3), (4)

Statement (1) is incorrect. In fact, $A = Z + N$

Statements (2), (3) and (4) are correct; they are the definitions of isobars, isotopes and isotones.

11. (1), (3)

The idea of magic number has led to the shell model and the nuclides with these number of protons or neutrons have been compared with the inert gases vis-à-vis stability in terms of closed shells.

12. (1), (2), (3)

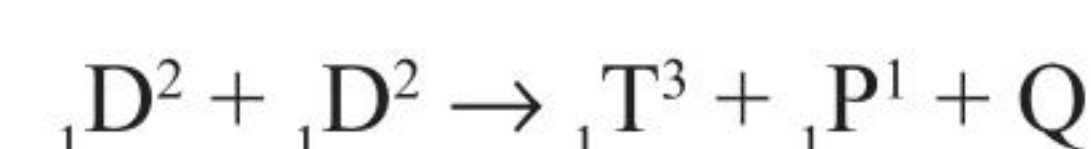
The last statement is incorrect because the amount of energy released per unit mass of the fuel is much more for fusion than for fission. Hence, (1), (2) and (3) are correct.

13. (2), (3)

Statement (1) is incorrect. The ${}_2\text{He}^4$ nucleus (or the α -particle) is exceptionally stable and has a much higher value of BE per nucleon than that for most other light nuclei. Statement (2) is correct but the reason of decrease in binding energy is different for the cases of smaller and larger values of A . The reason for the decrease in the BE per nucleon for nuclei with large A is that with an increase in the number of protons, the Coulomb repulsion increases. On the other hand, the decrease in the BE per nucleon for nuclei with small A is due to a surface effect: the nucleons at the surface being less strongly bound than those in the interior.

Statement (3) is also correct. The energy required to remove one neutron (i.e., one nucleon) is the same as the binding energy per nucleon for a given isotope.

Statement (4) is incorrect. To ensure both charge and mass number conservation, a proton must be produced as a by-product of the reaction:



14. (1), (2), (3)

$$\text{Use } \frac{N}{N_0} = e^{-\lambda t}$$

15. (1), (4)

(1) True, Cd absorbs neutrons.

(2) No, concrete reflects, does not slow down.

(3) 'Moderate the activity' is not correct. 'Moderator' in the sense of slowing the neutrons is different.

(4) True, it is a fact.

16. (1), (2), (3)

We know, $N = N_0 e^{-\lambda t}$

...(i)

where N = number of decayed nuclei in the sample at time t ,
 N_0 = initial number of nuclei.

Hence, total number of undecayed nuclei = $(N_0 - N)$

Substituting it in (i), we get $N_0 - N = N_0 (1 - e^{-\lambda t})$

This shows that the total number of undecayed nuclei decays exponentially with time and total number of decayed nuclei grows exponentially with time. Now,

$$R = -\lambda N = \frac{dN}{dt} \quad (R = \text{activity})$$

Hence, activity (R) $\propto$ number of undecayed nuclei.

Therefore, (1), (2), (3) are correct answers.

17. (1), (3)

$$R = R_0 A^{1/3}$$

For O^{16} , $R = R_0 (16)^{1/3}$

For ${}_{54}X^{128}$, $R' = R_0 (128)^{1/3}$

$$R' = \left(\frac{128}{16}\right)^{1/3} R = 2R$$

$$\therefore V' = \frac{4}{3} \pi R'^3 = 8V$$

18. (1), (4)

In nuclear fusion, two or more lighter nuclei are combined to form a relatively heavy nucleus and thus, releasing the energy.

19. (3), (4)

Due to mass defect (which is finally responsible for the binding energy of the nucleus), mass of a nucleus is always less than the sum of masses of its constituent particles.

${}_{10}^{20}\text{Ne}$ is made up of 10 protons plus 10 neutrons.

Therefore, mass of ${}_{10}^{20}\text{Ne}$ nucleus, $M_1 < 10(m_p + m_n)$

Also, heavier the nucleus, more is the mass defect.

$$20(m_n + m_p) - M_2 > 10(m_p + m_n) - M_1$$

Thus, $10(m_n + m_p) > M_2 - M_1$

or $M_2 < M_1 + 10(m_p + m_n)$

Now, since $M_1 < 10(m_p + m_n)$

$$\therefore M_2 < 2M_1$$

20. (2), (4)

In fusion, two or more lighter nuclei combine to make a comparatively heavier nucleus. In fission, a heavy nucleus breaks into two or more comparatively lighter nuclei. Further, energy will be released in a nuclear process if total binding energy increases.

Hence, correct options are (2) and (4).

21. (1), (3)

When their number of nuclei are equal $4N_0 e^{-3\lambda t} = 2N_0 e^{-2\lambda t}$

$$\Rightarrow t = \frac{1}{\lambda} \ln 2$$

When their decay rate are equal $3\lambda \times 4N_0 e^{-3\lambda t} = 2\lambda \times 2N_0 e^{-2\lambda t}$

$$\Rightarrow t = \frac{\ln 3}{\lambda}$$

At $t = \frac{1}{\lambda} \ln 4$, decay rate of $A (3\lambda) 4N_0 e^{-3\lambda \frac{\ln 4}{\lambda}} = \frac{3\lambda N_0}{16}$

At $t = \frac{\ln 4}{\lambda}$, decay rate of $B = 2\lambda \times 4N_0 e^{-2\lambda \frac{\ln 4}{\lambda}} = \frac{\lambda N_0}{4}$

22. (1), (3)

$$\frac{K_\alpha}{K_\gamma} = \frac{m_\gamma}{m_\alpha} = \frac{M - m}{m}$$

$$r = \frac{\sqrt{2mK_\alpha}}{qB} \Rightarrow K_\alpha = \frac{q^2 B^2 r^2}{2m}$$

$$K_\gamma = \frac{m}{(M - m)} K_\alpha = \frac{q^2 B^2 r^2}{2(M - m)}$$

$$\text{Energy released} = K_\gamma + K_\alpha = \frac{q^2 B^2 r^2}{2m} \left(\frac{M}{M - m} \right)$$

23. (2), (3)

Here, $N_A = N_0 e^{-\lambda t}$, $N_B = N_0 (1 - e^{-\lambda t})$

$$\text{Now } \frac{N_B}{N_A} = e^{\lambda t} - 1$$

If $t \ll T$; then $\frac{N_B}{N_A} = 1 + \lambda t - 1$ ($e^x = 1 + x$ for small x)

$$\frac{N_B}{N_A} = \lambda t$$

Linked Comprehension Type

For Problems 1–3

1. (1), 2. (2), 3. (3)

$$\frac{dN_X}{dt} = K - \lambda N_X$$

$$N_X = \frac{1}{\lambda} [K - K - \lambda N_0] e^{-\lambda t}$$

$$\frac{dN_Y}{dt} = \lambda N_X$$

$$N_Y = Kt + \left(\frac{K - \lambda N_0}{\lambda} \right) e^{-\lambda t} - \frac{K - \lambda N_0}{\lambda}$$

For Problems 4–6

$$4. (1) \quad \frac{dN}{dt} = q_0 t - \lambda N; \quad \frac{dN}{dt} + \lambda N = q_0 t$$

$$5. (1) \quad N = \frac{q_0 t}{\lambda} - \frac{q_0}{\lambda^2} + \frac{q_0}{\lambda^2} e^{-\lambda t}$$

$$P_{\text{inst}} = \lambda N E_0 = \left[q_0 t - \frac{q_0}{\lambda} + \frac{q_0}{\lambda} e^{-\lambda t} \right] E_0$$

$$6. (1) \quad P_{\text{av}} = \frac{\int_0^t \left[q_0 t - \frac{q_0}{\lambda} + \frac{q_0}{\lambda} e^{-\lambda t} \right] E_0 dt}{\int_0^t dt}$$

$$= \left[\frac{q_0 t}{2} - \frac{q_0}{\lambda} + \frac{q_0}{\lambda^2 t} - \frac{q_0}{\lambda^2 t} e^{-\lambda t} \right] E_0$$

For Problems 7–9

7. (3) From the graph and the fact that the n/p (= no. of neutrons/no. of protons) ratio for magnesium is 27/12, which is greater than 1 (= unit slope).

$$8. (2) \quad {}_{90}^{230}\text{Th} \rightarrow {}_{83}^{214}\text{Bi} + x {}_{-1}^0\text{e}^- + y {}_2^4\text{He}^{2+} + z \text{ (gamma ray)}$$

Since the sum of the atomic numbers and mass numbers on either side of the equation must be equal (matter cannot be created or destroyed), we get

$$90 = 83 - x + 2y \quad \text{and} \quad 230 = 214 + 4y$$

Solving, we get $y = 4$ and $x = 1$

The order of the reactions is irrelevant (i.e., alpha, beta,...). Since gamma rays have no atomic number or mass number, the value of z does not affect this particular calculation.

9. (4) Alpha radiation consists of the largest particles (helium nuclei with a mass number of 4, thus the greatest inertia) and are the slowest (about 1/3 times the speed of the light). Beta radiation consists of smaller (electrons, 1/1370 times lighter than a proton) and faster particles (about 4/5 times the speed of light). Gamma radiation consists of the smallest particles (photons, no mass) which travel at the greatest speed (at the speed of light)

For Problems 10–12

10. (2) In equilibrium, rate of decay = rate of production
 11. (4) As rate of decay = Rate of production

$$P = \lambda N \Rightarrow N = \frac{P}{\lambda} = \frac{Pt_{1/2}}{\ln 2} = 1.8 \times 10^{15}$$

12. (3) As $N = \frac{Pt_{1/2}}{\ln 2}$, it is dependent upon P and $t_{1/2}$. Initial number of ^{56}Mn nuclei will make no difference as in equilibrium, rate of production equals rate of decay. Large initial number will only make equilibrium come sooner.

For Problems 13–15

13. (4) Radioactivity is independent of all external conditions. When a nucleus undergoes an α -decay, its atomic number decreases by 2 and in beta decay, atomic number increases by 1.

14. (1) $T_M = \frac{1}{\lambda}$ and $T_H = \frac{0.693}{\lambda}$

$$\text{Now, } \frac{T_M}{T_H} = \frac{1}{\lambda} \times \frac{\lambda}{0.693} = \frac{1}{0.693}$$

$$\text{i.e., } T_M > T_H$$

From the above explanation, it is clear that choice (1) is correct and other choices (2), (3) and (4) are wrong.

15. (3) $A = \frac{dN}{dt} = \lambda N$; $n = \lambda N$

$$n = \frac{0.693}{T} N$$

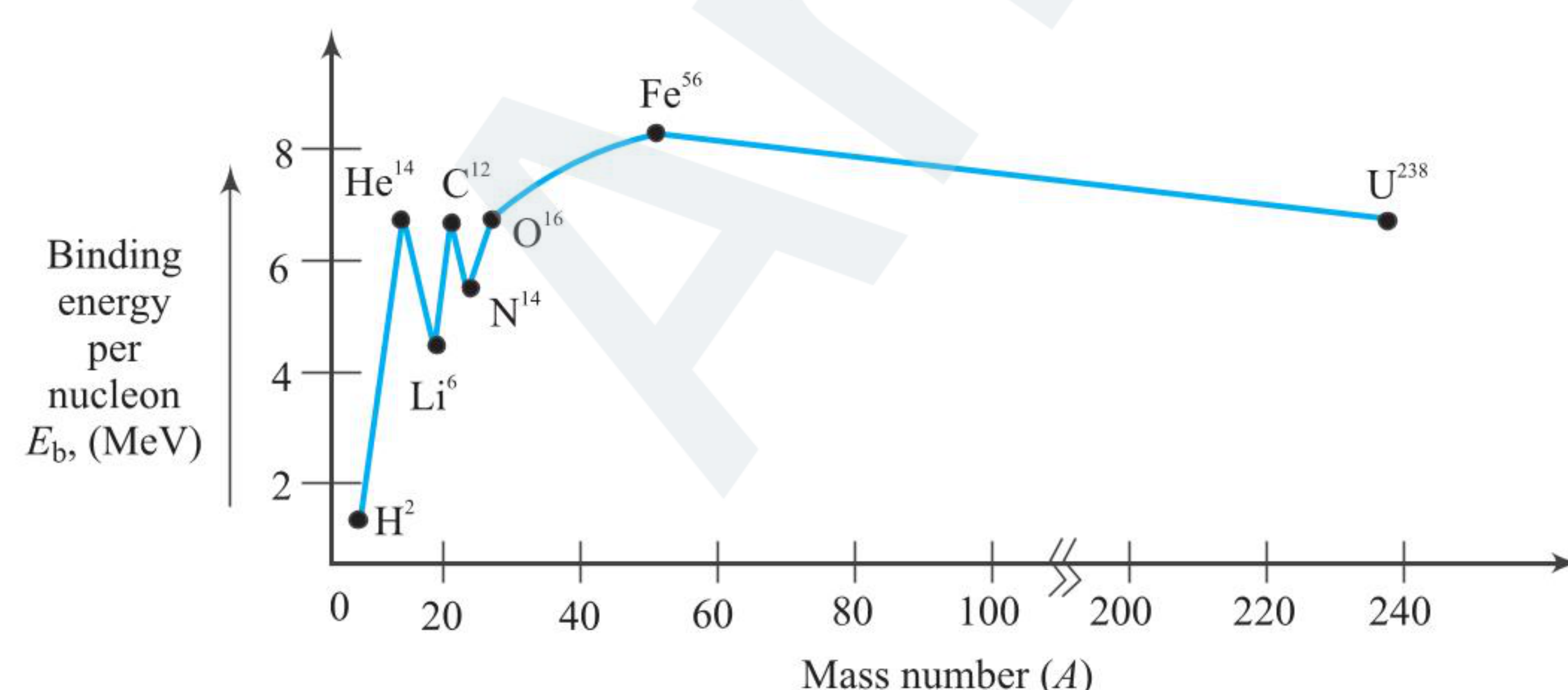
For Problems 16–18

16. (1), (2), (3), (4)

All options are basic properties of nuclear forces.

So, all options are correct.

17. (3) From graph, it is obvious that binding energy per nucleon (i.e., $\bar{B} = \frac{B}{A}$) is maximum for middle order element and its value is around 8.8 MeV per nucleon.



18. (4) Radius (R) of nucleus is related with mass number (A) as
- $$R \propto A^{1/3}$$

$$\text{Now, Volume} \propto R^3 \propto A$$

For Problems 19–22

19. (4) According to the passage, subatomic reactions do not conserve mass. So, we cannot find the third particle's mass by setting m_{neutron} equal to $m_{\text{proton}} + m_{\text{electron}} + m_{\text{third particle}}$.

By contrast, the total energy in this case, the sum of 'mass energy' and kinetic energy, is conserved. If E denotes total energy, then

$$E_{\text{neutron}} = E_{\text{proton}} + E_{\text{electron}} + E_{\text{third particle}}$$

The neutron has energy 949.97 MeV. The proton has energy 939.67 MeV + 0.01 MeV = 939.69 MeV. The electron has energy 0.51 MeV = 0.39 MeV = 0.90 MeV. Therefore, the third particle has energy

$$E_{\text{third particle}} = E_{\text{neutron}} - E_{\text{proton}} - E_{\text{electron}}$$

We just found the third particle's total energy, the sum of its mass energy and kinetic energy. Without more information, we cannot figure out how much of that energy is mass energy.

20. (1) As just shown, energy conservation allows us to calculate the third particle's total energy. But we don not know what percentage of that total is mass energy vs, kinetic energy.
 21. (3) The term ' c^2 ' in $E = mc^2$ can mislead you into thinking that E varies with the square of mass. But E varies linearly with m . Indeed, we can write Einstein's equation as $E = (\text{constant}) m$, where the constant happens to equal c^2 . By contrast, graphs (2) and (4) represent an equation such as $E = (\text{constant}) m^2$.

Since the particles in this ensemble all have positive kinetic energy, the total energy is positive even when the mass is zero [relativistic physics permits this, because kinetic energy no longer equals $(1/2)mv^2$]. So, the graph does not begin at the origin.

22. (2) A proton with only mass energy ($E_{\text{total}} = mc^2$) could never decay into a neutron and other particles, because a neutron has more mass energy than a proton does. Therefore, the reaction products would have more energy than the original proton, in violation of energy conservation.

But suppose the proton has additional energy, such as potential energy from its interactions with other particles in a nucleus. Then, the total energy of the proton— mc^2 plus potential energy—can equal the total energy of the products. In this case, the products (neutron and other particles) have more mass and more kinetic energy than the proton did. Therefore, during the reaction, the increase in mc^2 and the increase in kinetic energy must be offset by an equally big decrease in the system's potential energy. This happens when a nucleus undergoes β^+ decay.

For Problems 23–26

23. (4) Nuclear reactions conserve total charge and also conserve the total approximate mass (as measured by the atomic mass number). Therefore, since the uranium, xenon, and strontium nuclei have atomic masses 236, 140 and 94, the 'other particles' must have total atomic mass A such that

$$236 = 140 + 94 + A$$

So, $A = 2$. The other particles are two nucleons. This narrows down the answer to options (2), (3) and (4). For nuclei, the atomic number—i.e., the number of protons—tells us the charge. So, the other particles must have total charge Z such that

$$92 = 54 + 38 + Z \quad \text{or} \quad Z = 0$$

In summary, the other particles have total atomic mass 2 and total charge 0. Only option (4) fits this description.

24. (1) According to the passage, all nucleons attract each other with the same strong nuclear force. So, the strong nuclear force holds together three protons and one neutron (${}^4_3\text{Li}$) just as vigorously as it holds together two protons and two neutrons (${}^4_2\text{He}$). But other forces play a role, too. Specifically, protons electrostatically repel other protons. This repulsion 'tries' to make a nucleus fly apart. Since ${}^4_2\text{He}$ contains only two protons, the attractive strong nuclear force overcomes the repulsion of the protons. Therefore, the nucleus holds together. But in ${}^4_3\text{Li}$, the mutual repulsion of the three protons overcomes the strong nuclear attraction, and the nucleus falls apart (or undergoes radioactive decay into a more stable nucleus).

25. (2) Once the neutron gets sufficiently close to the nucleus, the strong nuclear force sucks it in. The same reasoning would apply to the proton, except for one thing; the proton feels electrostatically repelled by the six protons already inside the carbon nucleus. This repulsion prevents a 100 m s^{-1} proton from getting close enough to the nucleus for the strong force to 'kick in'. Remember, the strong force becomes important only at tiny distances.

26. (3) Graph (1) would correctly show the binding energy per nucleons, but this differs from the total binding energy of the nucleus. The total binding energy takes into account not only the binding energy per nucleon, but also the number of nucleons.

To understand this subtle point, compare ${}^{94}_{38}\text{Sr}$ to ${}^{236}_{92}\text{U}$. As the passage states, ${}^{94}_{38}\text{Sr}$ has a higher binding energy per nucleon. Here,

$${}^{94}_{38}\text{Sr} \text{ binding energy per nucleon} \approx 9 \text{ MeV}$$

$${}^{236}_{92}\text{U} \text{ binding energy per nucleon} \approx 7 \text{ MeV}$$

But ${}^{236}_{92}\text{U}$ still has a higher total binding energy, simply because it contains more nucleons. Indeed, since total binding energy of a nucleus = binding energy per nucleon $\times$ number of nuclei, the total binding energies are

$${}^{94}_{38}\text{Sr} \times (94 \text{ nucleons}) \approx 846 \text{ MeV}$$

$${}^{236}_{92}\text{U} \times (236 \text{ nucleons}) \approx 1650 \text{ MeV}$$

So, as atomic mass increases, so does the total binding energy. Uranium has more binding energy, even though it has less binding energy per nucleon.

Critically, however, the total binding energy does not increase linearly with atomic mass. It increases at a smaller and smaller rate, precisely because the binding energy per nucleon decreases. Indeed, since 'atomic mass number' specifies the number of nucleons, the binding energy per nucleon is simply the slope of a binding energy vs. atomic mass number graph. Between strontium and uranium, this slope decreases, as just explained. Thus, the answer must be (3) rather than (4).

For Problems 27–29

27. (3) If the particles are treated as point charges, $U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$

$$q_1 = 2e \text{ (alpha particle), } q_2 = 82e \text{ (gold nucleus),}$$

$$r = 6.5 \times 10^{-14} \text{ m}$$

$$\therefore U = (8.987 \times 10^8 \text{ N m}^2 \text{C}^{-2}) \times \frac{(2 \times 82)(1.602 \times 10^{-19} \text{ C})}{6.50 \times 10^{-14} \text{ m}}$$

$$= 5.82 \times 10^{-13} \text{ J}$$

$$\text{or } U = 5.82 \times 10^{-13} \text{ J} \times \left(\frac{1 \text{ eV}}{1.602 \times 10^{-19} \text{ J}} \right)$$

$$= 3.63 \times 10^6 \text{ eV} = 3.63 \text{ MeV}$$

28. (3) Apply conservation of energy: $K_1 + U_1 = K_2 + U_2$

Let point 1 be the initial position of the alpha particle and point 2 be where the alpha particle momentarily comes to rest. Alpha particle is initially far from the lead nucleus implies $r_1 \approx \infty$ and $U_1 = 0$. Alpha particle stops implies $K_2 = 0$. Conservation of energy thus says

$$K_1 = U_2 = 5.82 \times 10^{-13} \text{ J} = 3.63 \text{ MeV}$$

$$\mathbf{29. (2)} \quad K = \frac{1}{2}mv^2$$

$$\therefore v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(5.82 \times 10^{-13} \text{ J})}{6.64 \times 10^{-27} \text{ kg}}} = 1.32 \times 10^7 \text{ m s}^{-1}$$

For Problems 30–32

30. (1), 31. (2), 32. (4)

For Problems 33–35

$$\mathbf{33. (4)} \quad \frac{dN}{dt} = \lambda N(t)$$

From the given data, $20000 = \lambda N_0$

$$14800 = \lambda N(0.5 \text{ h}) = \lambda N$$

$$\frac{N}{N_0} = \frac{148}{200}$$

$$N = N_0 e^{-\lambda t}$$

$$\therefore e^{-\lambda t} = \frac{148}{200}$$

$$\text{or } \lambda = \frac{\left(\ln \frac{200}{148} \right)}{t} \approx 1.6 \times 10^{-4} \text{ decays s}^{-1}$$

$$\text{Half-life, } T = \frac{0.693}{\lambda} = 4340 \text{ s} = 1.2 \text{ h}$$

$$\mathbf{34. (3)} \quad \frac{dN}{dt} = \lambda N_0$$

Number of radioactive nuclei that were present in the sample at $t = 0$,

$$N_0 = \frac{20000}{1.6 \times 10^{-4}} = 1.25 \times 10^8$$

$$\mathbf{35. (3)} \quad N(t) = N_0 e^{-\lambda t}$$

$$\frac{dN}{dt} = \lambda N(t)$$

$$N(t) = \frac{300}{1.6 \times 10^{-4}} = 1.875 \times 10^6$$

Matrix Match Type

1. i $\rightarrow$ a.; ii. $\rightarrow$ d.; iii. $\rightarrow$ c.; iv. $\rightarrow$ b.

(i) Thermal energy of air molecules at room temperature:

$$kT = 1.38 \times 10^{-23} \times 300 \text{ J} = 0.025 \text{ eV}$$

(ii) Binding energy of heavy nuclei per nucleon $\approx 7 \text{ MeV}$

(iii) X-ray wavelength $\approx 1 \text{ \AA}$

$$E = \frac{hc}{\lambda} \approx 12 \text{ keV}$$

(iv) For visible light: wavelength $\approx 6000 \text{ \AA}$

$$E = \frac{hc}{\lambda} \approx 2 \text{ eV}$$

2. i. → a., b., c., d.; ii. → a., b., c., d.; iii. → a., b., c., d.; iv. → a., b., c., d.

In all the reactions in Column II:

Mass of products will be less than original mass of the system. The mass converts into energy, hence binding energy increases.

Basically, in all four reactions mentioned in Column II, energy is released and hence for all

$$m_{\text{products}} > m_{\text{original system}}$$

As energy is released in all 4 reactions, BE/nucleon increases in all.

Mass number and charge number are conserved in all processes.

3. i. → b., c., d.; ii. → b., c., d.; iii. → b., c., d.; iv. → a., c., d.

(i) In the given spontaneous radioactive decay, the number of protons remain constant and all conservation principles are obeyed.

(ii) In fusion reaction of two hydrogen nuclei, a proton is decreased as a positron shall be emitted in the reaction. All the three conservation principles are obeyed.

(iii) In the given fission reaction, the number of protons remain constant and all conservation principles are obeyed.

(iv) In beta negative decay, a neutron transforms into a proton within the nucleus and the electron is ejected out.

4. i. → a.; ii. → b.; iii. → c.; iv. → d.

For all types of waves, sound wave, light wave, string wave the term related is frequency, which is given only in one option. Other phenomenon are properly matching.

Photoelectric effect proves photon character of light.

γ -rays can only be produced from nucleus.

In case of k capture X-rays are emitted.

5. i. → a.; ii. → d.; iii. → e.; iv. → b.

Binding energy per nucleon for middle order element is maximum because middle order element is most stable. So, (i) → (b)

Nuclear force depends only on spin of nucleons. So, (ii) → (d)

For nuclear fission, $\frac{Z^2}{A}$ is greater than 15. So, (iii) → (e)

Magic numbers are explained by Shell model. So, (iv) → (b)

6. i. → c., d., e.; ii. → b.; iii. → a.; iv. → a.

Stability of nucleus is decided by

(i) Mass defect → greater → stability greater

(ii) Neutron-proton ratio, i.e., $\frac{N}{P} \approx 1 = 1 \rightarrow$ More stable

(iii) Packing fraction = negative → more stable

(iv) Binding energy per nucleon greater → greater stability

For radioactive substance binding energy per nucleon is minimum. So, they are unstable.

For bound orbit, total energy is always negative.

Stopping potential is the particular negative potential when no electron reaches the plate (i.e., anode).

7. i. → a., b., e.; ii. → a., c.; iii. → d.; iv. → b., d.

In nuclear fusion, two lighter nuclei fuse and make big nuclei. In this, mass defect is converted into energy according to $E = mc^2$.

In nuclear fission, heavy nuclei split into two or more than two smaller nuclei. In this process, mass is converted into energy according to $E = mc^2$.

In β -decay, neutron proton ratio decreases, so nucleus becomes more stable.

Both nuclear fission and nuclear fusion are exothermic reactions.

8. i. → c.; ii. → d.; iii. → a.; iv. → b.

$$E_1 = [2m({}_1\text{H}^2) - m({}_1\text{H}^3) - m({}_1\text{H}^1)] \times 931.5 \text{ MeV} = 4 \text{ MeV}$$

$$E_2 = [-m({}_2\text{He}^4) - m({}_0\text{n}^1) + m({}_1\text{H}^3) + m({}_1\text{H}^2)] \times 931.5 \text{ MeV} = 17.6 \text{ MeV}$$

$$E_3 = [-m({}_2\text{He}^3) - m({}_0\text{n}^1) + 2m({}_1\text{H}^2)] \times 931.5 \text{ MeV} = 3.3 \text{ MeV}$$

$$E_4 = [m({}_2\text{He}^3) - m({}_1\text{H}^2) - m({}_2\text{He}^4) - m({}_1\text{H}^1)] \times 931.5 \text{ MeV} = 18.3 \text{ MeV}$$

9. (3) Before β decay, neutron is at rest. Hence $E_n = m_n c^2$, $p_n = 0$

After β decay, from conservation of momentum: $p_n = p_p + p_e$

$$\text{Or } p_p + p_e = 0 \Rightarrow |p_p| = |p_e| = p$$

$$\text{Also, } E_e = (m_e^2 c^4 + p_e^2 c^2)^{\frac{1}{2}} = (m_e^2 c^4 + p^2 c^2)^{\frac{1}{2}}$$

From conservation of energy:

$$(m_p^2 c^4 + p^2 c^2)^{\frac{1}{2}} + (m_e^2 c^4 + p^2 c^2)^{\frac{1}{2}} = m_n c^2$$

$$m_p c^2 = 936 \text{ MeV}, m_n c^2 = 938 \text{ MeV}, m_e c^2 = 0.51 \text{ MeV}$$

Since the energy difference between n and p is small, pc will be small, $pc \ll m_p c^2$, while pc may be greater than $m_e c^2$.

$$\Rightarrow m_p c^2 + \frac{p^2 c^2}{2m_p^2 c^4} \simeq m_n c^2 - pc$$

To first order $pc \simeq m_n c^2 - m_p c^2 = 938 \text{ MeV} - 936 \text{ MeV} = 2 \text{ MeV}$

This gives the momentum. Then,

$$E_p = (m_p^2 c^4 + p^2 c^2)^{\frac{1}{2}} = \sqrt{936^2 + 2^2} \simeq 936 \text{ MeV}$$

$$E_e = (m_e^2 c^4 + p^2 c^2)^{\frac{1}{2}} = \sqrt{(0.51)^2 + 2^2} \simeq 2.06 \text{ MeV}$$

Numerical Value Type

1. (6) We have to find the time at which $\lambda_A N_A = \lambda_B N_B$

$$\left(\frac{\ln 2}{T_A}\right)(4N_0 e^{-\lambda_A t}) = (N_0)\left(\frac{\ln 2}{T_B}\right)(e^{-\lambda_B t})$$

$$e^{(\lambda_A - \lambda_B)t} = 8$$

$$(\lambda_A - \lambda_B)t = \ln 8 = 3(\ln 2)$$

$$\left(\frac{\ln 2}{1} - \frac{\ln 2}{2}\right)t = 3 \ln(2) \Rightarrow t = 6 \text{ min}$$

2. (4) We have $\frac{t}{t_{1/2}} = \frac{40 \text{ hours}}{20 \text{ hours}} = 2$

$$\text{Thus, } A = \frac{A_0}{2^{t/t_{1/2}}} = \frac{A_0}{2^2} = \frac{A_0}{4}$$

So one fourth of the original activity will remain after 40 hours.

3. (2) In one half-life the number of active nuclei reduces to half the original number. Thus, in two half-lives the number is reduced

to $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$ of the original number. The number of remaining active nuclei is, therefore, $8.0 \times 10^{18} \times \left(\frac{1}{2}\right) = 2 \times 10^{18}$.

4. (2) The activity of the sample at time t is given by $A = A_0 e^{-\lambda t}$

where λ is the decay constant and A_0 is the activity at time $t = 0$ when the capacitor plates are connected. The charge on the capacitor at time t is given by $Q = Q_0 e^{-t/CR}$

where Q_0 is the charge at $t = 0$ and $C = 100 \mu\text{F}$

$$\text{Thus, } \frac{Q}{A} = \frac{Q_0}{A_0} \frac{e^{-t/CR}}{e^{-\lambda t}}$$

It is independent of t if $\lambda = \frac{1}{CR}$

$$\text{Or } R = \frac{1}{\lambda C} = \frac{t_{av}}{C} = \frac{20 \times 10^{-3} \text{ s}}{100 \times 10^{-6} \text{ F}} = 200 \Omega$$

$$5. (8) \quad \begin{array}{c} x \xrightarrow{t_1} \alpha \\ t_2 \downarrow \beta \end{array}$$

$$6h = 3(t_{eq})$$

$$\Rightarrow N = \frac{N_0}{(2)^3} \Rightarrow \frac{N_0}{N} = 8$$

6. (7) x and y are number of α -decays and β -decays respectively.

$$92 - 2x + y = 85$$

$$\text{or } 2x - y = 7 \quad \dots(i)$$

$$\text{Similarly, } 238 - 4x = 210 \quad \dots(ii)$$

$$x = 7, \text{ put in (i) we get } y = 7$$

7. (6) Effective decay constant will be sum of all different decay constants.

$$\text{So } \lambda_{\text{eff}} = \lambda + 2\lambda + 3\lambda = 6\lambda, \text{ hence } n = 6$$

$$8. (12) \text{ From, } R = R_0 \left(\frac{1}{2} \right)^n \text{ We have, } 1 = 64 \left(\frac{1}{2} \right)^n$$

$$\text{or } n = 6 = \text{number of half-lives}$$

$$t = n \times t_{1/2} = 6 \times 2 = 12 \text{ h}$$

$$9. (4) \text{ Activity of } S_1 = \frac{1}{2} \text{ (activity of } S_2)$$

$$\text{or } \lambda_1 N_1 = \frac{1}{2} (\lambda_2 N_2) \quad \text{or } \frac{\lambda_1}{\lambda_2} = \frac{N_2}{2N_1}$$

$$\frac{T_1}{T_2} = \frac{2N_1}{N_2} \quad \left(T = \text{half-life} = \frac{\ln 2}{\lambda} \right)$$

$$\text{Given } N_1 = 2N_2, \frac{T_1}{T_2} = 4$$

$$10. (8) \quad \frac{dN_x}{dt} = \frac{dN_y}{dt}$$

$$\lambda_x N_x = \lambda_y N_y$$

$$\frac{\ln 2}{1} (8N_0 e^{-\lambda_x t}) = \frac{\ln 2}{2} (N_0 e^{-\lambda_y t})$$

$$t = \frac{4 \ln 2}{\lambda_x - \lambda_y} = \frac{4 t_x t_y}{t_y - t_x} = 8$$

11. (12) Radioactive decay equation is

$$N = N_0 e^{-\lambda t} = N_0 e^{-\ln(2)t/T} \quad \left(\because \lambda = \frac{\ln 2}{T} \right)$$

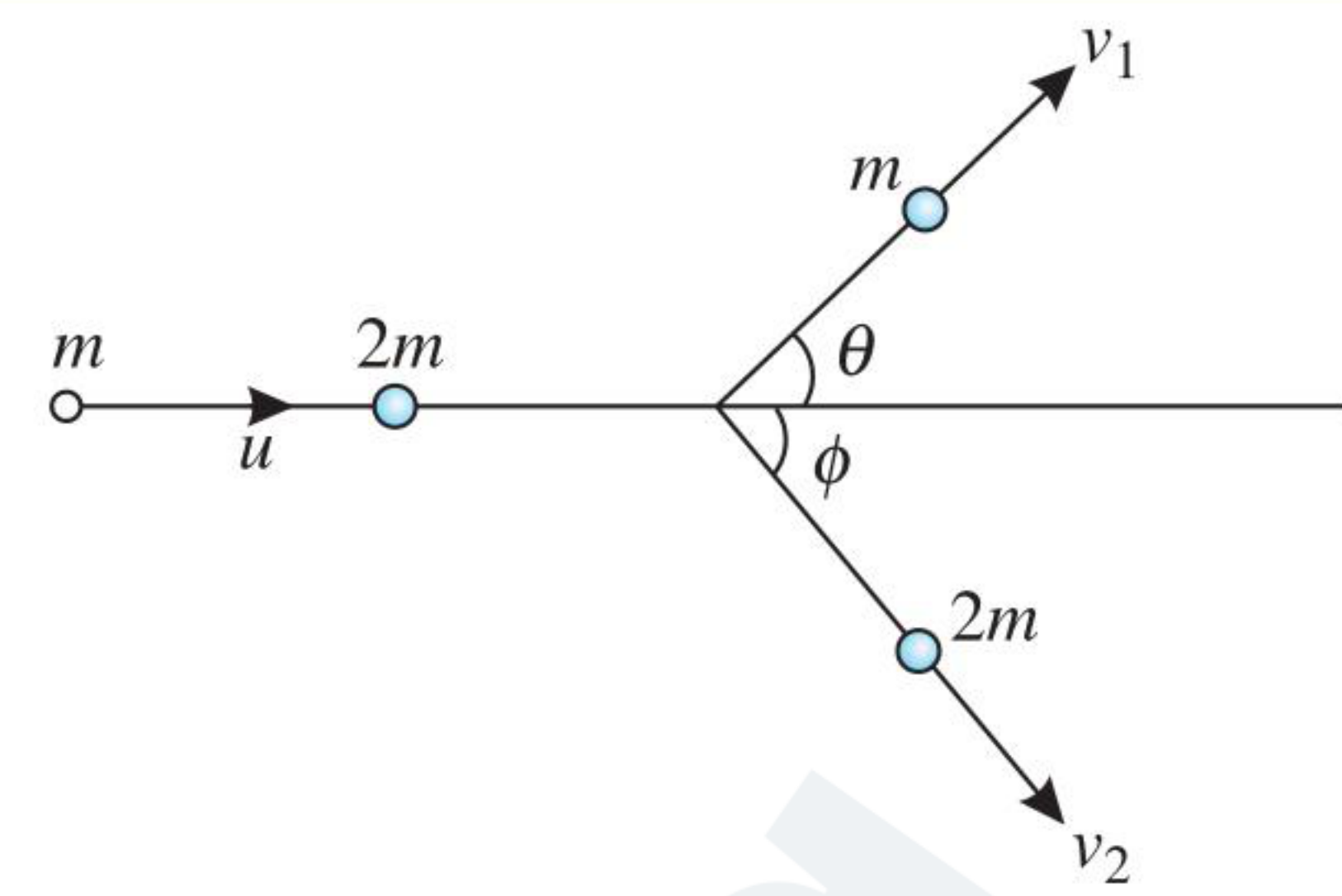
After decay of 99% of the initial sample, only 1% will be left, i.e., $N/N_0 = 1\%$.

$$\therefore \frac{N}{N_0} = \frac{1}{100} = e^{-\ln(2)t/T}$$

If we take the natural logarithm, we have $-\ln 100 = -\frac{\ln 2 \times t}{T}$ which on solving for t yields

$$t = \frac{\ln 100}{\ln 2} \times T = \frac{\log 100}{\log 2} \times T = \frac{2}{0.3010} \times 10 = 731 \text{ min} = 12.2 \text{ h}$$

12. (90)



$$mu = mv_1 \cos \theta + 2mv_2 \cos \phi \quad \dots(i)$$

$$mv_1 \sin \theta = 2mv_2 \sin \phi \quad \dots(ii)$$

$$\frac{2}{3} \left(\frac{1}{2} mu^2 \right) = \frac{1}{2} 2mv_2^2 \quad \dots(iii)$$

$$\frac{1}{3} \left(\frac{1}{2} mu^2 \right) = \frac{1}{2} mv_1^2 \quad \dots(iv)$$

$$\text{Solving equations, } v_1 = v_2 = u / \sqrt{3}$$

$$\theta + \phi = 120^\circ \text{ and } \theta = 90^\circ$$

Archives

JEE Advanced

Single Correct Answer Type

$$1. (3) \quad A = A_0 2^{-t/T_H}$$

$$\frac{A_0}{64} = A_0 2^{-t/T_H} \Rightarrow 6 = \frac{t}{T_H} \Rightarrow t = 6T_H = 108 \text{ days}$$

$$2. (3) \quad E = \frac{3}{5} \frac{Z(Z-1)e^2}{4\pi\epsilon_0 R}$$

$$n + {}^{15}_8\text{O} \longrightarrow {}^{15}_7\text{N} + {}^1_1\text{H}$$

$$Q = (M_n + M_{{}^{15}_8\text{O}} - M_{{}^{15}_7\text{N}} - M_{{}^1_1\text{H}})c^2$$

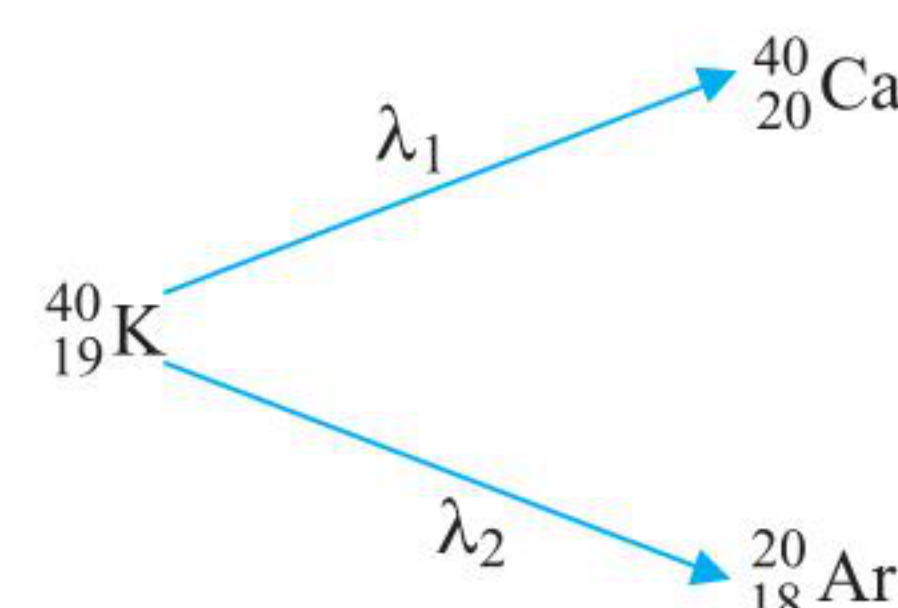
$$= 0.003796 \times 931.5 = 3.5359 \text{ MeV}$$

$$\Delta E = \frac{3}{5} \times \frac{e^2}{4\pi\epsilon_0} \times \frac{1}{R} (8 \times 7 - 7 \times 6)$$

$$= \frac{3}{5} \times (1.44 \text{ MeV fm}) \times \frac{1}{R} \times 14 = 3.5359 \text{ MeV}$$

$$R = 3.42 \text{ fm}$$

3. (1)



Decay constant

$$\lambda = \lambda_1 + \lambda_2 = 5 \times 10^{-10} \text{ per year}$$

Number of radioactive ${}^{40}_{19}\text{K}$ present at time ' t ',

$$N = N_0 e^{-\lambda t}$$

$$\Rightarrow N_0 - N = N_{\text{stable}}$$

and $N = N_{\text{radioactive}}$

$$\frac{N_0}{N} - 1 = \frac{N_{\text{stable}}}{N_{\text{radioactive}}} = 99$$

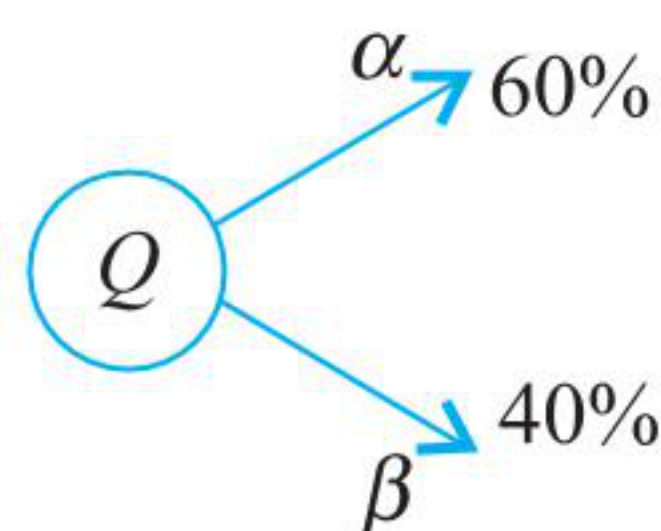
$$\frac{N_0}{N} = 100$$

$$\frac{N}{N_0} = e^{-\lambda t} = \frac{1}{100}$$

$$\Rightarrow \lambda t = 2 \ln 10 = 4.6$$

$$t = 9.2 \times 10^9 \text{ years}$$

4. (4) A heavy nucleus Q undergoes alpha-decay and beta-decay.



Initially $Q = 1000 = N$

Half-life of $Q = 20$ minutes

After 1 hr = 20 + 20 + 20 minutes

i.e., After 3 half life

$$\text{Number of nuclei left. } n = \frac{N}{2^3} = \frac{1000}{2^3} = 125$$

$$\text{Number of nuclei decayed} = 1000 - 125 = 875$$

So, number of nuclei decayed by α decay

$$875 \times 0.6 = 525$$

Multiple Correct Answers Type

1. (1), (3)



${}_{90}^{232}\text{Th}$ is converting into ${}_{82}^{212}\text{Pb}$

$$\text{Change in mass number (A)} = 232 - 212 = 20$$

$$\therefore \text{Number of } \alpha \text{ particles } x = \frac{20}{4} = 5$$

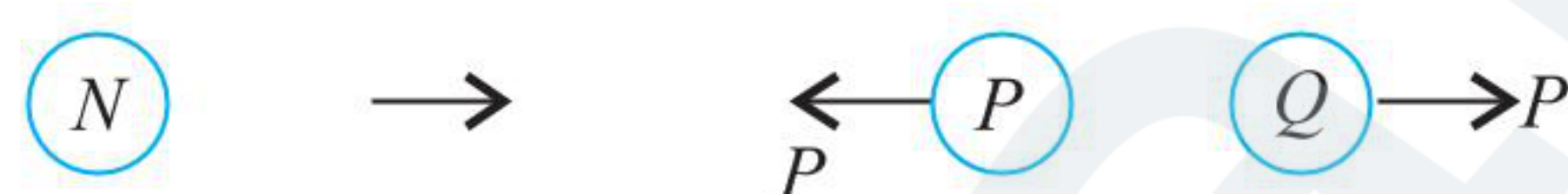
Due to 5 α particles, Z will change by 10 units. Since given change is 8, place at this location. Hence $2x - y = 8$

$$\text{or } 2 \times 5 - y = 8$$

$$\text{or } y = 2$$

2. (1),(3),(4)

The linear momentum should be conserved.



So momentum of one nucleus is P in forward direction then momentum of the other nucleus will be P in backward direction

$$\text{Energy released} = (\Delta m)C^2 = \delta C^2 = KE_P + KE_Q$$

$$KE_P = \frac{P^2}{2m_P}, KE_Q = \frac{P^2}{2m_Q}$$

$$\Rightarrow KE_P : KE_Q = \frac{1}{m_P} : \frac{1}{m_Q} = m_Q : m_P$$

$$KE_P = \left(\frac{m_Q}{m_P + m_Q} \right) \delta C^2$$

$$KE_Q = \left(\frac{m_P}{m_P + m_Q} \right) \delta C^2$$

$$KE_P + KE_Q = \delta C^2$$

$$\frac{P^2}{2m_P} + \frac{P^2}{2m_Q} = \delta C^2$$

$$P = C \sqrt{2 \left(\frac{m_P m_Q}{m_P + m_Q} \right) \delta}$$

Linked Comprehension Type

For Problems 1–3

1. (4) The very high temperature maintained in the plasma causes fusion.

2. (1) Given $k = 8.6 \times 10^{-3} \text{ eV/K}$, $\frac{e}{4\pi\epsilon_0} = 1.44 \times 10^{-9} \text{ eVm}$

$$\text{KE of two deuterons} = 2 \times 1.5 \text{ kT} = 3 \text{ kT}$$

This KE has to overcome the potential energy of the nucleus.

$$\Rightarrow 3kT = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r} \text{ or } T = \frac{1}{k} \cdot \frac{1}{3 \times r} \cdot \frac{e^2}{4\pi\epsilon_0}$$

$$\therefore T = \frac{1.44 \times 10^{-9}}{8.6 \times 10^{-5}} \times \frac{1}{3 \times 4 \times 10^{-15}}$$

$$\Rightarrow T = 1.39 \times 10^9 \text{ K}$$

3. (2) The product of the deuteron density and confinement time t_0 should be greater than Lawson criteria, $5 \times 10^{14} \text{ s cm}^{-3}$.

$$\text{Deuteron density} = 8.0 \times 10^{14} \text{ cm}^{-3}$$

$$\text{Confinement time} = 9.0 \times 10^{-1} \text{ s}$$

$$\therefore \text{Lawson criteria} = 8.0 \times 10^{14} \times 9.0 \times 10^{-1} = 7.2 \times 10^{14}$$

It is not enough if the density is very high but the confinement time also should be high enough for the reaction.

For Problems 4–5

4. (4) Total energy remains conserved. Energy is shared by antineutrino, proton and electron. Kinetic energy of electron has continuous spectrum and it is maximum when antineutrino does not share any kinetic energy. So total energy is shared with proton and electron only.

$$\therefore K \leq 0.8 \times 10^6 \text{ eV}$$

Kinetic energy of electron will be minimum or zero when total energy is shared by proton and antineutrino.

$$\therefore 0 \leq K \leq 0.8 \times 10^6 \text{ eV}$$

5. (3) $K_p + K_e + K_{\bar{\nu}} = 0.8 \times 10^6 \text{ eV}$

When electron has zero kinetic energy is shared by antineutrino and proton.

$$\text{Then, } K_p + K_{\bar{\nu}} = 0.8 \times 10^6 \text{ eV}$$

As antineutrino is very high mass in comparison to proton so it will have almost contribution in total energy.

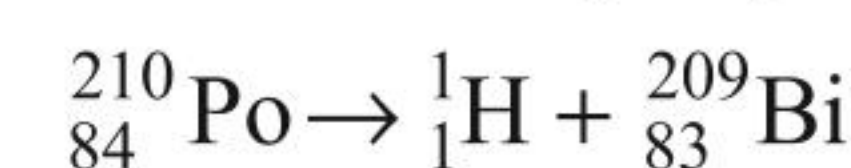
$$\therefore \text{Its energy is almost } 0.8 \times 10^6 \text{ eV}$$

For Problems 6–7

6. (3) ${}^6_3\text{Li} \rightarrow {}^4_2\text{He} + {}^2_1\text{H}$

$$\frac{Q}{c^2} = 6.015123 - 4.002603 - 2.014102 = -0.001582 < 0$$

So no α -decay is possible.



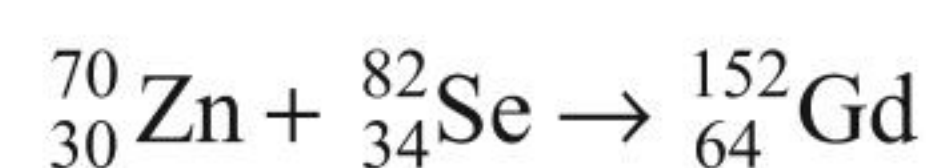
$$\frac{Q}{c^2} = 209.9828766 - 1.007825 - 208.980388 = -0.005337 < 0$$

So, this reaction is not possible.



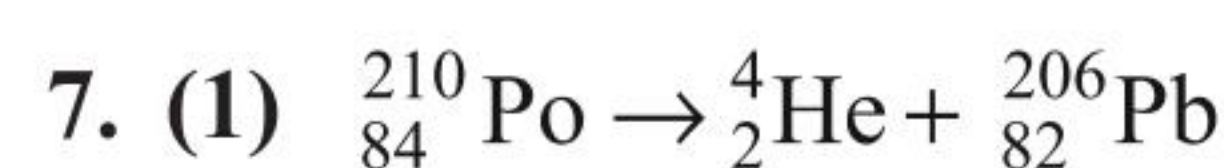
$$\frac{Q}{c^2} = 2.014102 + 4.002603 - 6.015123 = 0.001582 > 0$$

So, this reaction is possible.



$$\frac{Q}{c^2} = 69.925325 + 81.916709 - 151.919803 = -0.077769 < 0$$

So this reaction is not possible.



$$Q = (209.982876 - 4.002603 - 205.97455)c^2 = 5.422 \text{ MeV}$$

From conservation of momentum, $\sqrt{2K_1(4)} = \sqrt{2K_2(206)}$

$$\therefore K_1 = \frac{103}{2} K_2$$

$$K_1 + K_2 = 5.422 \quad \text{or} \quad K_1 + \frac{2}{103} K_1 = 5.422$$

$$\Rightarrow \frac{105}{103} K_1 = 5.422$$

$$\therefore K_1 = 5.319 \text{ MeV} = 5319 \text{ keV}$$

Matrix Match Type

1. a. $\rightarrow$ p., q., t.; b. $\rightarrow$ q.; c. $\rightarrow$ s.; d. $\rightarrow$ s.

(a) $\rightarrow$ (p): The energy of a capacitor is increased when connected to a battery.

(a) $\rightarrow$ (q): By compressing a gas adiabatically, the internal energy increases.

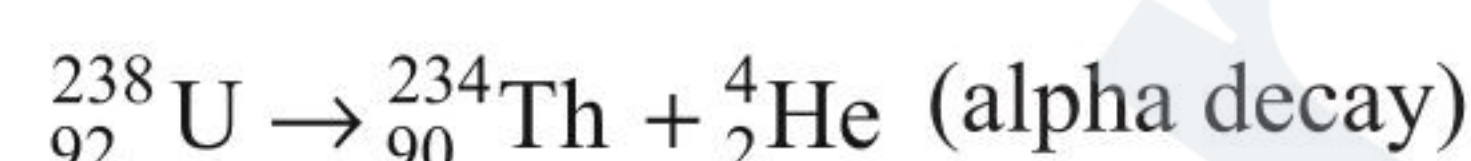
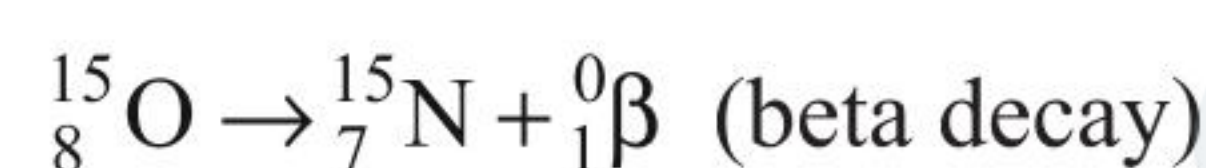
(a) $\rightarrow$ (t): By placing a loop of wire in a time varying magnetic field perpendicular to its plane $\mathcal{E} = -\left(\frac{d\phi}{dt}\right)$ an e.m.f. is generated. This increases the energy of the loop.

(b) $\rightarrow$ (q): Mechanical energy of pushing the piston, does work on a gas, increasing its energy by increasing the velocity of random motion.

(c) $\rightarrow$ (s): Internal energy of the system is converted into mechanical energy. Energy internal to system, is converted to energy of motion, temperature and photons in nuclear fission (natural radioactivity) of heavy fragments.

(d) $\rightarrow$ (s): The increase in the energy of the products of fission or radioactivity comes from a decrease in the initial mass.

2. (3)



Numerical Value Type

1. (8) $N = N_0 e^{-\lambda t}$

$$\ln|dN/dt| = \ln(N_0\lambda) - \lambda t$$

$$\text{From graph } \lambda = \frac{1}{2} \text{ per year}$$

$$\frac{t_1}{2} = \frac{0.693}{1/2} = 1.386 \text{ year}$$

$$4.16 \text{ year} = 3t_{1/2}$$

$$p = 8$$

2. (1) $N = N_0 e^{-\lambda t}$

$$\frac{dN}{dt} = 10^{10} = N_0(\lambda) e^{-10^{-9}t}$$

$$\text{At } t = 0, 10^{10} = N_0 10^{-9}$$

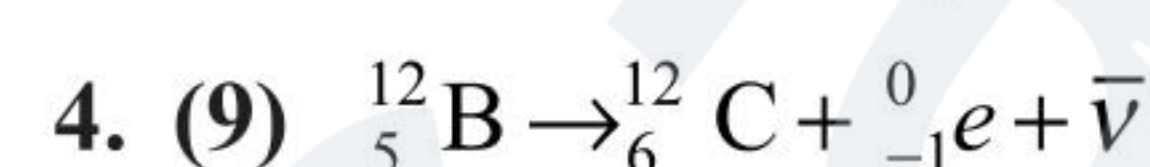
$$N_0 = 10^{19}$$

$$\begin{aligned} \text{Mass of sample} &= N_0 \times (\text{mass of atom}) \\ &= 10^{19} \times 10^{-25} \text{ kg} = 10^{-6} \text{ kg} = 1 \text{ mg} \end{aligned}$$

3. (4) $f = (1 - e^{-\lambda t}) = 1 - e^{-\lambda t} \approx 1 - (1 - \lambda t) = \lambda t$

$$f = 0.04$$

$$\text{Hence \% decay} \approx 4\%$$



$$\text{Mass defect} = (12.014 - 12) u$$

$$\therefore \text{Released energy} = 13.041 \text{ MeV}$$

$$\therefore \text{Energy used for excitation of } {}_{6}^{12}\text{C} = 4.041 \text{ MeV}$$

$$\therefore \text{Energy converted to KE of electron} = 13.041 - 4.041 = 9 \text{ MeV}$$

5. (5) Suppose total volume of blood is V ml

$$\therefore 2.5 \text{ ml of blood activity} = 115 \text{ Bq}$$

$$\therefore 1 \text{ ml of blood activity} = \frac{115}{2.5}$$

$$\therefore \text{Activity of whole blood} = \left(\frac{115}{2.5}\right)V \text{ Bq}$$

$$\text{Activity } A = A_0 e^{-\lambda t}$$

$$\text{Here, } \lambda = \frac{\ln 2}{8} (\text{days})^{-1}$$

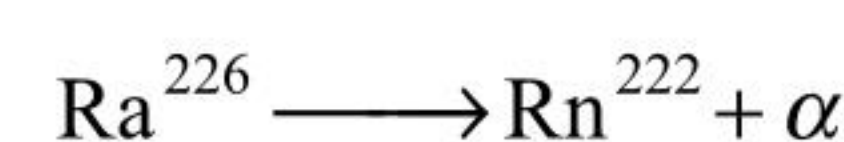
$$\left(\frac{115V}{2.5}\right) = 2.4 \times 10^5 e^{-\lambda t} = 2.4 \times 10^5 e^{-\frac{\ln 2}{8 \times 24} \times 11.5}$$

$$= 2.4 \times 10^5 e^{-\frac{0.7 \times 11.5}{8 \times 24}} = 2.4 \times 10^5 e^{-\frac{1}{24}} = 2.4 \times 10^5 \left(1 - \frac{1}{24}\right)$$

$$V = 2.4 \times 10^5 \times \frac{23}{24} \times \frac{2.5}{115} \approx 5000 \text{ ml}$$

$$\therefore \text{Total volume of blood} = 5 \text{ liter}$$

6. (135.00)



$$Q = (226.005 - 222 - 4) 931 \text{ MeV} = 4.655 \text{ MeV}$$

$$K_\alpha = \frac{A-4}{A} (Q - E_\gamma)$$

$$4.44 \text{ MeV} = \frac{222}{226} (Q - E_\gamma)$$

$$Q - E_\gamma = (4.44) \left(\frac{226}{222}\right) \text{ MeV}$$

$$E_\gamma = 4.655 - 4.520 = 0.135 \text{ MeV} = 135 \text{ keV}$$